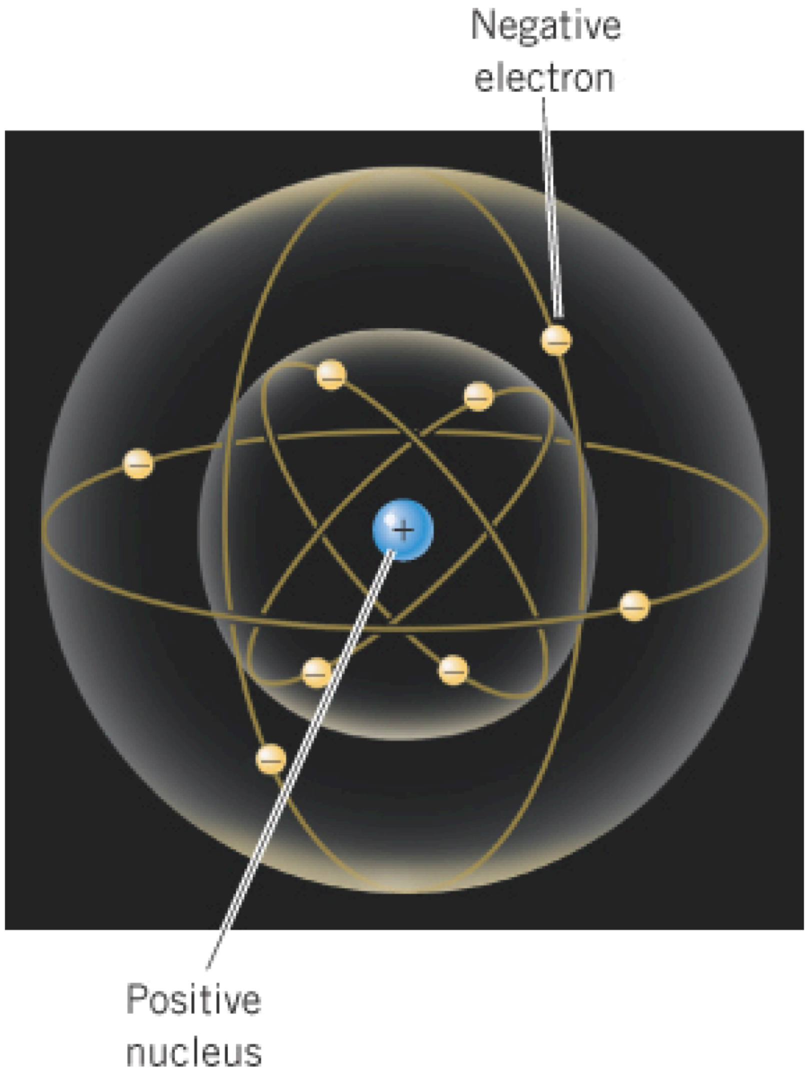
An Atom

In its natural state, an atom is electrically neutral.

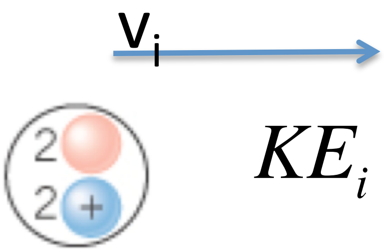
If there are Z electrons in the neutral atom,

The charge of the nucleus is $e \cdot Z$

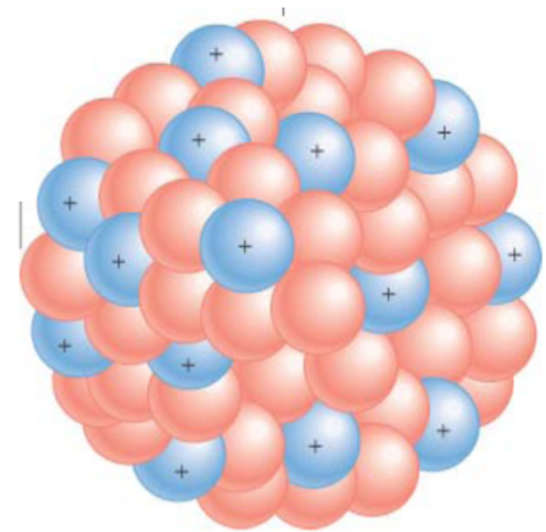
$e = 1.6 \cdot 10^{-19}$ C



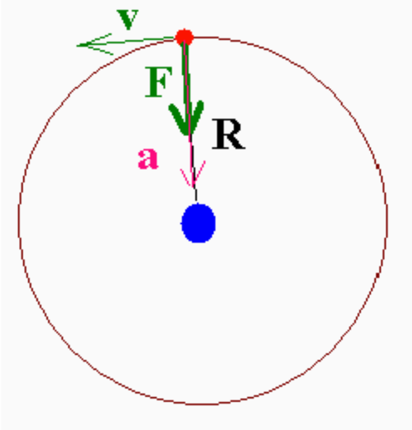
The Rutherford Experiment



$$KE_i + 0 = 0 + k\frac{2e * 79e}{r}$$



http://micro.magnet.fsu.edu/electromag/java/rutherford/



The Bohr Model

$$k\frac{e^2}{R^2} = m\frac{V^2}{R} \qquad E = \frac{mV^2}{2} - k\frac{e^2}{R}$$

We can solve the equations and find

Energy: $$E = -k\frac{e^2}{2R}$$

In addition we can find other variables describing the motion of the electron:

Linear momentum

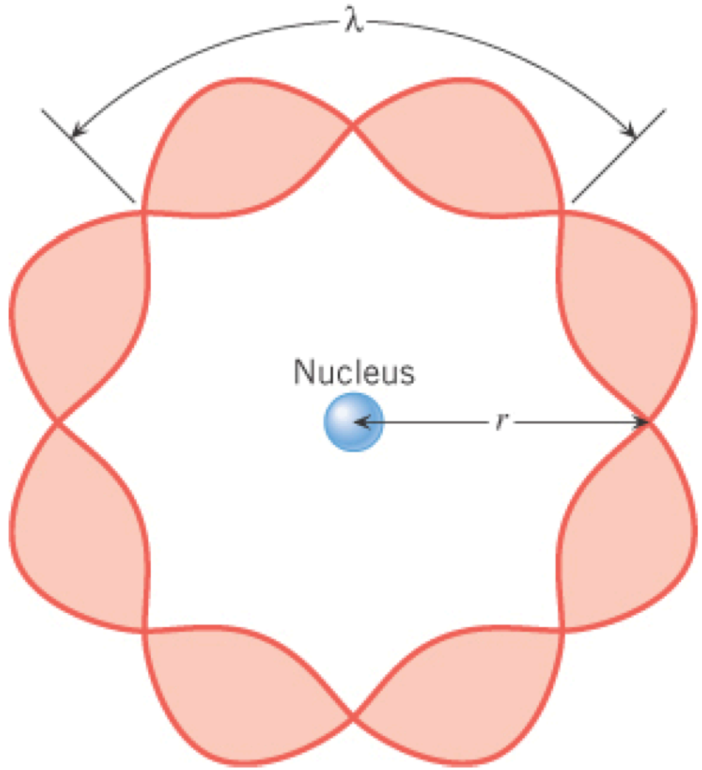
$$p = mV = e\sqrt{\frac{km}{R}}$$

Angular momentum:

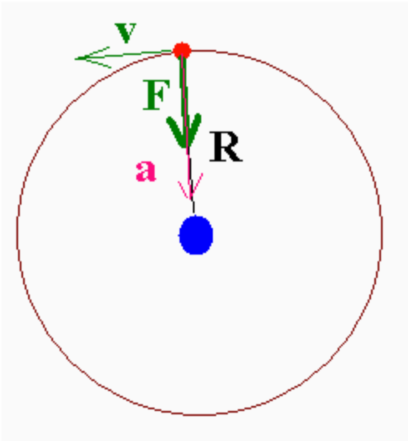
$$L = pR = e\sqrt{kmR}$$

$$2\pi r = n\lambda \quad n = 1, 2, 3, \ldots$$

$$\lambda = \frac{h}{p}$$



Electron orbits must be standing waves!



The Bohr Model

$$E = -k\frac{e^2}{2R} \qquad p = mV = e\sqrt{\frac{km}{R}}$$

$$L = pR = e\sqrt{kmR}$$

$$2\pi R = n\cdot\lambda, \quad n = 1, 2, 3, 4, \ldots \qquad \lambda = \frac{h}{p} = \frac{h}{mv}$$

We can solve this system of equation and get this!

$$p = \frac{2\pi k e^2 m}{h\cdot n} \qquad L = n\frac{h}{2\pi}$$

$$R = \frac{h^2}{4\pi^2 kme^2}n^2 \qquad E = -\frac{2\pi^2 k^2 me^4}{h^2}\cdot\frac{1}{n^2}$$

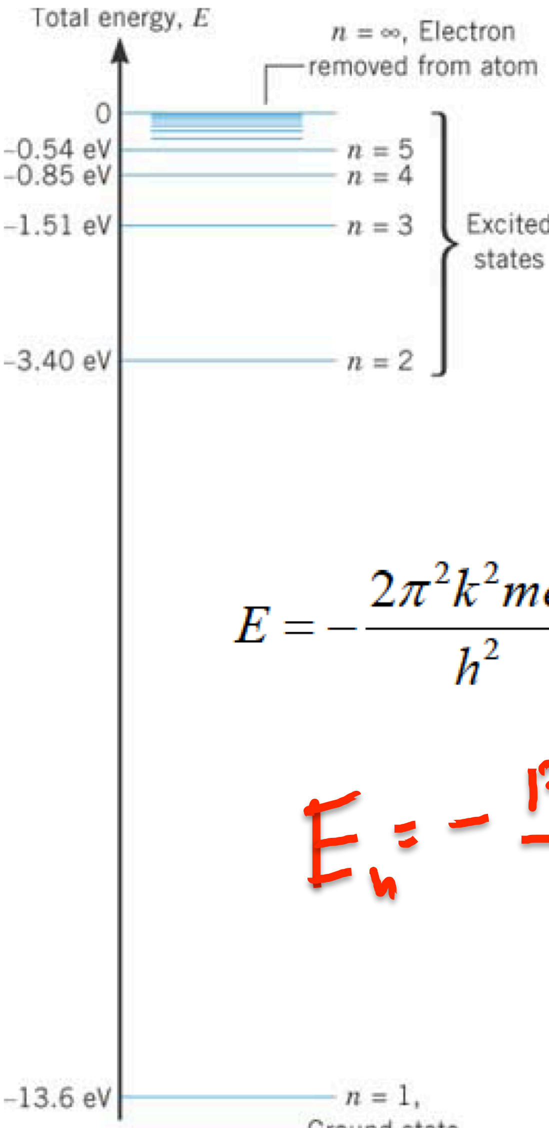
Everything is quantized!

(What about K or U?)

The Quantum World

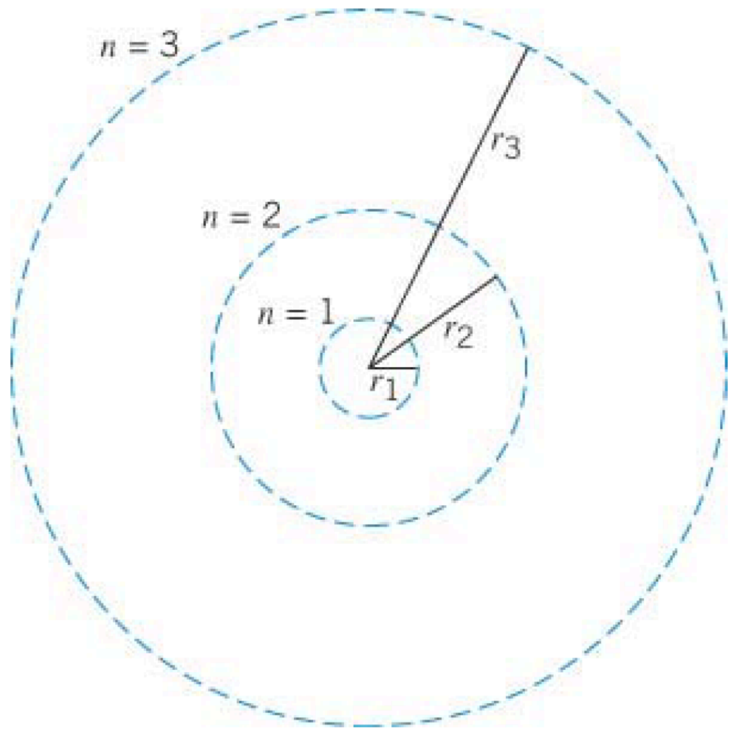
The quantum world is the world of very small objects which usually can have only certain values for their variables (or, in other words, certain values are *restricted* from existence!).

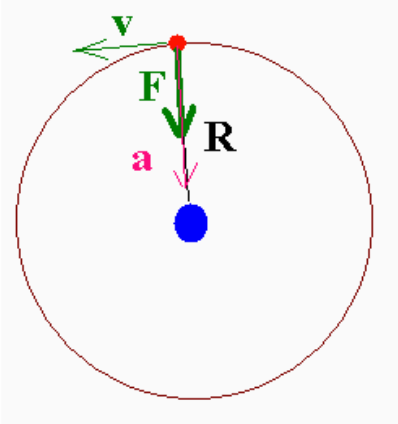
ENERGY LEVEL DIAGRAMS



$$E = -\frac{2\pi^2 k^2 m e^4}{h^2} \cdot \frac{1}{n^2}$$

$$E_n = -\frac{13.6\ \text{eV}}{n^2}$$





The Bohr Model

$$E = -\frac{2\pi^2 k^2 m e^4}{h^2} \cdot \frac{1}{n^2}$$

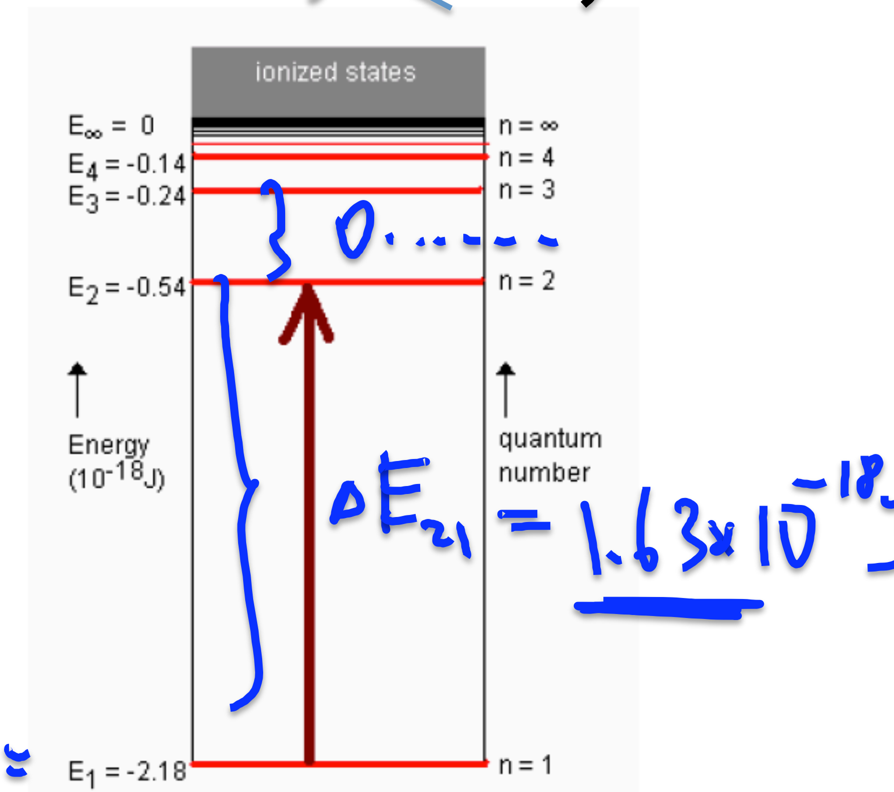
Can H atom absorb 1.6×10^{-18} J?

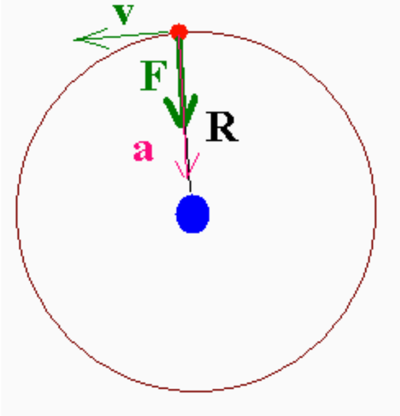
1) Yes   2). No

Normally an atom is in its ground state.

If we hit or heat the atom, it adds some energy, nut only ***certain*** amounts of energy can be added, which are exactly equal to the *difference between two energy levels*.

-13.6eV =





The Bohr Model

$$E = -\frac{2\pi^2 k^2 m e^4}{h^2} \cdot \frac{1}{n^2}$$

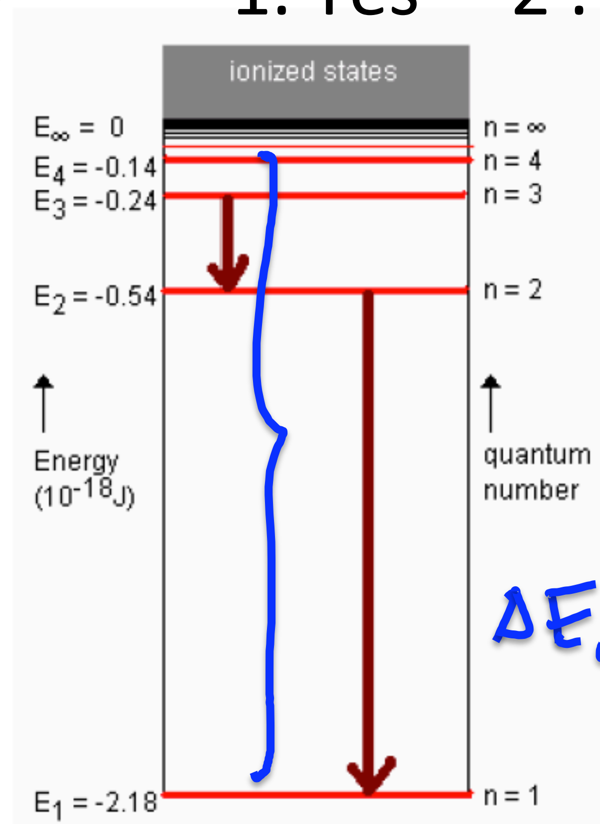
Can H atom release 0.3×10^{-18} J?

1. Yes 2. No

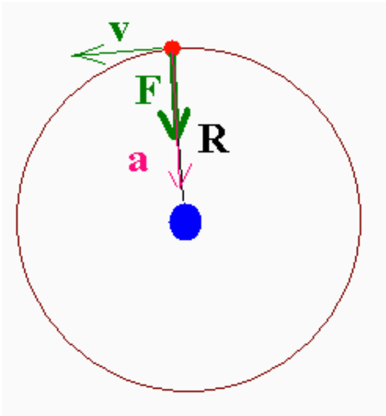
If the atom is initially in the excited state, it cannot be in this state forever, electron gives up some energy (which can have only a certain value) and what ?

What does take the energy away?

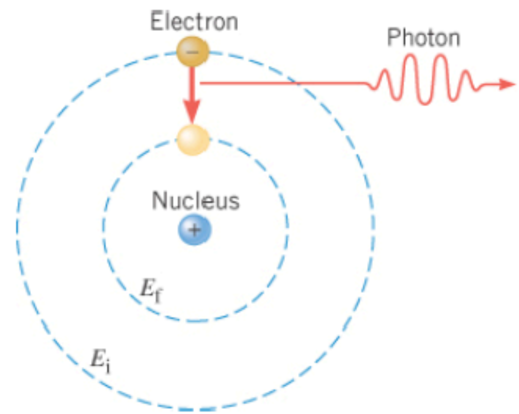
Clearly, it cannot disappear.



$\Delta E_{nm} =$



The Bohr Model



If the atom is initially in the exited state, it cannot be in this state forever, electron gives up some energy and **the atom emits light!**

The energy of the light is exactly equal to the energy difference of the levels because of

The law of conservation of energy!

$$E_{light} = E_i - E_f = \Delta E$$

The Bohr Model

$$E_{photon} = \frac{hc}{\lambda} = hf \quad E_{photon} = E_i - E_f$$

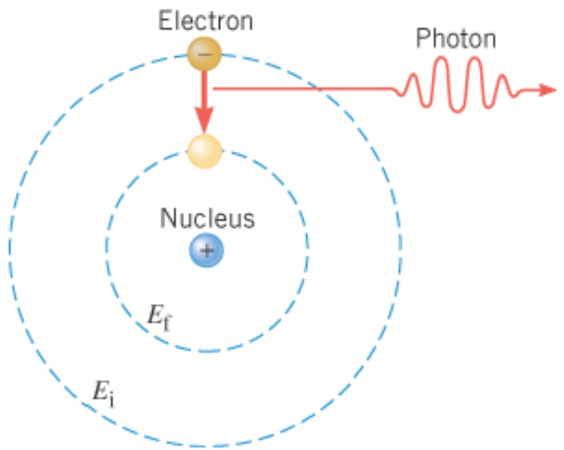
$$E_{i,f} = -\frac{2\pi^2 k^2 m e^4}{h^2} \cdot \frac{1}{n_{i,f}^{\ 2}}$$

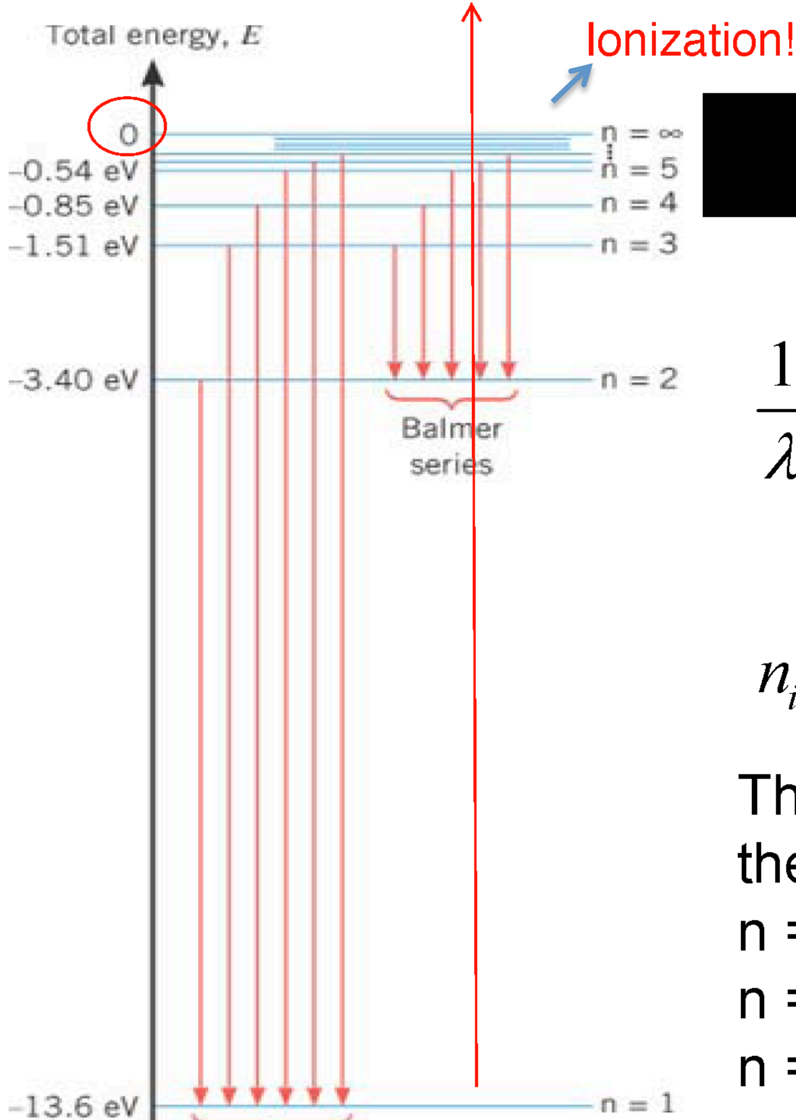
The wavelength of ***a photon:***

$$\frac{1}{\lambda} = R_E\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

the Rydberg constant

$$R_E = \frac{2\pi^2 k^2 m e^4}{ch^3} = 1.097 \cdot 10^7 \text{ 1/m}$$





$$\frac{1}{\lambda} = \frac{2\pi^2 m k^2 e^4}{h^3 c} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

$$n_i, n_f = 1, 2, 3, \ldots \qquad n_i > n_f$$

The first three wavelengths in the Balmer series are:

n = 3, l = 656 nm

n = 4, l = 486 nm

n = 5, l = 434 nm

If electron falls from 4th level, how many lines can be generated?

Sample Problem

The electron in a hydrogen atom is in the n = 2 state.
When it drops to the ground state a photon is emitted. What is the wavelength of the photon? Is this in the visible spectrum?

$$E_n = -13.6\text{eV}/n^2$$

$$E_i = E_2 = -13.6\text{eV}/4 = -3.4\text{eV}$$

$$E_f = E_1 = -13.6\text{eV}/1 = -13.6\text{eV}$$

$$E_{photon} = E_i – E_f = 10.2\text{eV}$$

Converting this to joules gives:

$$E_{photon} = 10.2\text{eV}*1.60\text{x}10^{-19}\text{J/eV} = 1.632\text{x}10^{-18}\text{ J}$$

For a photon $E_{photon} = hf = hc/\lambda$ => $\lambda = hc/E_{photon}$ =

$$= 6.63\text{x}10^{-34}*3\text{x}10^{8}/1.632\text{x}10^{-18} = 1.22\text{x}10^{-7}\text{ m} = 122\text{ nm}$$

This is in the ultraviolet part of the spectrum, so it would not be visible to us.

A shortcut

$E_{photon} = hf = hc/\lambda$ $\quad hc \cong 1240\, eV \cdot nm$

$$E_{photon} = \frac{1240}{\lambda(nm)}(eV)$$

$$\lambda(nm) = \frac{1240}{E_{photon(eV)}} = \frac{1240}{10.2} = 121.5\, nm$$

The nucleus

A nucleus consists of protons and neutrons; these are known as nucleons. Each nucleus is characterized by two numbers: *A*, the atomic mass number (the total number of nucleons); and *Z*, the atomic number (the number of protons). The number of neutrons is *N*, so $A = N + Z$.

Any nucleus can be written in a form like this:

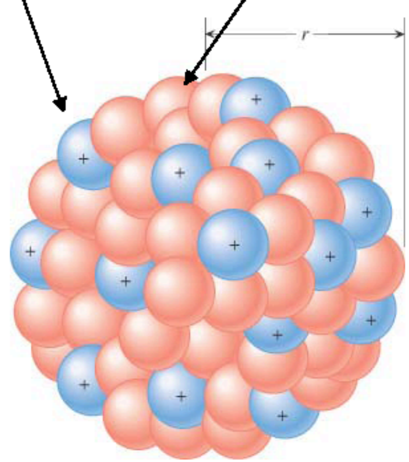
$${}^{A}_{Z}\mathrm{X} \qquad {}^{27}_{13}\mathrm{Al} \qquad {}_{Z}X^{A} \qquad {}^{A}X_{Z}$$

The X stands for the chemical symbol. On the right is a particular isotope of aluminum, aluminum-27. Isotopes of an element have the same *Z*, but a different number of neutrons.

atomic mass number

atomic number

$$\underbrace{A}_{\text{Number of protons and neutrons}} = \underbrace{Z}_{\text{Number of protons}} + \underbrace{N}_{\text{Number of neutrons}}$$



$^{A}_{Z}X$

- A: Number of protons and neutrons
- Z: Number of protons

$^{13}_{6}X$ $^{14}_{6}X$

The mutual electrical repulsion of the protons tends to push the nucleus apart.

What then, holds the nucleus together?

Electrostatic force? 1

Magnetic force? 2

Force of friction? 3

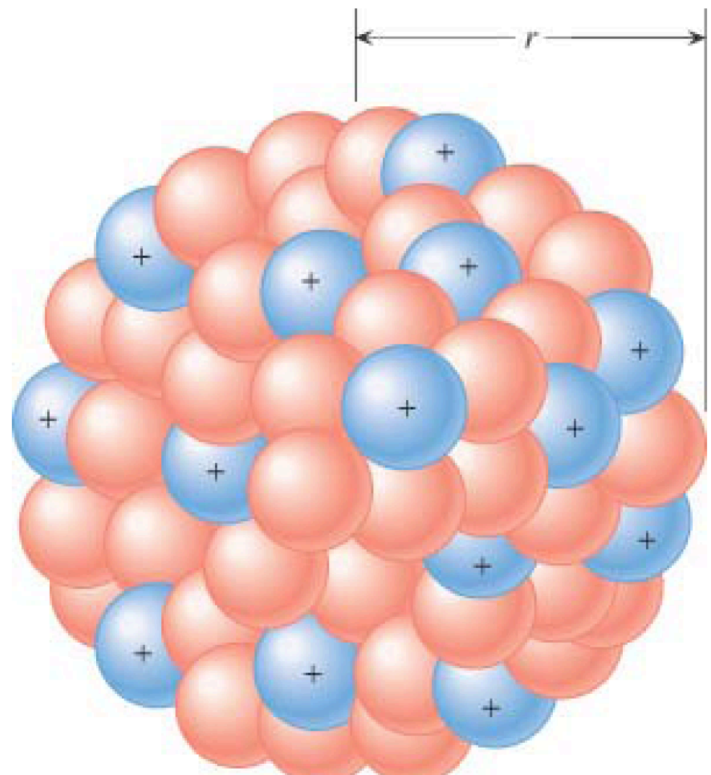
Normal force? 4

Elastic force? 5

Force of gravity? 6

Force of love attraction? 7

None of the above? 8



Holding the nucleus together

The nucleus is tiny, so the protons are all very close together. The gravitational force attracting them to each other is much smaller than the electric force repelling them, so there must be another force keeping them together. This other force is known as the strong nuclear force.

The strong nuclear force is a very strong attractive force for protons and neutrons separated by a few femtometers, but it is basically negligible for larger distances.

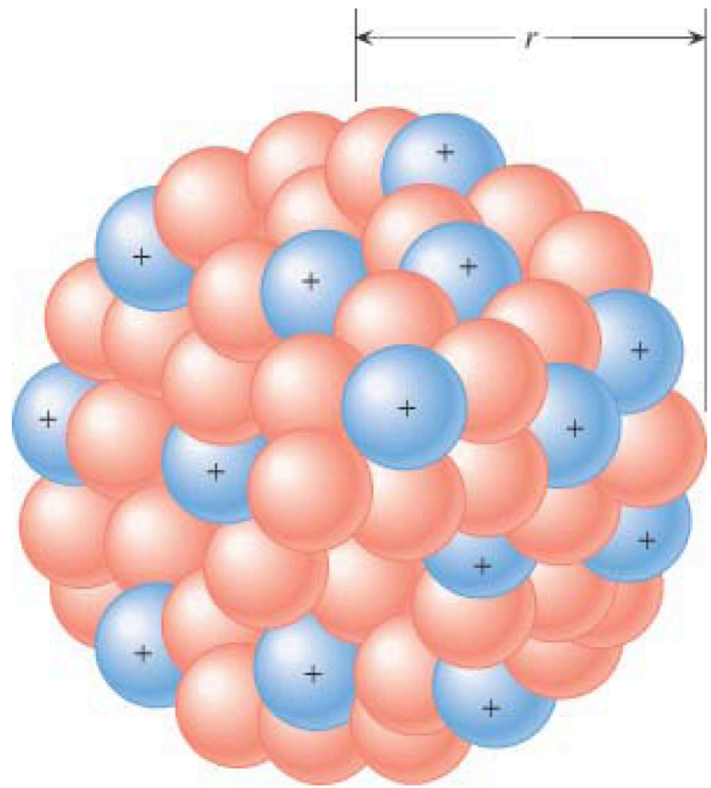
The mutual electrical repulsion of the protons tends to push the nucleus apart.

What then, holds the nucleus together?

The strong nuclear force.

A new force which does not have an analog in the world of large objects.

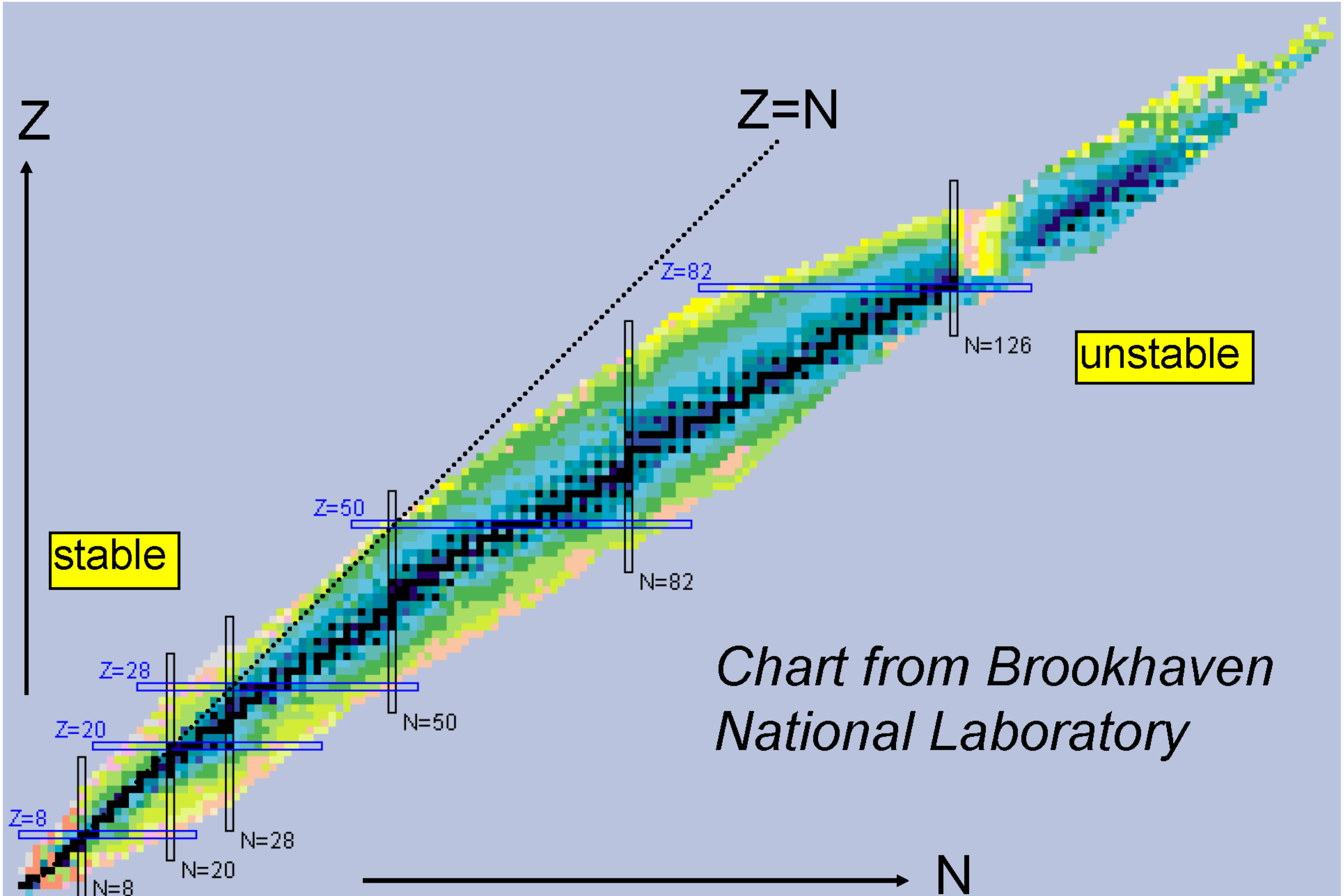
repulsion vs. attraction
what wins?



The strong force

The tug-of-war between the attractive force of the strong nuclear force and the repulsive electrostatic force between protons has interesting implications for the stability of a nucleus. Atoms with very low atomic numbers have about the same number of neutrons and protons. As Z gets larger, however, stable nuclei will have more neutrons than protons. Eventually, a point is reached beyond which there are no stable nuclei: the bismuth nucleus with 83 protons and 126 neutrons is the largest stable nucleus. Unstable nuclei eventually break up - this is known as radioactive decay.

Chart of the nuclides



Radioactivity

It is impossible to predict when an unstable particle or nucleus will decay. However, radioactive decay is governed by statistics, so we can predict the decay pattern of a large number of radioactive nuclei.

The rate at which nuclei decay is proportional to *N*, the number of nuclei there are.

$$R = -\frac{\Delta N}{\Delta t} = \lambda N \qquad |R| = \frac{|\Delta N|}{\Delta t} = \lambda N$$

where λ is the decay constant.

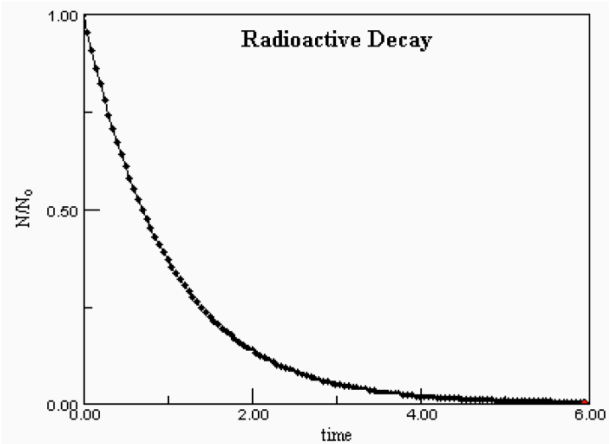
Radioactivity

Whenever the rate at which something occurs is proportional to the number of objects, the decay happens exponentially.

$$N = N_0 \, e^{-\lambda t}$$

Number remaining after a time t

Initial number at t = 0



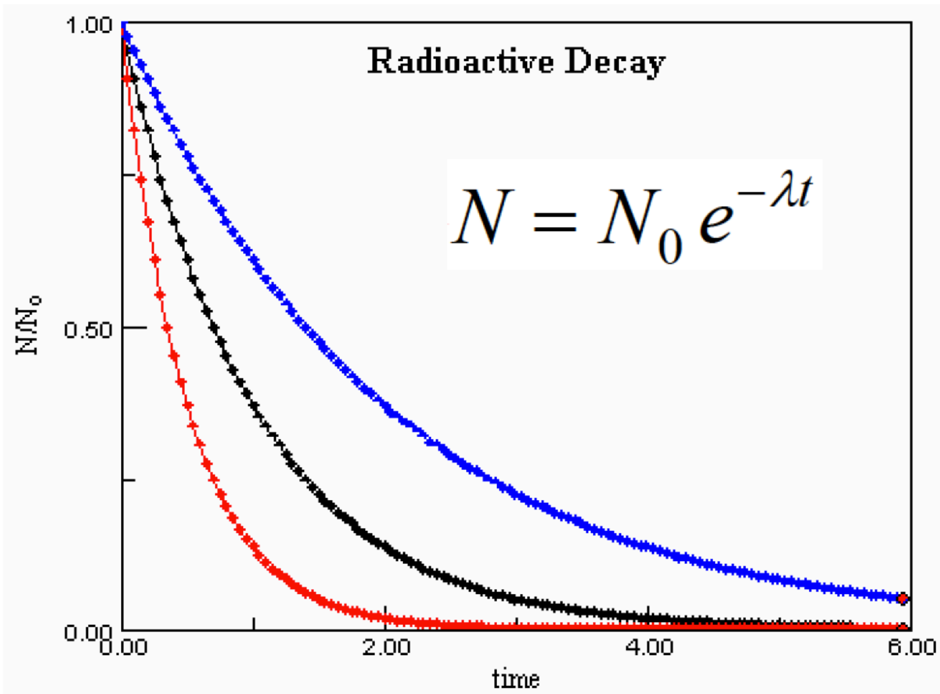
λ is the decay constant

it tells how fast atoms decay (on average!)

Increasing the decay constant

The black curve represents the decay curve for a particular radioactive sample. Which curve represents the decay of a second sample that has a larger decay constant?

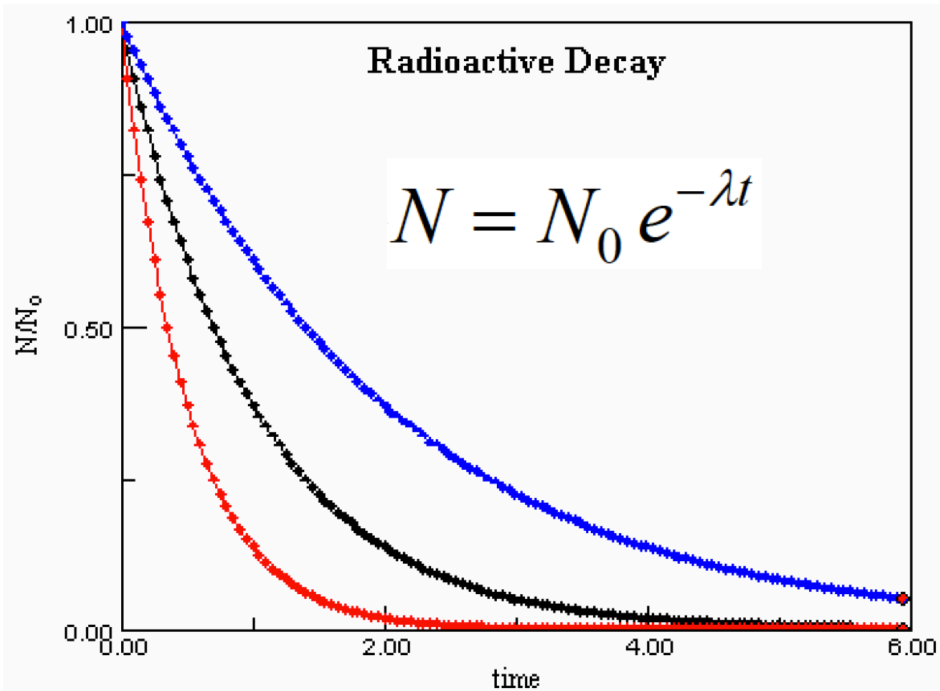
1. the red curve
2. the blue curve



Increasing the decay constant

A larger decay constant means the decay happens more quickly. The red curve represents a larger decay constant, and a shorter half-life.

1. the red curve



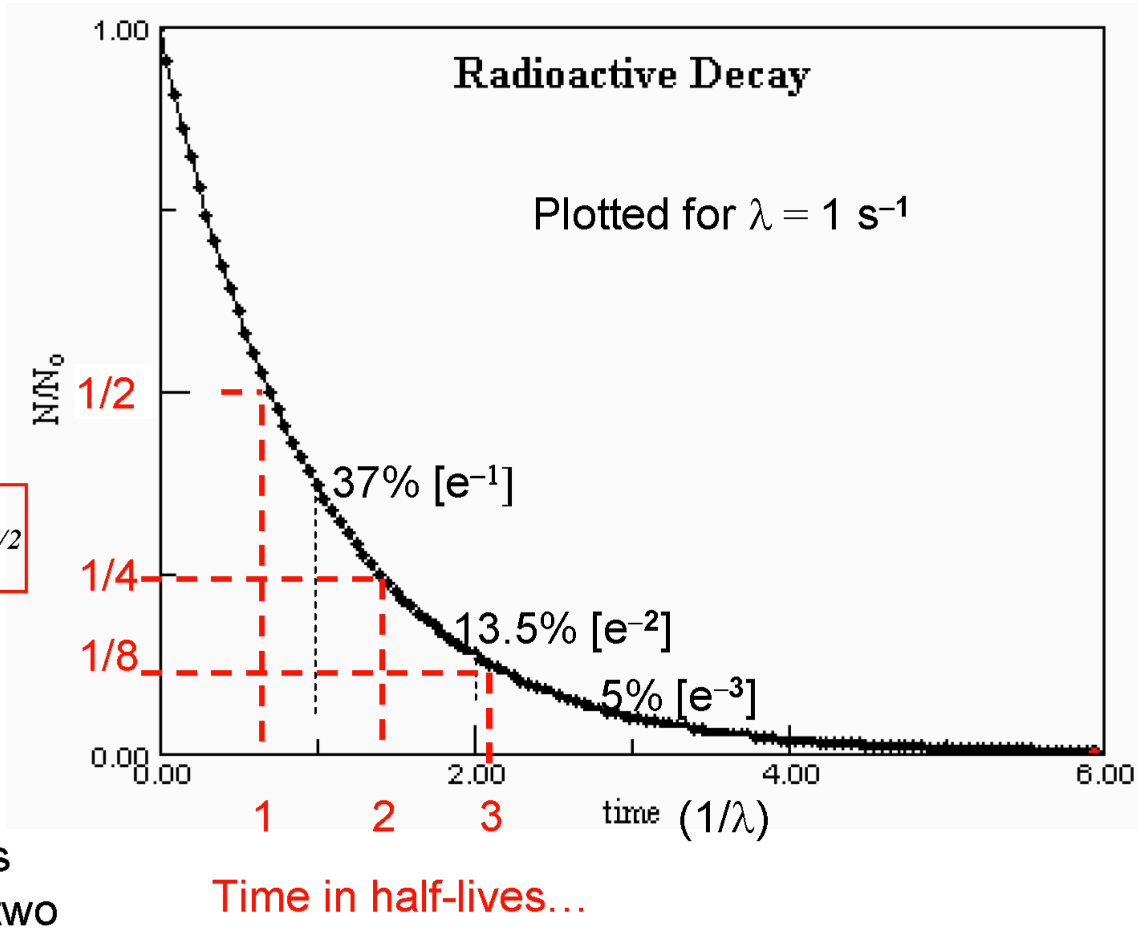
Radioactivity

$$N = N_0 e^{-\lambda t}$$

Or

$$N = N_0 2^{-t/T_{1/2}}$$

Where $T_{1/2}$ is the half life

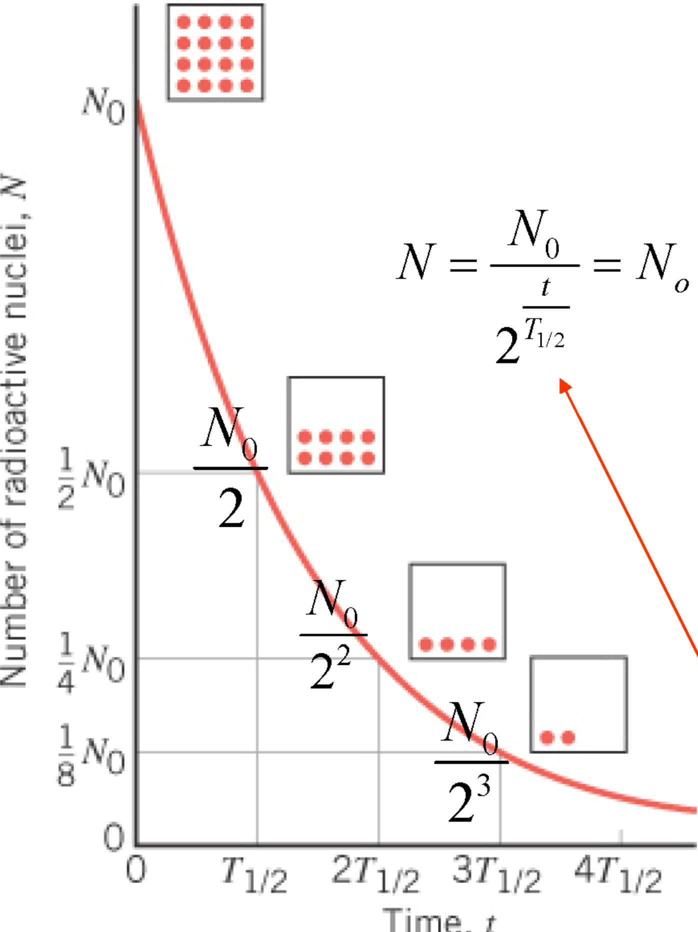


Note: Three ½-lives are approximately two 1/e-lives; 2/3 vs ln(2)

Radioactive Decay and Activity

The ***half-life*** of a radioactive decay is the time in which ½ of the radioactive nuclei disintegrate.

$$N = \frac{N_0}{2^{\frac{t}{T_{1/2}}}} = N_o \cdot 2^{-\frac{t}{T_{1/2}}} = N_0 \cdot \exp\{\ln(2^{-\frac{t}{T_{1/2}}})\} = N_0 \cdot \exp(-\lambda t)$$



$$\frac{N_0}{2^n}$$

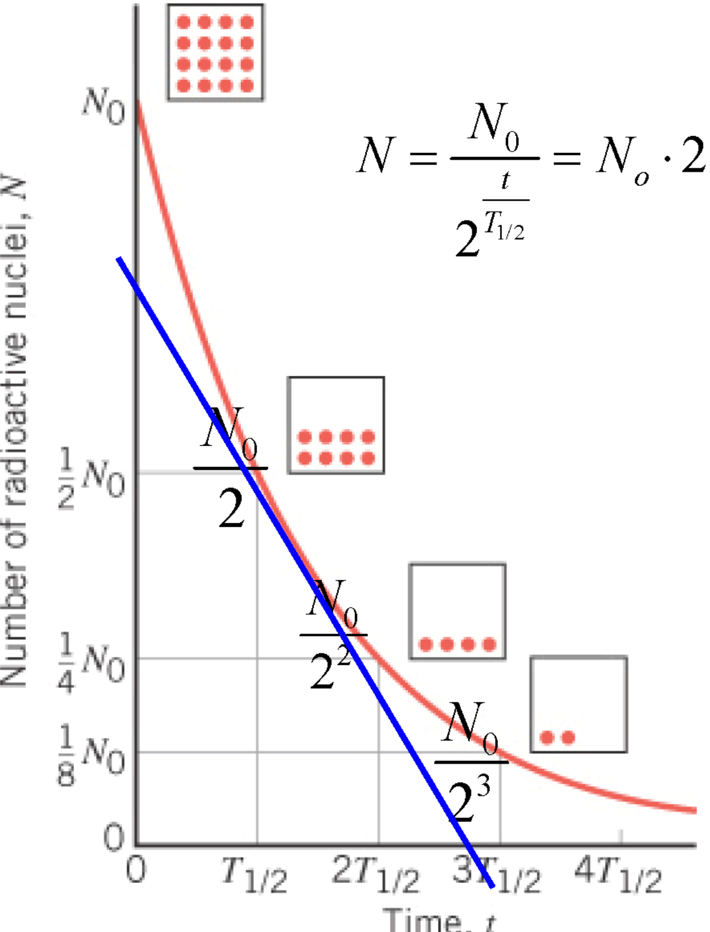
$$N = N_o e^{-\lambda t}$$

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

$$t = n \cdot T_{1/2}$$

Radioactive Decay and Activity

The ***half-life*** of a radioactive decay is the time in which ½ of the radioactive nuclei disintegrate.



$$\frac{N_0}{2^n}$$

$$N = \frac{N_0}{2^{\frac{t}{T_{1/2}}}} = N_o \cdot 2^{-\frac{t}{T_{1/2}}} = N_0 \cdot \exp\{\ln(2^{-\frac{t}{T_{1/2}}})\} = N_0 \cdot \exp(-\lambda t)$$

$$N = N_o e^{-\lambda t}$$

Activity (instantaneous; at *t*) is the rate of the decay (= |slope|)

$$R = \left|\frac{\Delta N}{\Delta t}\right| = \lambda \cdot N$$

λ is the decay constant

$$N = \frac{N_0}{2^n}$$

$$N = N_0 2^{-n}$$

$$n = \frac{t}{T_{1/2}}$$

Some Half-Lives for Radioactive Decay

Isotope		Half-Life
Polonium	$^{214}_{84}Po$	1.64×10^{-4} s
Krypton	$^{89}_{36}Kr$	3.16 min
Radon	$^{222}_{86}Rn$	3.83 d
Strontium	$^{90}_{38}Sr$	29.1 yr
Radium	$^{226}_{88}Ra$	1.6×10^{3} yr
Carbon	$^{14}_{6}C$	5.73×10^{3} yr
Uranium	$^{238}_{92}U$	4.47×10^{9} yr
Indium	$^{115}_{49}In$	4.41×10^{14} yr

Units

The activity of a sample of radioactive material is measured in disintegrations per second, the SI unit for this being the becquerel (Bq).

A more common unit is the curie (Ci). This is the activity of one gram of Radium.

1 Ci = 3.7 x 10^{10} decays/s = 3.7 x 10^{10} Bq

If NOW we have 64 radioactive atoms and in 1 hour we have only 32 of those atoms left, how long should we wait from NOW until the *last* atom decays?

Hint:

$2^0 = 1$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 32$

$2^6 = 64$

$2^7 = 128$

$2^8 = 256$

$2^9 = 512$

$2^{10} = 1024$