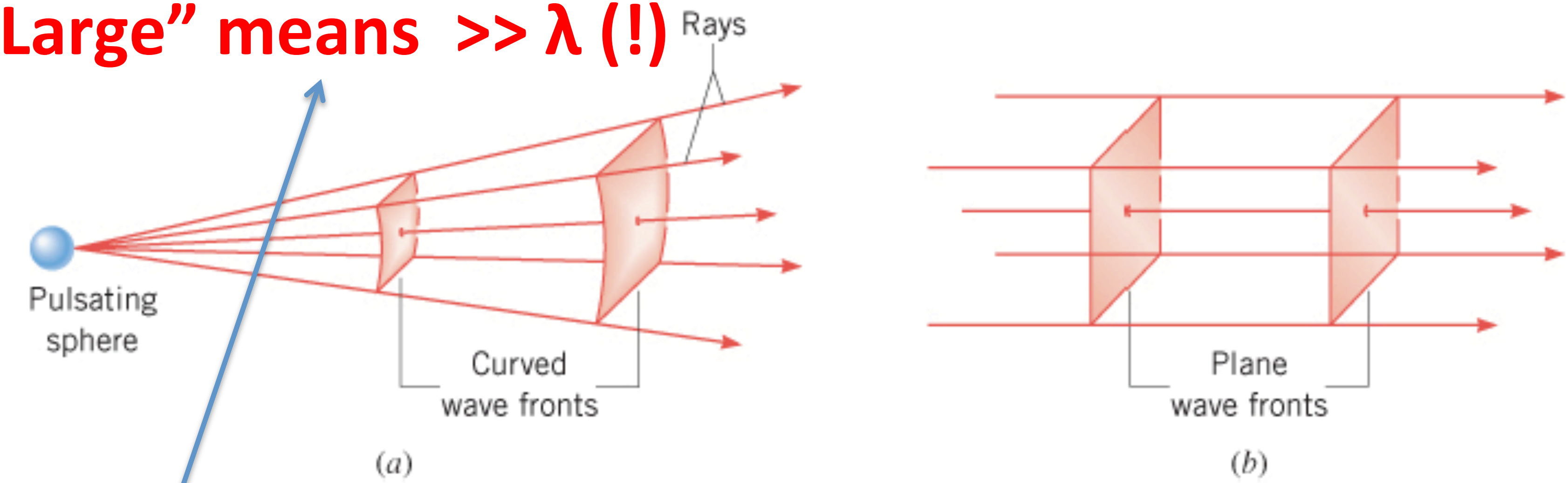
“Large” means >> λ (!)

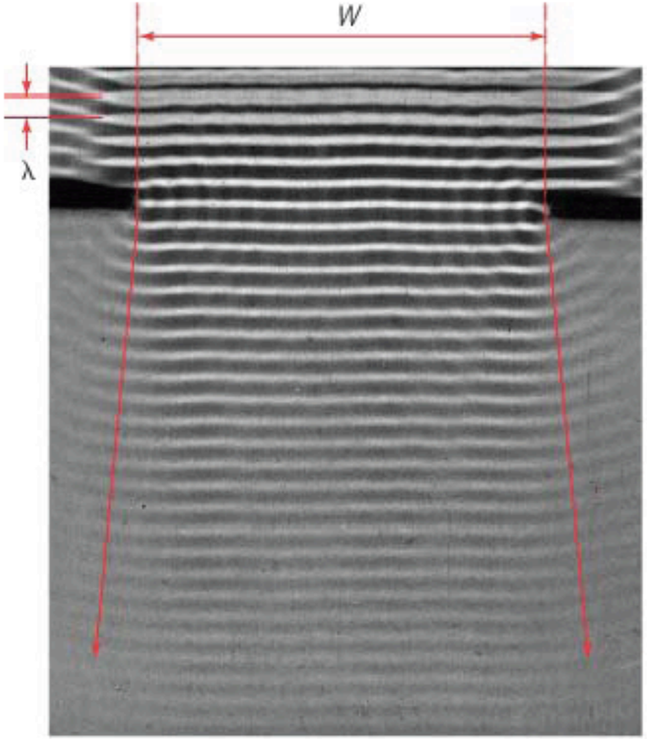


Physical optics vs. Geometrical optics

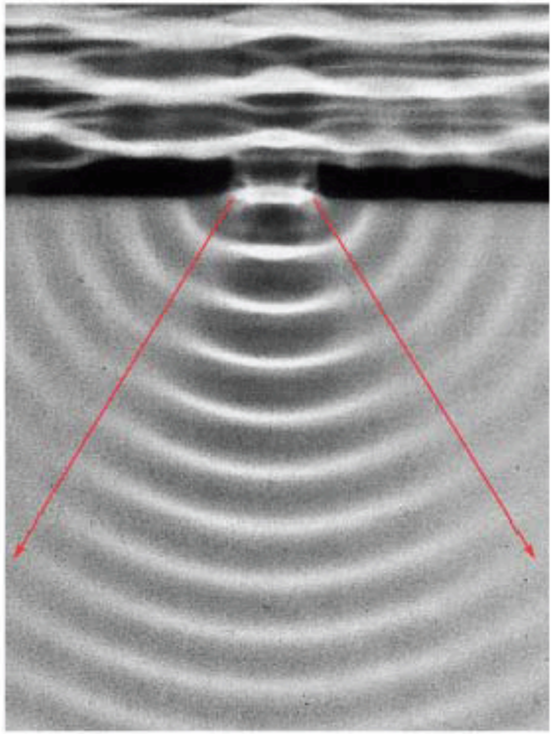
At large distances from the source, the wave fronts become less and less curved.

At a *large* distance from a source we observe a simple plane wave.

This is *when* we can use light rays to represent propagation of light



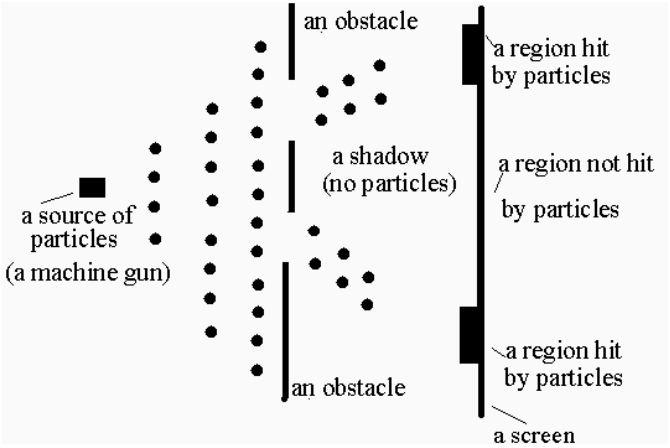
(*a*) Smaller value for λ/*W*, less diffraction.



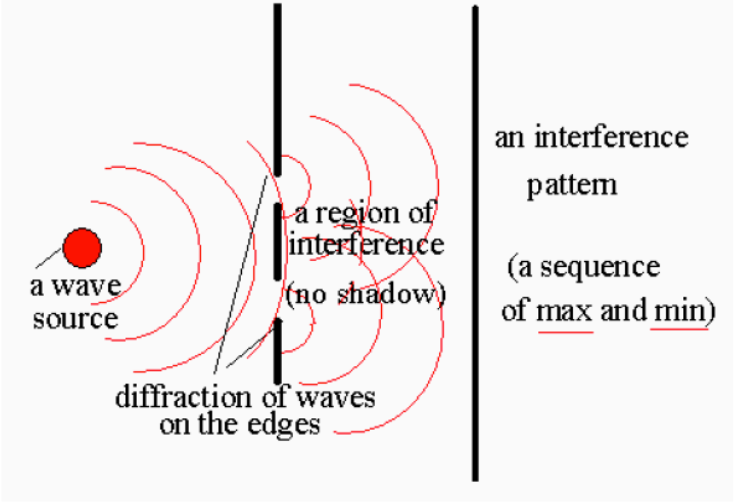
(*b*) Larger value for λ/*W*, more diffraction.

The extent of the diffraction increases as the ratio of the wavelength to the width of the opening increases.

Particles vs. Waves

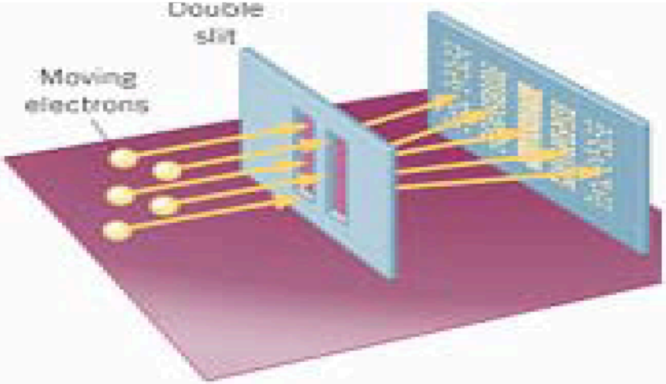


From study light we know there is the difference between particles and waves when they encounter a double slit.

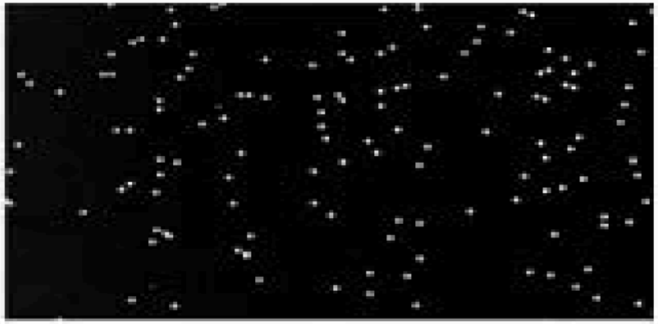


Particles would make a shadow between two bright regions.

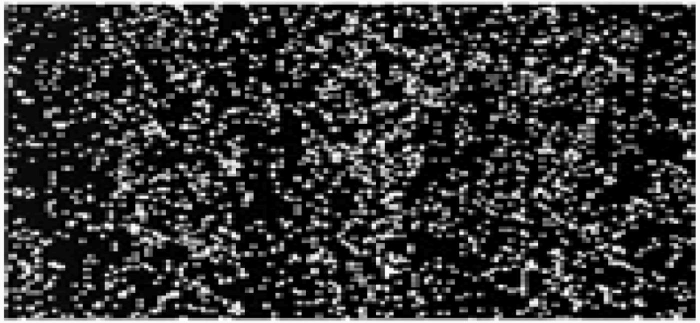
Waves make a sequence of bright and dark fringes.



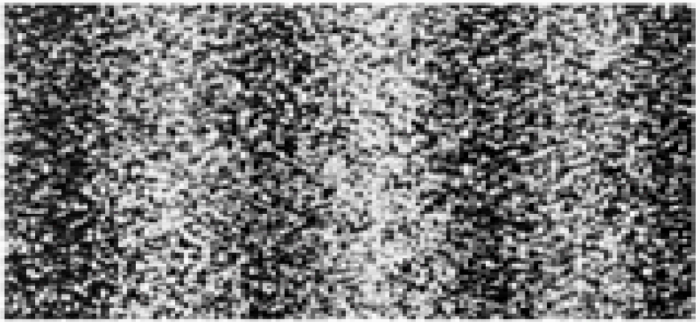
(a)



(b) After 100 electrons



(c) After 3000 electrons



(d) After 70 000 electrons

Each individual electron is located at a specific place on the screen, but we cannot predict the location, we can predict only the probability of finding an electron at a given location, and that probability has a wave-like distribution.

Small particles behave like both, classical big particles and classical waves!

To describe the "particle-like" properties of small objects we can use such variables as

Mass, energy, location, velocity, momentum, etc.
(but have to be very careful)

To describe the "wave-like" properties of small objects we can use such variables as

Wavelength, amplitude, period, etc.
(but have to be very careful)

We need to find a way to connect wave-like and particle-like properties of quantum objects!

The de Broglie wavelength

From experiments on diffraction of electrons and other small particles is found that to every object of mass having the momentum *p* we can assign a wavelength λ

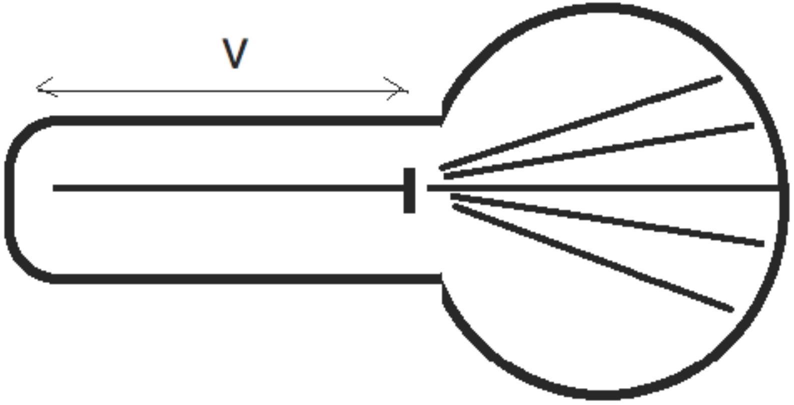
$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}}$$

The universal constant $h = 2\pi \cdot 1.0546 \cdot 10^{-34}\ \mathrm{J \cdot s}$

or $\boxed{h = 6.626 \times 10^{-34}\,\mathrm{J \cdot s}}$

is named as Plank's constant.

Its value is so small that the wave-like properties of objects cannot be measured for large objects, even like a droplet of dust.



Electrons in an electron gun are accelerated by electric field with generating potential difference V = 4000 V. Calculate the de Broglie wavelength of the electrons

We define **1 eV = energy of electron accelerated by 1 V**

or 1 eV = 1.6×10^{-19} J => KE_e = 4000 eV (!)

$$\lambda_e = \frac{h}{\sqrt{2m_e eV}} = \frac{h}{\sqrt{2m_e \cdot KE_{e(eV)} \cdot 1.6 \cdot 10^{-19}}} = \frac{1.23\,(nm)}{\sqrt{KE_e(eV)}}$$ = 0.02 nm

Wave-particle duality

Light, like all quantum objects, exhibits both wave-like behavior and particle-like behavior.

Some experiments (double-slit, thin films) can be explained in terms of light acting as a wave.

Some experiments (photoelectric effect, and others) can be explained in terms of light acting as a particle.

Einstein's realization

In 1900 Max Planck introduced the idea of discrete energy levels.

Later in 1905 Einstein came to the idea that electromagnetic waves have a particle nature.

The energy lost by the atoms is given off as an electromagnetic wave in the form of single photons.

Energy of a photon : $E = hf$, where f is the frequency

Photons

The speed of light is v = c !

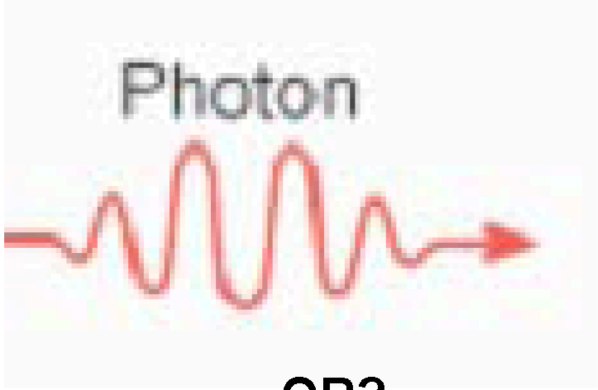
Momentum is $p = mv = mc$

$$\lambda = \frac{h}{mc} \qquad m = \frac{h}{\lambda c} \qquad !$$

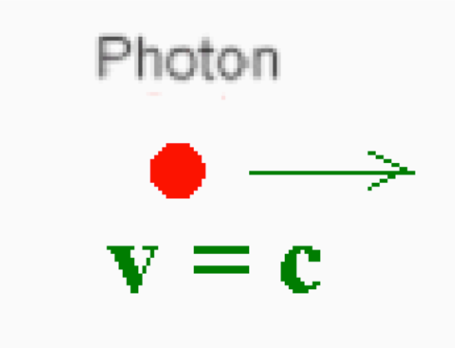
$$E = mc^2$$

(Another famous equation from Einstein!)

$$E_{light} = \frac{h}{\lambda c} c^2 = \frac{hc}{\lambda} = hf$$



OR?



Both!

The energy of photons

An electron volt is the amount of energy associated with accelerating an electron through a potential difference of 1 V.

$$\underline{1\ \text{eV} = 1.60 \times 10^{-19}\ \text{J}}$$

$$\boxed{h \sim 4 \times 10^{-15}\ \text{eV s}}$$

We can use $\boxed{c = f\lambda}$ and $\boxed{E = hf}$ to determine the frequency and energy for photons in the visible spectrum.

Wavelength	Frequency	Energy
400 nm (violet)	7.5×10^{14} Hz	**? (eV)**
700 nm (red)	4.3×10^{14} Hz	**? (eV)**

Intensity of light

$E_{photon} = hf$ this formula describes an energy of a ***single*** photon $E_{ph} = hf = 1240/\lambda_{(nm)}$

h = 0.4×10^{-14} eVs

$c = f\lambda$ this formula describes a ***wave***, i.e. a ***vast*** number of photons.

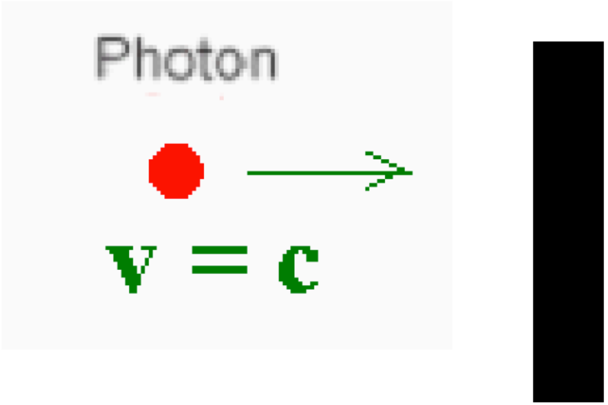
Wavelength	Frequency	Energy
400 nm (violet)	7.5×10^{14} Hz	~ 3 eV
700 nm (red)	4.3×10^{14} Hz	1.8 eV

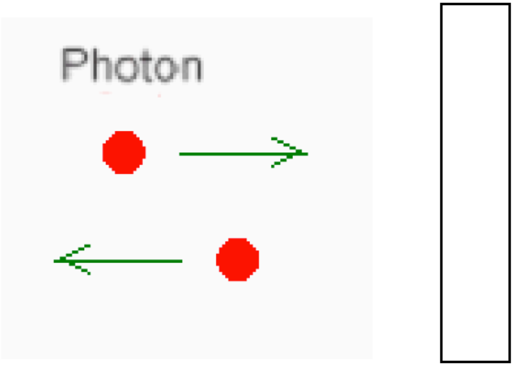
I = Intensity of light = intensity of light wave = N^*E_{photon}

N is the number of photons passing through 1 m^2 per 1 s

Photons

Momentum is $p = mc$



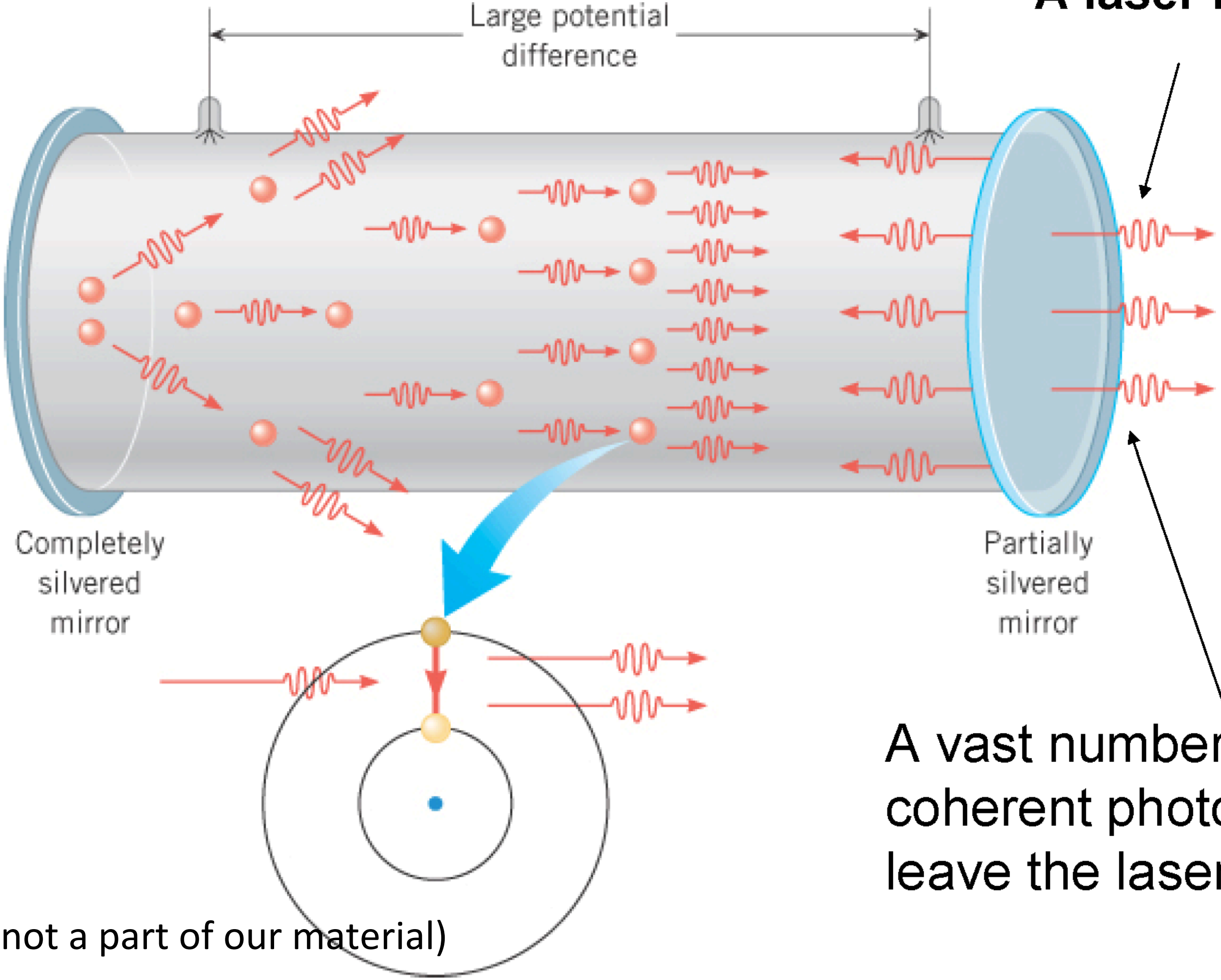


(not a part of our material)

We can explain the pressure of light as the result of many photons hitting the surface.

The light pressure on the black surface is a half of the pressure on the white surface.

A laser beam



A vast number of coherent photons leave the laser

(not a part of our material)

The Compton Effect

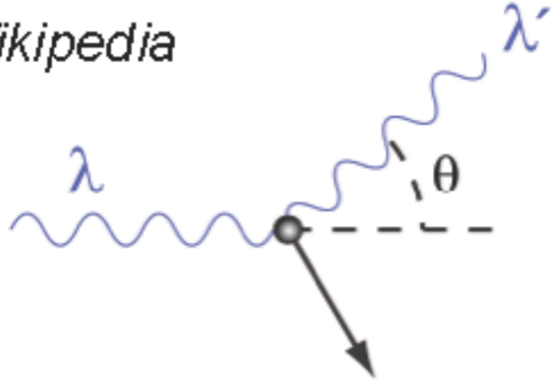


Figure from Wikipedia

(not a part of our material)

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta)$$

The combination of factors $$\frac{h}{m_e c} = 2.43\times10^{-12}\ \mathrm{m}$$

is known as the ***Compton wavelength***.

The collision causes the photon wavelength to increase by somewhere between 0 (for a scattering angle of 0°) and twice the Compton wavelength (for a scattering angle of 180°).

The Photoelectric Effect

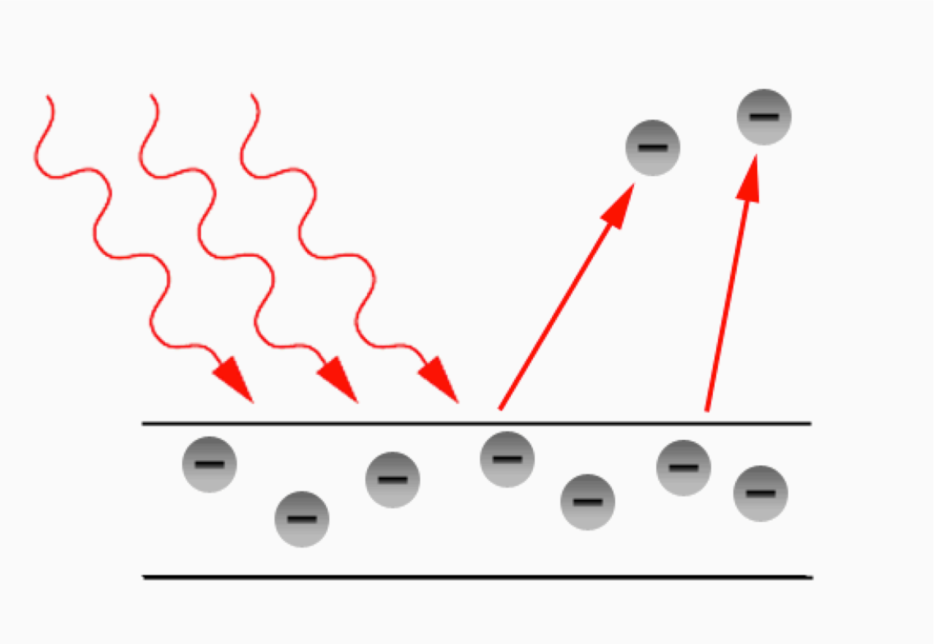
DEMO

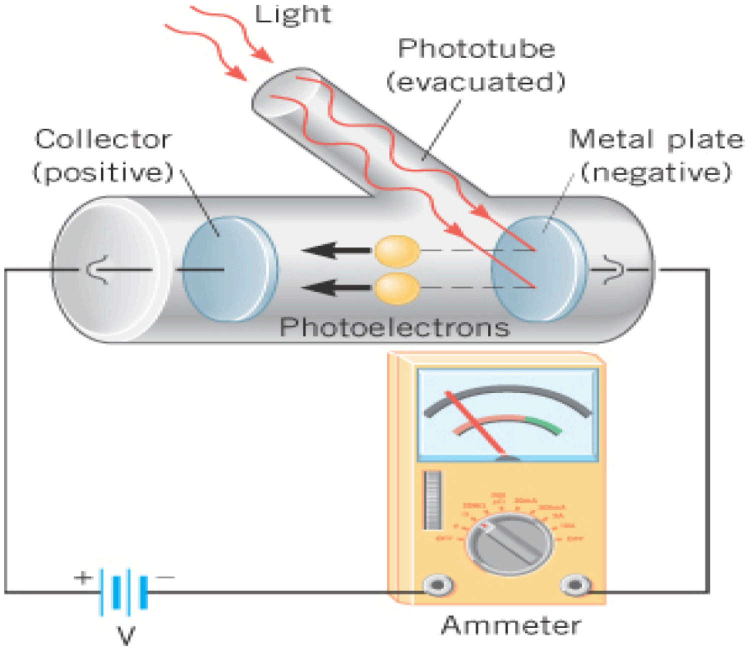
In 1921, Einstein won the Nobel Prize for Physics not for his work on relativity, but for explaining the photoelectric effect.

The photoelectric effect occurs when light shines on a metal – this may cause electrons to be emitted.

This is closely related to how solar panels generate electricity.

Figure from Wikipedia





Photoelectric effect:

A photon hits an electron;

The energy of photon is *hf*;

The energy needed to take an electron off the metal is $W_{0;}$

If $hf < W_0$ nothing happens;

If $hf = W_0$ an electron leaves the metal and even a week electric field picks it up => we observe current in the circuit; but if we reverse the field the current disappears.

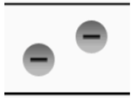
If $hf > W_0$ an electron leaves the metal and *has* some kinetic energy, so current is observed with no additional electric field, but if we introduce the field and reverse it, the current decreases, and at a certain critical value of the voltage the current disappears.

The full explanation of the photoelectric effect can be seen ***only*** in terms of light being made up of photons:

1. To eject one electron from the metal takes one photon.

2. Electrons are bound to the metal by a binding energy we call the work function, W_o, which differs from metal to metal (but measured for all of them). If the photon energy is less than the work function, no electrons are emitted.

3. The cutoff (minimum) frequency $\boxed{f_o = f_{min}}$ is where the photon energy $\boxed{hf_{min} = W_o}$



4. Above the cutoff frequency the photons have more energy than what is needed to eject an electron. The extra energy shows up as the electron's kinetic energy:

$$\boxed{K_{max} = hf - W_o}$$

The photoelectric effect

From ***energy conservation***, the energy of the photon goes into

1). liberating the electron from the metal (this energy is called ***the work function of the metal***), and

What happens to W_0?

2). whatever energy is left takes the form of the electron’s kinetic energy.

SIM.

$$hf = W_0 + K_{\max}$$

Maximum kinetic energy the electron can obtain

Energy of the photon which strikes the electron

The work function of the metal (measured for all metals)

$$K_{\max} = \frac{mV_{\max}^2}{2}$$

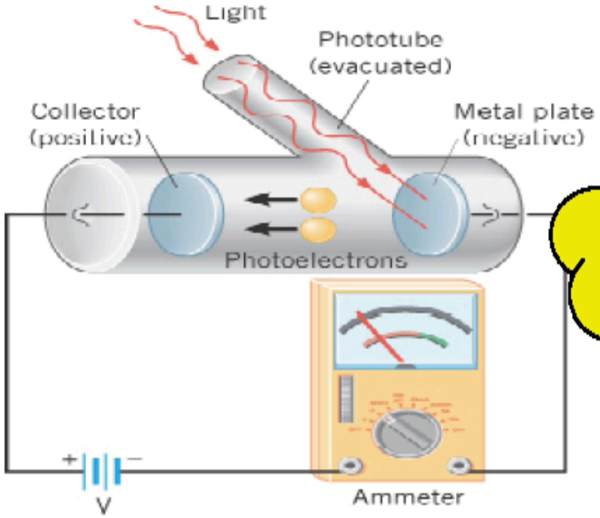
K_{max} > or = 0 ONLY if E_{photon} > or = W_0 !!

Electron Work Functions of The Elements

Reference: CRC handbook on Chemistry and Physics.

Note: Work function can change for crystaline elements based upon the orientation. For example Ag:4.26, Ag(110):4.64, Ag(110):4.52, Ag(111):4.74

Element	eV	Element	eV	Element	eV	Element	eV	Element	eV	Element	eV
Ag	4.26	Al	4.28	As	3.75	Au	5.1	B	4.45	Ba	2.7
Be	4.98	Bi	4.22	C	5	Ca	2.87	Cd	4.22	Ce	2.9
Co	5	Cr	4.5	Cs	2.14	Cu	4.65	Eu	2.5	Fe	4.5
Ga	4.2	Gd	3.1	Hf	3.9	Hg	4.49	In	4.12	Ir	5.27
K	2.3	La	3.5	Li	2.9	Lu	3.3	Mg	3.66	Mn	4.1
Mo	4.6	Na	2.75	Nb	4.3	Nd	3.2	Ni	5.15	Os	4.83
Pb	4.25	Pt	5.65	Rb	2.16	Re	4.96	Rh	4.98	Ru	4.71
Sb	4.55	Sc	3.5	Se	5.9	Si	4.85	Sm	2.7	Sn	4.42
Sr	2.59	Ta	4.25	Tb	3	Te	4.95	Th	3.4	Ti	4.33
Tl	3.84	U	3.63	V	4.3	W	4.55	Y	3.1	Zn	4.33
Zr	4.05	Photons with energy of 3 eV shine at a metal plate made of ... Does PEE happen?									



Photoelectric effect:

$$hf = W_0 + K_{\max}$$

$I = P/A$!

I = N*E$_{\text{photon}}$ = hf **N**

(per m^2 per s)

If $f < W_o/h$ => no electrons => no current

If $f > W_o/h$ => there is current

For a case $f > W_o/h$ and f = const:

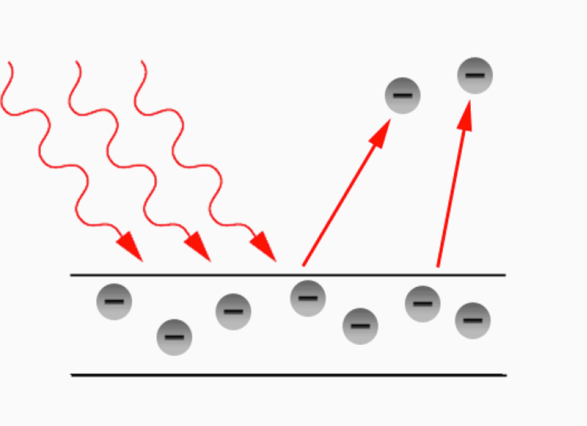
Increasing intensity I means increasing number of photons N, hence the number of free electrons, hence the current increases.

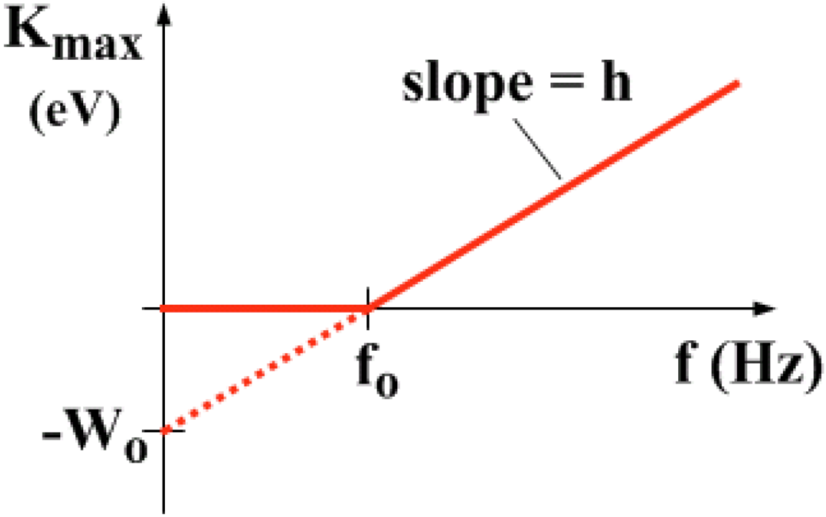
The photoelectric effect – a graph

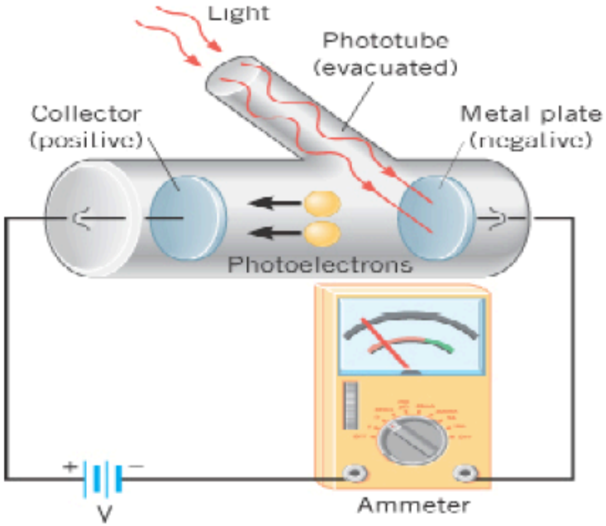
A graph of *K_{max}* vs. photon frequency gives a line with a slope of Planck's constant, a *y*-intercept equal to the negative of the work function, and an *x*-intercept of the threshold frequency.

$$hf = W_0 + K_{\max}$$

$$K_{\max} = hf - W_0 \underline{\geq 0} \quad !$$







Stopping potential

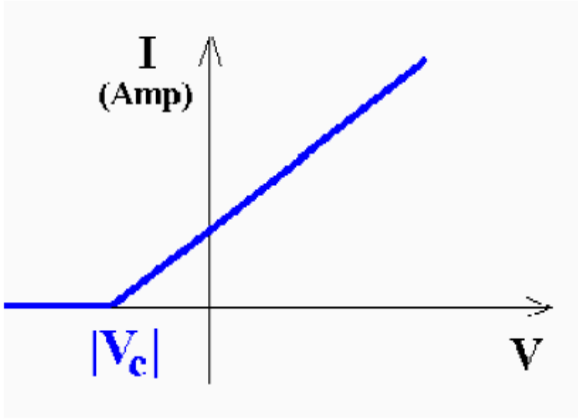
$$hf = W_0 + K_{\max}$$

$$I = N^{*}E_{\mathrm{photon}} = hf\,\mathbf{N}$$

For a case $f > W_o/h$ and f = const:

changing the voltage changes the current.

$$e \cdot |V_c| = K_{\max}$$



Sample problem

Iron has a work function of 4.50 eV.

(a) What is the minimum frequency of light necessary to cause electrons to be ejected from an iron plate?

(b) Is this in the visible spectrum?

(c) If the iron plate is exposed to light with a frequency of 1.50×10^{15} Hz, what is the maximum kinetic energy of the ejected electrons?

(d) what is the minimum voltage which “kills” the current?

Sample problem

Iron has a work function of 4.50 eV.

(a) What is the minimum frequency of light necessary to cause electrons to be ejected from an iron plate?

$$hf = W_0 + \cancel{K_{\max}}$$

The minimum frequency is when the photon energy exactly matches the work function: $hf_{\min} = W_0$

$$f_{\min} = \frac{W_0}{h} = \frac{(4.50\ \text{eV})(1.60\times 10^{-19}\ \text{J/eV})}{6.63\times 10^{-34}\ \text{J s}} = 1.09\times 10^{15}\,\text{Hz}$$

Sample problem

Iron has a work function of 4.50 eV.

(a) What is the minimum frequency of light necessary to cause electrons to be ejected from an iron plate?

A shortcut

The minimum frequency is when the photon energy exactly matches the work function: $hf_{\min} = W_0$

$$E_{ph} = hf = 1240/\lambda_{(nm)} \Rightarrow \quad \lambda_{ph(nm)} = \frac{1240}{E_{ph\,(eV)}} = \frac{1240}{4.5} = 276\ nm$$

$$\Rightarrow \quad f_{ph}^{cutoff} = \frac{c}{\lambda} = \frac{3*10^8}{276*10^{-9}} = \frac{300}{276}*10^{15}\,Hz$$

(b) Is this in the visible spectrum?

Let’s find the corresponding wavelength.

$$=> \quad \lambda_{ph(nm)} = \frac{1240}{E_{ph\,(eV)}} = \frac{1240}{4.5} = 276\,nm$$

The smallest wavelength in the visible spectrum is 400 nm, so this is ultraviolet light.

(c) If the iron plate is exposed to light with a frequency of 1.50 x 10^{15} Hz, what is the maximum kinetic energy of the ejected electrons?

$$\lambda = \frac{c}{f} = 200\,nm \qquad E_{ph} = \frac{1240}{\lambda} = 6.2\,eV$$

$$K_{\max} = hf - W_0$$

$$= 6.22 \text{ eV} - 4.50 \text{ eV}$$

$$= 1.72 \text{ eV} \qquad (\text{or } 2.75\times10^{-19} \text{ J})$$

What happens if you double the frequency of the light?
or intensity of the light?

What happens if you double the frequency of the light?
or intensity of the light?

$$K_{\text{max}} = hf - W_0$$

(using SI units)

$$= \frac{(6.63\times10^{-34}\ \text{J s})\times(1.50\times10^{15}\ \text{Hz})}{1.60\times10^{-19}\ \text{J/eV}} - 4.50\ \text{eV}$$

$$= 6.22\ \text{eV} - 4.50\ \text{eV}$$

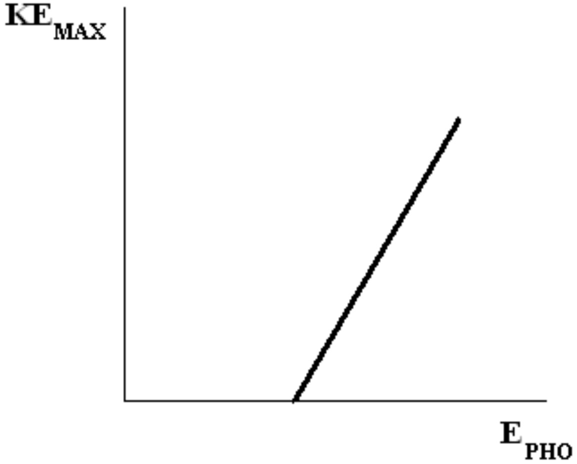
$$= \underline{1.72\ \text{eV}} \qquad (\text{or } 2.75\times10^{-19}\ \text{J})$$

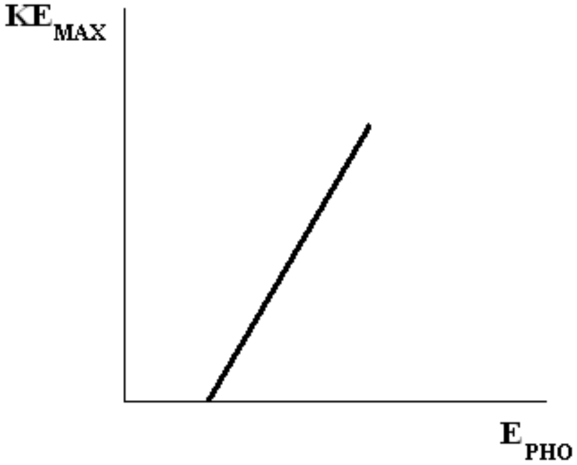
(d) what is the minimum voltage which “kills” the current?

(Stopping potential = ?)

$$e\cdot|V_c| = K_{\text{max}}$$

$$|V_c| = \frac{K_{\text{max}}}{e} = 1.72\ V$$





Which of the graphs (on the left or on the right) represents the photoelectric effect for a metal with a larger work function?

Estimate the ratio of the work functions. 2

What is the value of the slope of the lines? 1