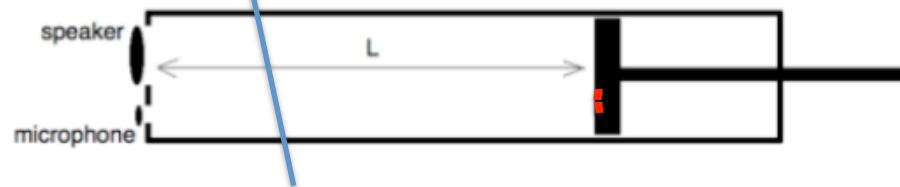


As shown in the Figure, the height of an air column in a particular pipe is adjusted by changing the water level in the pipe. In a traditional experiment, a tuning fork is placed over the pipe, and the height of the air column is adjusted, by moving a reservoir of water up and down, until the pipe makes a loud sound, which is when the pipe's fundamental frequency matches the frequency of the tuning fork. If the speed of sound is 340 m/s, and an

air column of 22.4 cm produces the loudest sound, what is the frequency of the tuning fork?



$$f_n = \frac{nv}{2L}, \text{ where } n = 1, 2, 3, \dots$$

$$f_n = \frac{nv}{4L}, \text{ where } n = \underline{1}, 3, 5, \dots$$

This tube is ... 1 opened

2 closed

A particular pipe that is open at both ends has a fundamental frequency of 442 Hz. When it, and a second pipe, have their fundamental frequencies excited simultaneously, a beat frequency of 8 Hz is observed. What is the ratio of the length of the first pipe to that of the second pipe if the second pipe is (a) also open at both ends, and (b) closed at one end.

$$f_n = \frac{nv}{2L}, \text{ where } n = 1, 2, 3, \dots$$

$$f_n = \frac{nv}{4L}, \text{ where } n = 1, 3, 5, \dots$$

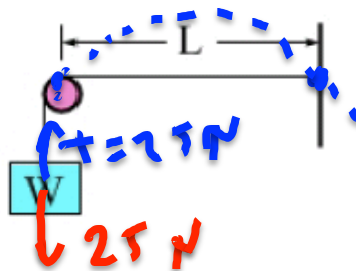
A particular pipe that is open at both ends has a fundamental frequency of 442 Hz. When it, and a second pipe, have their fundamental frequencies excited simultaneously, a beat frequency of 8 Hz is observed. What is the ratio of the length of the first pipe to that of the second pipe if the second pipe is (a) also open at both ends, and (b) closed at one end.

$$\frac{442}{450} = \frac{\frac{1 \cdot v}{2L_0}}{\frac{1 \cdot v}{4 \cdot L_c}} = \frac{4L_c}{2L_0} = 2 \frac{L_c}{L_0}$$

$$\frac{L_0}{L_c} = \frac{2 \cdot 450}{442}$$

$$f_n = \frac{nv}{2L}, \text{ where } n = 1, 2, 3, \dots$$

$$f_n = \frac{nv}{4L}, \text{ where } n = 1, 3, 5, \dots$$



As shown in the Figure, a string passing over a pulley supports the weight of a **25 N** block that hangs from the string. The other end of the string is fixed to a wall. The string has a mass per unit length of 75 grams per meter.

The part of the string between the wall and the pulley is observed to oscillate with a fundamental frequency of 44 Hz. (a) What is the speed of waves on the string? (b) What is the distance, L, from the wall to the pulley? (c) If the weight hanging from the string is doubled, what will be the fundamental frequency of the part of the string between the wall and the pulley?

$$\mu = \frac{m}{L} = \frac{0.075 \text{ kg}}{1 \text{ m}} \quad \checkmark$$

$$V = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{25}{0.075}} = 18.3 \text{ m/s}$$

$$f_n = \frac{nv}{2L}, \text{ where } n = 1, 2, 3, \dots$$

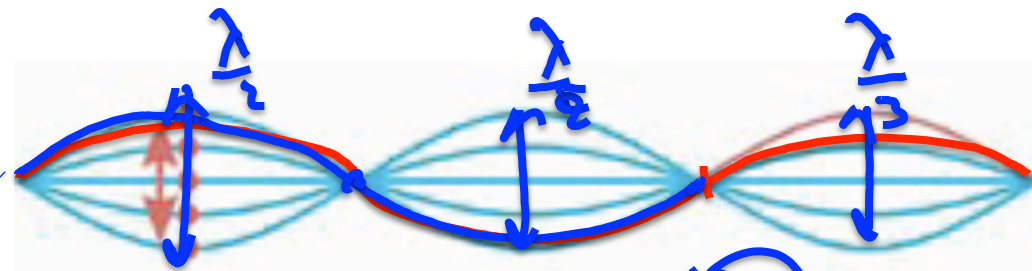
1. A 2. T 3. ~~f~~ $V = \frac{1}{2} \lambda \cdot f$
 $18.3 = 44 \cdot \lambda$

$$f_n = \frac{nv}{4L}, \text{ where } n = 1, 3, 5, \dots$$

$$\frac{1}{2} \lambda = L$$



A particular guitar string has a length of 75 cm, and a mass per unit length of 80 grams/meter. You hear a pure tone of 1320 Hz when a particular standing wave, represented by the sequence of images shown in the Figure, is excited on the string. (a) What is the wavelength of this standing wave? (b) What is the speed of waves on this string? (c) What is the tension in the string? (d) What is the fundamental frequency of this string?



$$\hookrightarrow 50 \text{ cm} = \lambda_d$$

1. y
2. n

$$\left(\frac{\lambda}{2}\right) = ?$$

$$75 \text{ cm} = 3 \frac{\lambda}{2}$$

$$\lambda = \frac{75 \cdot 2}{3} = 100 \text{ cm}$$

A particular guitar string has a length of 75 cm, and a mass per unit length of 80 grams/meter. You hear a pure tone of 1320 Hz when a particular standing wave, represented by the sequence of images shown in the Figure, is excited on the string. (a) What is the wavelength of this standing wave? (b) What is the speed of waves on this string? (c) What is the tension in the string? (d) What is the fundamental frequency of this string?

$$1320 = \frac{3 \cdot v}{2 \cdot 0.75} \rightarrow v = \sqrt{\frac{F_T}{\mu}}$$

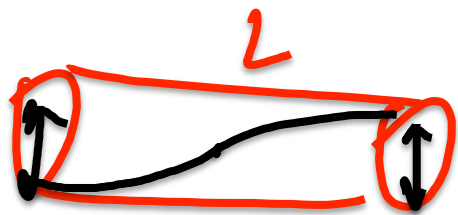
$v \cdot \mu = f \lambda$

$$f_n = \frac{nv}{2L}, \text{ where } n = 1, 2, 3, \dots$$

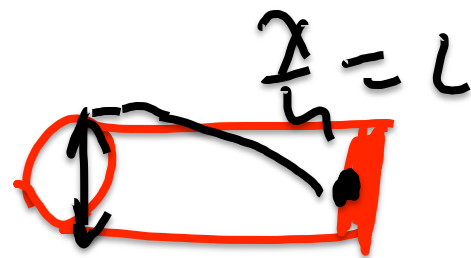
$$\cancel{f_n = \frac{nv}{4L}}, \text{ where } n = 1, 3, 5, \dots$$



$$\frac{\lambda_1}{2} = L$$



$$\lambda_4 = 2 \cdot L$$

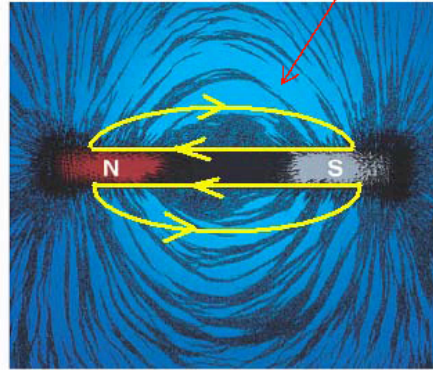
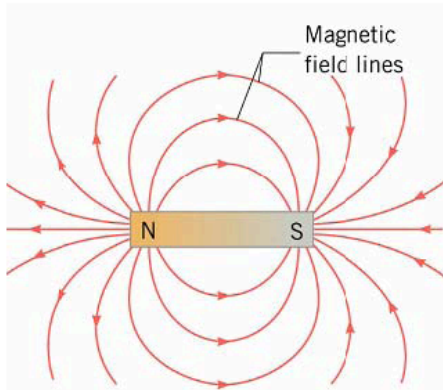


$$\lambda_4 = \frac{1}{4} \cdot L$$

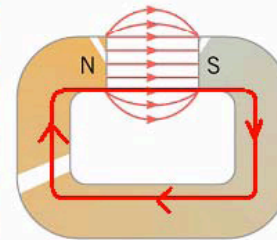
1
2
3
4

Magnetic field lines are continuous loops.

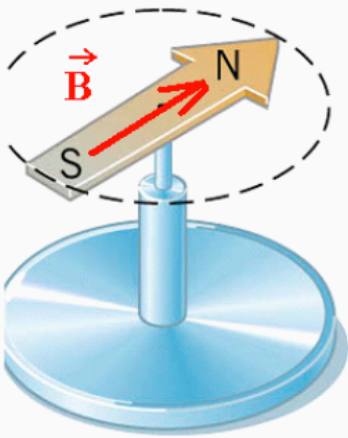
Magnetic field line NEVER can go away to infinity,
they can only make loops!



(b)



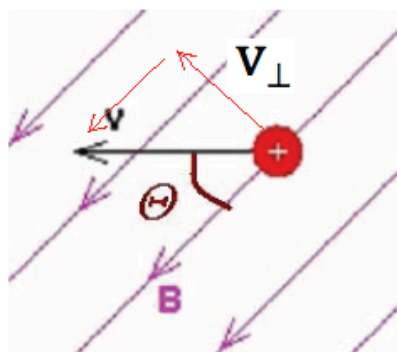
(c)



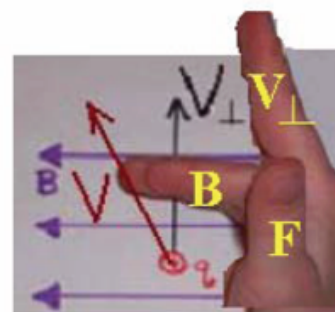
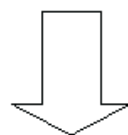
The direction of the magnetic field can be measured (found) by the compass a (small magnetic needle on a pivot).

Magnetic field points in the direction of the force experienced by a north pole (it points from S to N *inside* the magnet!).

A moving charge in a uniform magnetic field



A charge q is moving at the velocity \mathbf{v} in the magnetic field \mathbf{B}



The charge experiences a force \mathbf{F} acting on it.

The force has its magnitude and direction, which have to be found *independently* from each other!

The magnitude of the force:

$$F = |\vec{F}| = |q \cdot \mathbf{v} \cdot \mathbf{B} \cdot \sin(\theta)| = |q| \cdot |\mathbf{v}| \cdot |\mathbf{B}| \cdot |\sin(\theta)|$$

The magnitude of the force is *the same* for positive and negative charges!

$$|\mathbf{F}| = |q| \mathbf{B} v_\perp$$

The magnetic force always remains perpendicular to the velocity and is directed toward the center of the circular path.

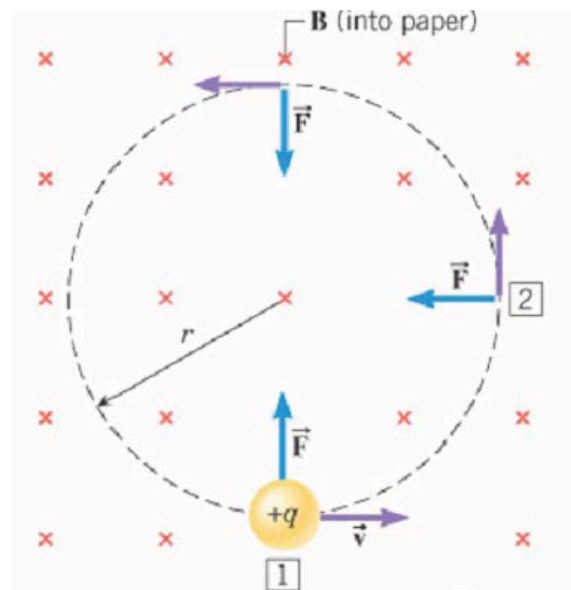
$$F = ma \qquad a = a_c = \frac{v^2}{r}$$

$$F = m \frac{v^2}{r}$$

$$F = |qvB \sin(90^\circ)| = |q| vB$$

$$|q| vB = m \frac{v^2}{r}$$

$$r = \frac{mv}{|q| B}$$



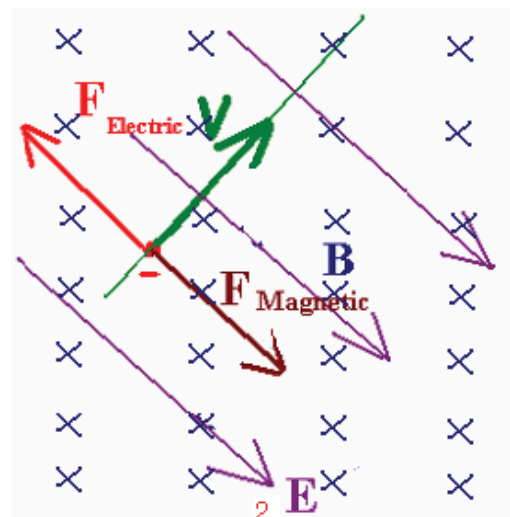
$$T = \frac{2\pi r}{v} = \frac{2\pi m}{|q| B}$$

An electron in an electromagnetic field

V = const!

$$F_{\text{magnetic}} = - F_{\text{electric}}$$

The forces have to have the same magnitude and opposite direction!



$$|F_{\text{magnetic}}| = |F_{\text{electric}}|$$

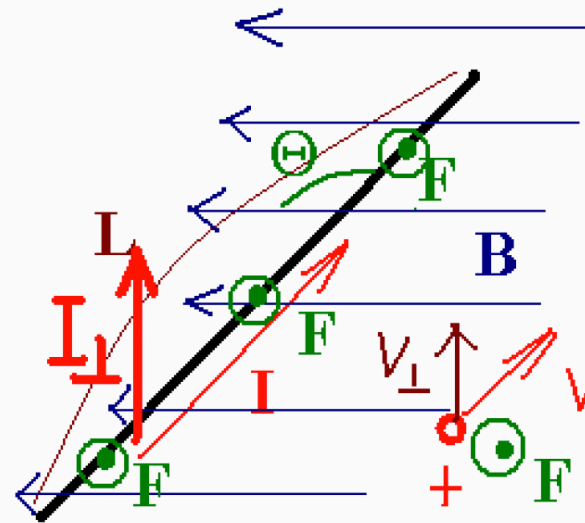
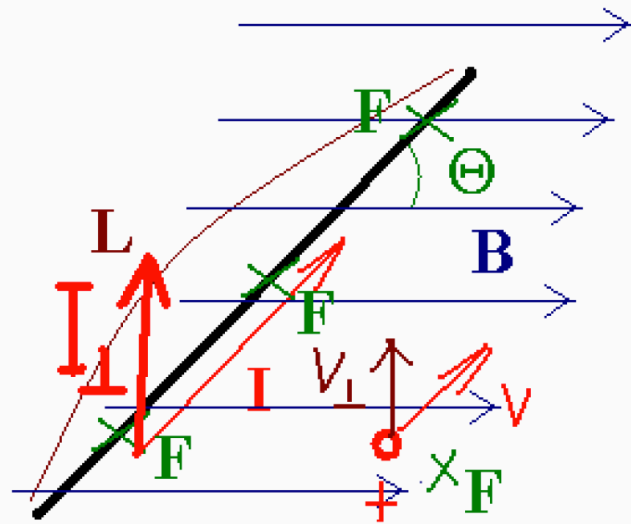
$$|evB\sin(90^\circ)| = |eE|$$

$$E = v*B \quad (!)$$

The answer does *not* depend
on the value of the charge!

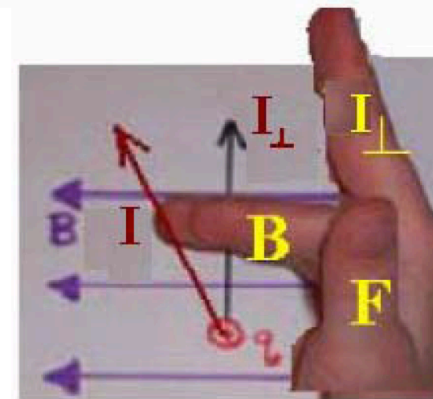
For the *electron* the electric field has the direction *opposite* to the force (because $F = eE$, and $e < 0$).

The force on a current-carrying wire



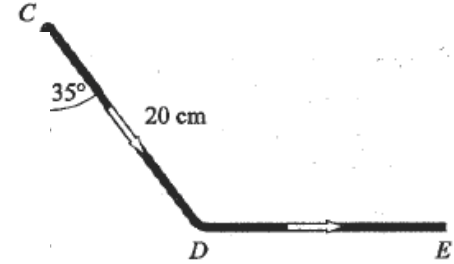
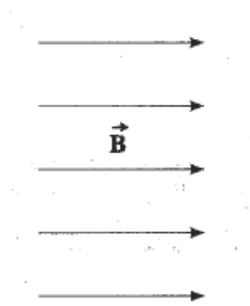
$$|F| = ILB |\sin \theta|$$

The right-hand rule!



Find the net force on the wire shown in the picture below if $B = 0.15 \text{ T}$. Assume the current in the wire to be 5.0 A .

$$|F| = ILB |\sin \theta|$$



1. 0 N

2. 0.6 N

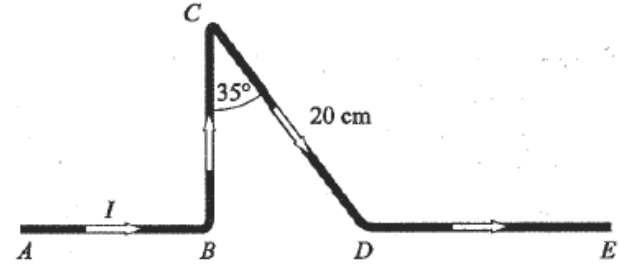
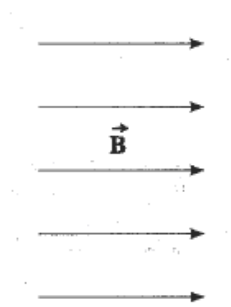
3. 0.12 N

4. 0.18 N

5. ??

Find the net force on the wire shown in the picture below if $B = 0.15 \text{ T}$. Assume the current in the wire to be 5.0 A .

$$|F| = ILB |\sin \theta|$$



1. 0 N 2. 0.6 N 3. 0.12 N 4. 0.18 N 5. ??

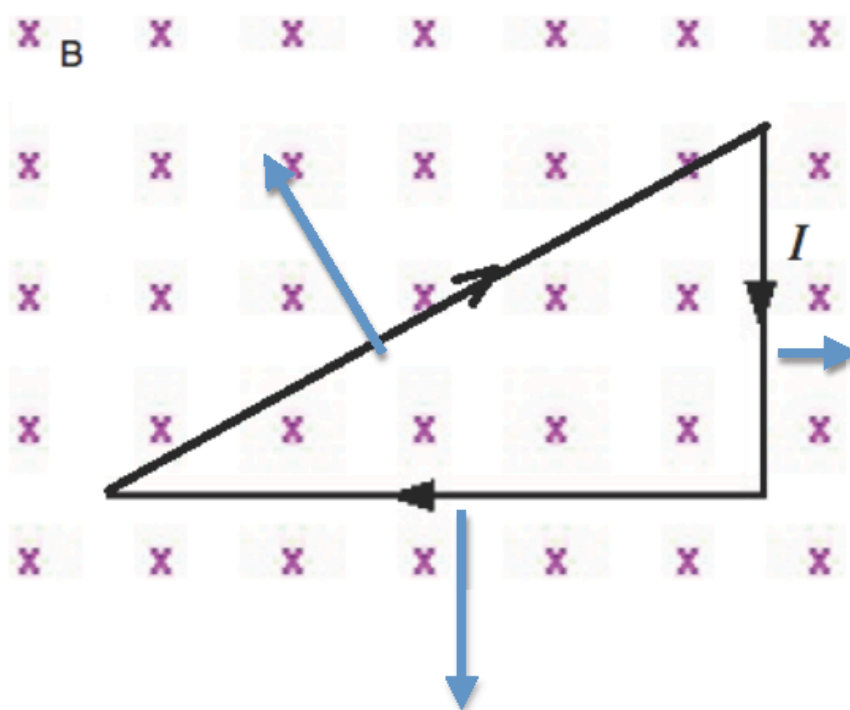
The force on a current-carrying loop

A wire loop carries a clockwise current in a uniform magnetic field directed into the page.

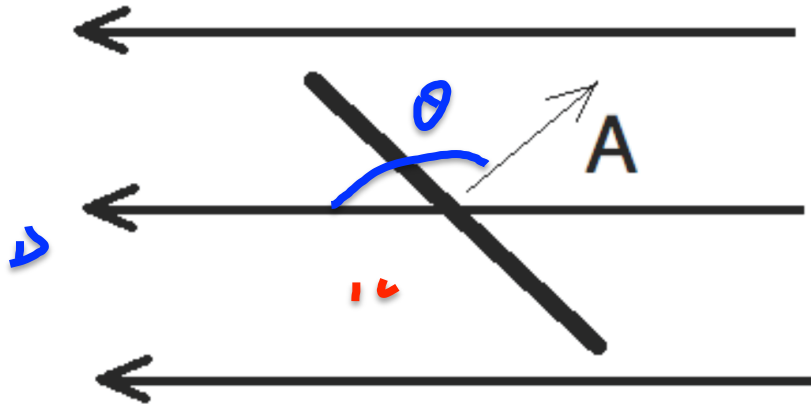
In what direction is the net force on the loop?

The net force is zero

For ANY shape, as long as the field is uniform.



B 1.20 T - Mol 120
 $\theta = ?$



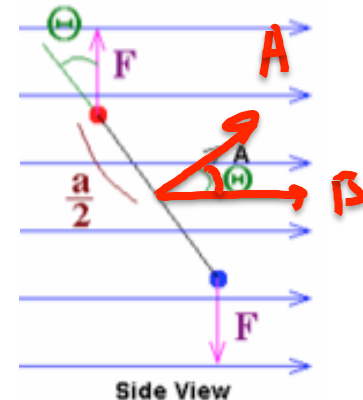
The picture shows magnetic field B (points to the left) and a loop with a current in it so the area vector A points as shown.

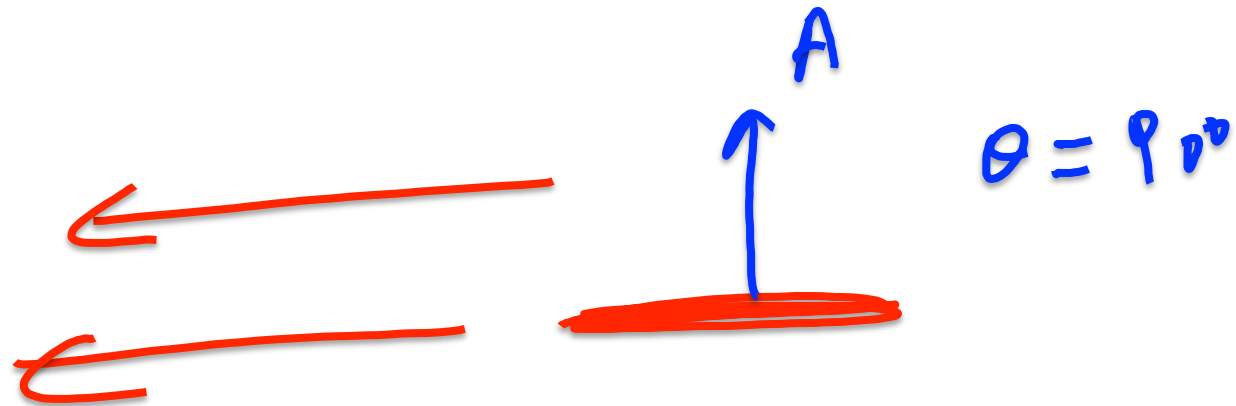
If we release the loop from rest, in which

direction is it going to start rotating?

$$\tau = IAB \sin \theta \cdot N$$

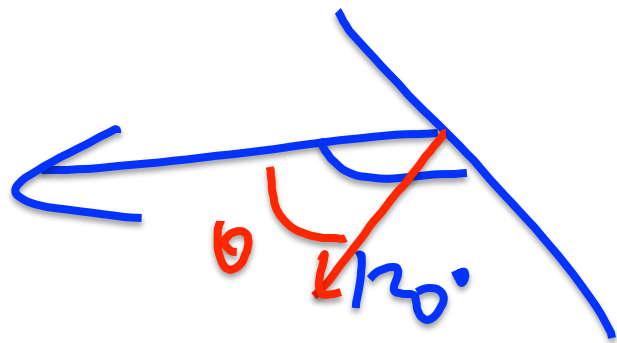
$$\tau_{\max} = IAB \cdot N$$





$$\tau = A \cdot J \cdot B \cdot \sin 90 = \tau_{\max}$$





The magnetic field from a long straight wire

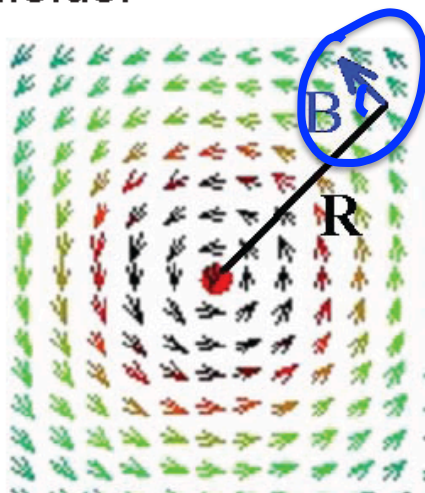
The long straight current-carrying wire, for magnetism, is analogous to the point charge for electric fields.

The magnetic field a distance r from a wire with current I is:

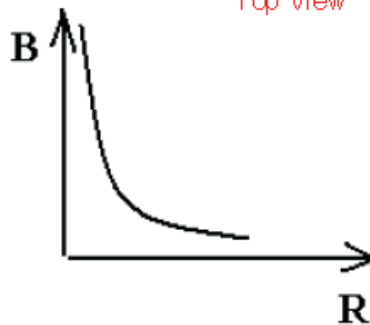
$$B = \frac{\mu_0 I}{2\pi r}$$

μ_0 , the permeability of free space, is:

$$\mu_0 = 4\pi \times 10^{-7} \text{ Tm/A}$$



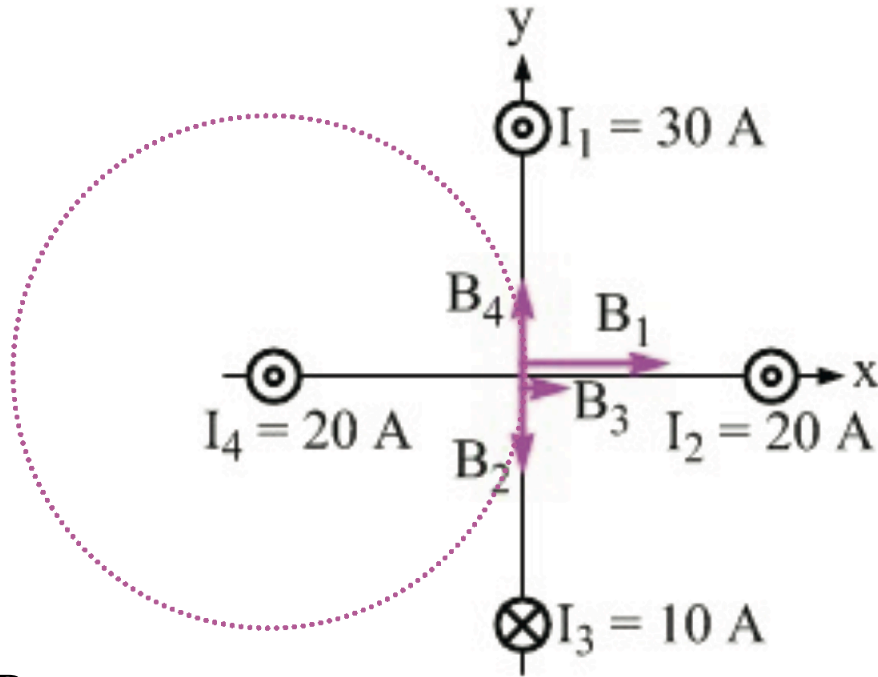
Top View



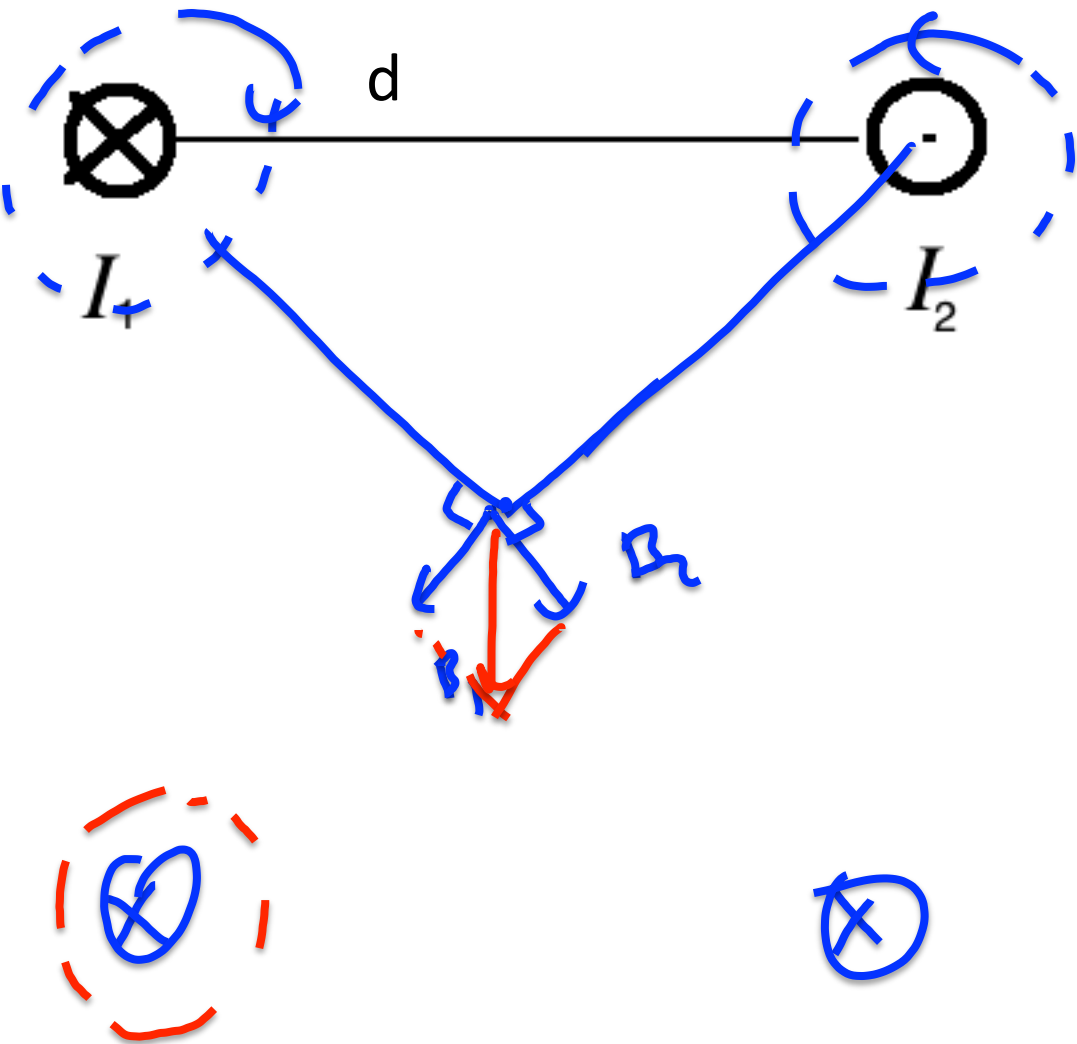
The net magnetic field

We add the individual fields to find the net field, which is directed right.

$$B = \frac{\mu_0 I}{2\pi r}$$



$$\text{If } \frac{\mu_0}{2\pi r} \Rightarrow B_{\text{net}} = \dots$$

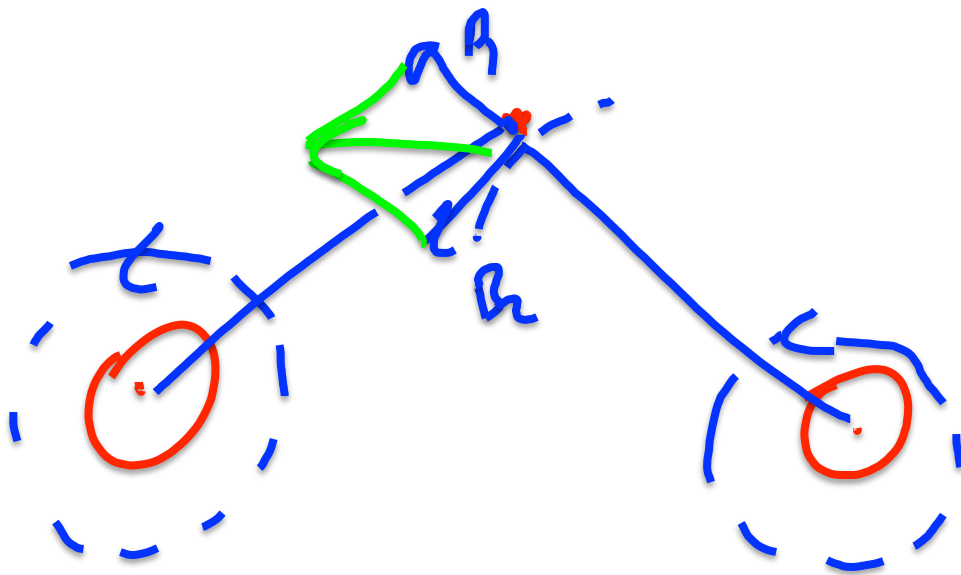


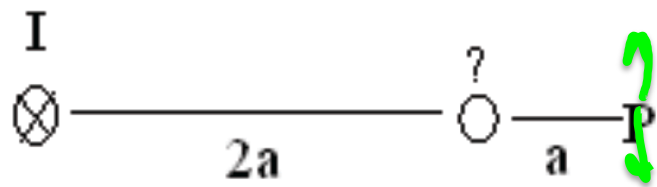
$$B = \frac{\mu_0 I}{2\pi r}$$

$$I_1 = I_2 = 2 \text{ Amp};$$

$$d = 60 \text{ cm};$$

$$r_1 = r_2 = 40 \text{ cm}.$$





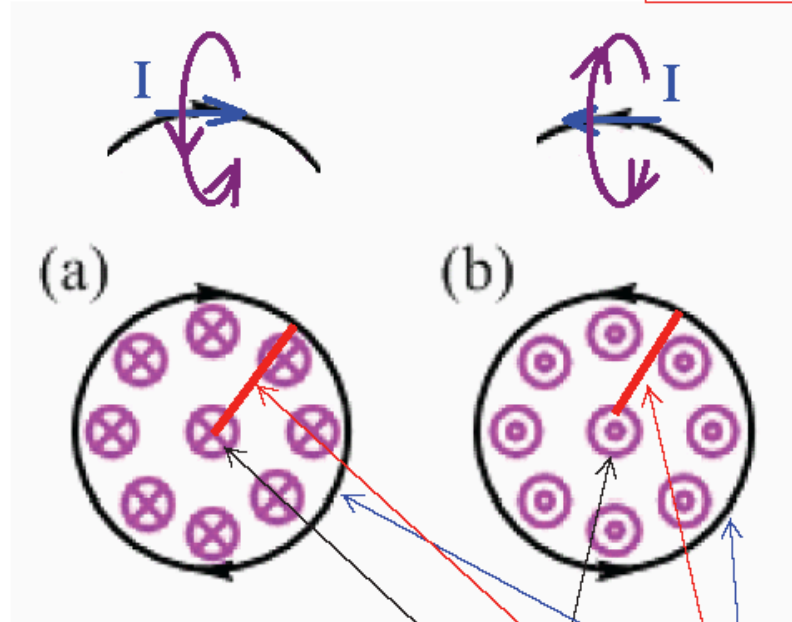
Two wires carry a current. What current ($2I$, $3I$ or etc.) should be carried by the second wire to make the net magnetic field at the point P be zero? What should be the direction of that current?

$$B = \frac{\mu_0 I}{2\pi r}$$

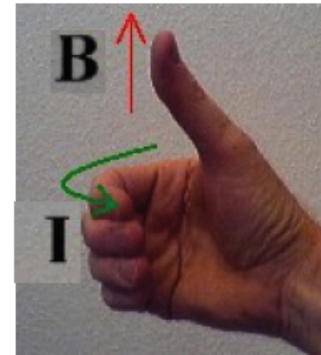
1. left
2. right
3. up
4. down
5. into
- 6 out

A field in the loop

$$\text{At the center of the loop} \quad |B| = \frac{\mu_0 I}{2R}$$



$$\text{At the center of the loop} \quad |B| = \frac{\mu_0 I}{2R}$$



($\times N$ for a flat loop with N turns)

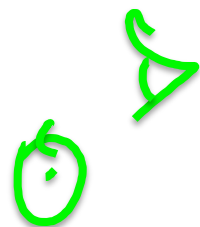
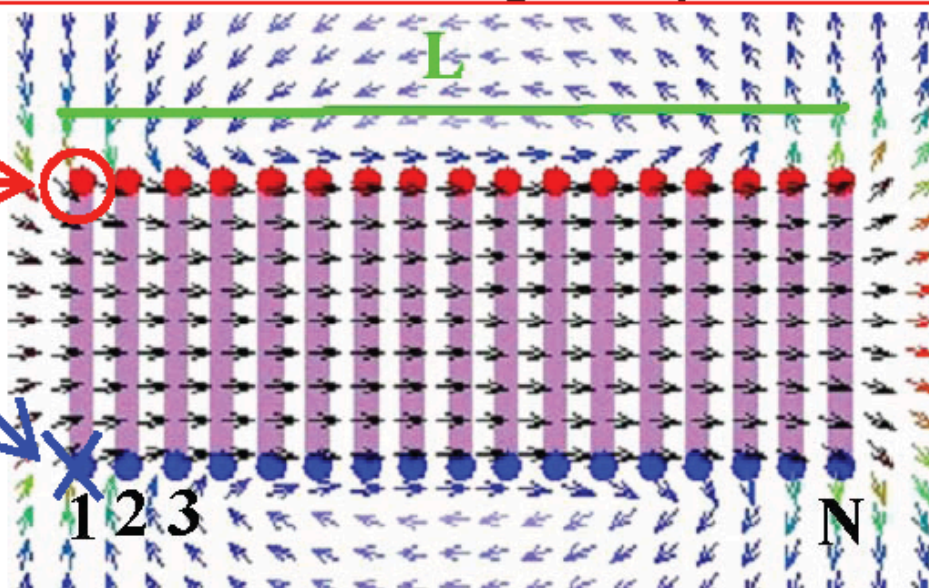
The field from a solenoid

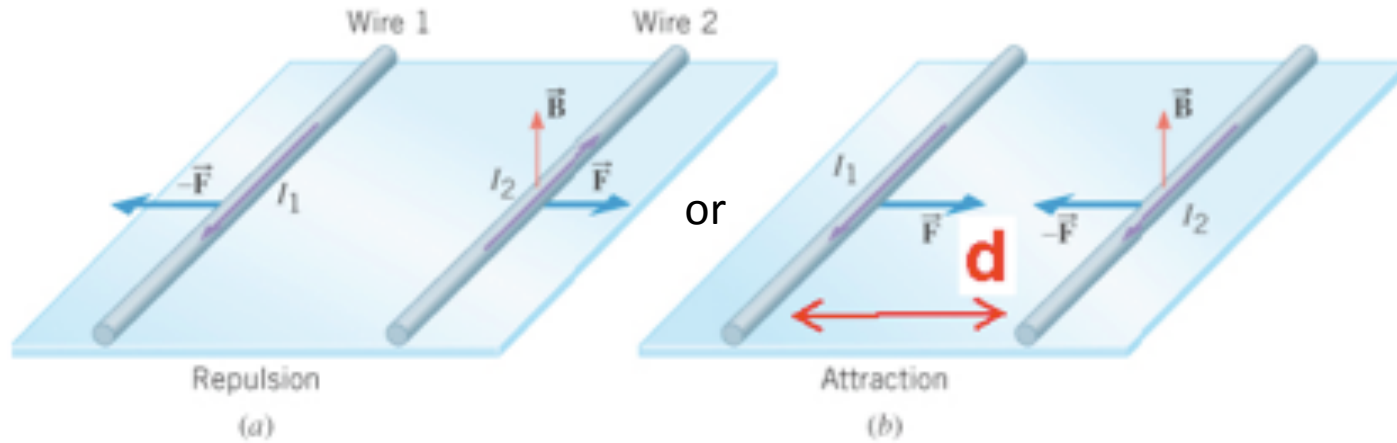
A solenoid is simply a coil of wire with a current going through it. It's basically a bunch of loops stacked up. Inside the coil, the field is very uniform (not to mention essentially identical to the field from a bar magnet).

For a solenoid of length L , current I , and total number of turns N , the magnetic field inside the solenoid is given by:

$$B = \frac{\mu_0 N I}{L}$$

$$N/L = n$$





Wires \neq charges!

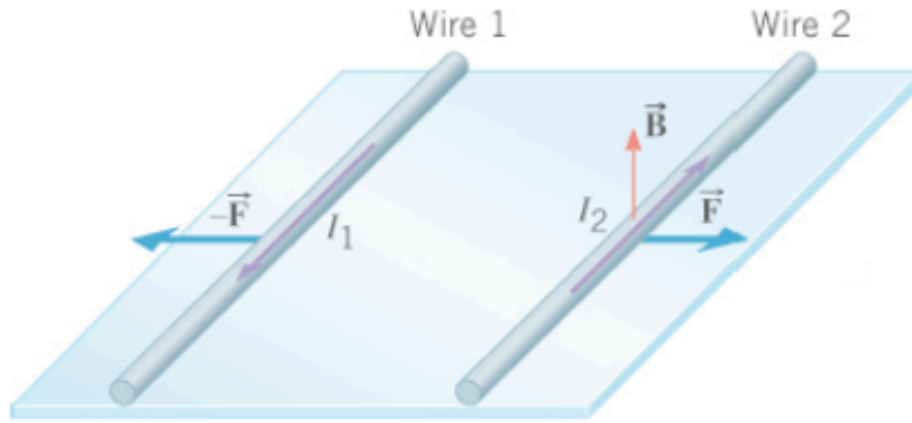
For two straight wires:

$$|F| \propto B_1 I_2 \propto B_2 I_1 \propto \frac{I_1 I_2}{d} = C \frac{I_1 I_2}{d}$$

(per unit length, i.e. per 1 m)

$\frac{\mu_0}{2\pi}$

A "universal" constant



$$|F| = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d} L$$

If we double the current in the second wire but increase the distance between the wires four times, find the change in the force between the wires.

1. $F_f = F_i$

2. $F_f > F_i$

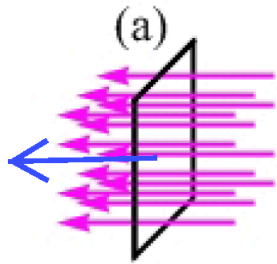
3. $F_f < F_i$

Magnetic flux

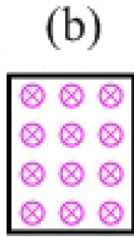
$$\Phi_B = BA \cos \theta$$

$$\Phi = \vec{A} \cdot \vec{B} = AB \cos \theta$$

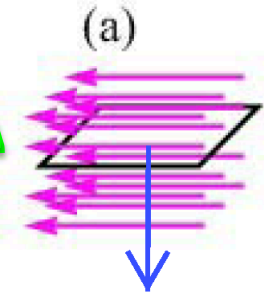
The more field lines pass through an area, the larger the flux.



Lots of flux.



$\theta = 0$
 $\Phi_{\text{max}} = BA$

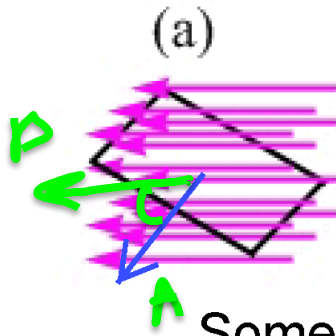


No flux.

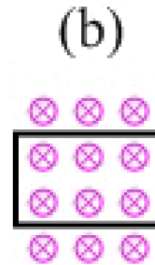


$\theta = 90$

θ is the angle between the field and the "normal" to the area (\perp)



Some flux.

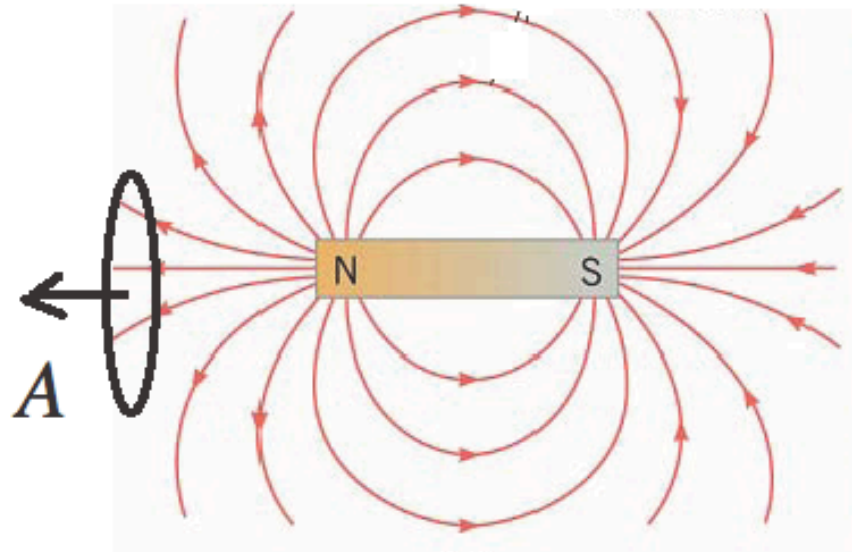
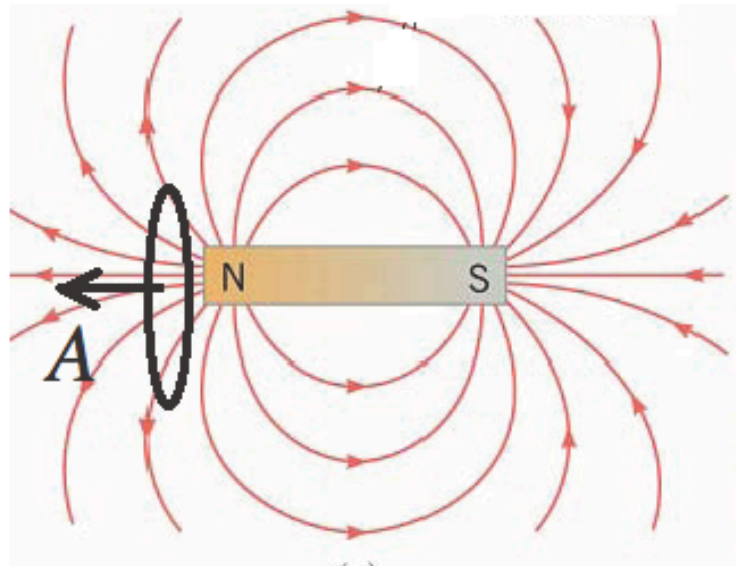


Comparing flux

1. the flux in the loop is positive
2. the flux in the loop is negative

$$\Phi_B = BA \cos \theta$$

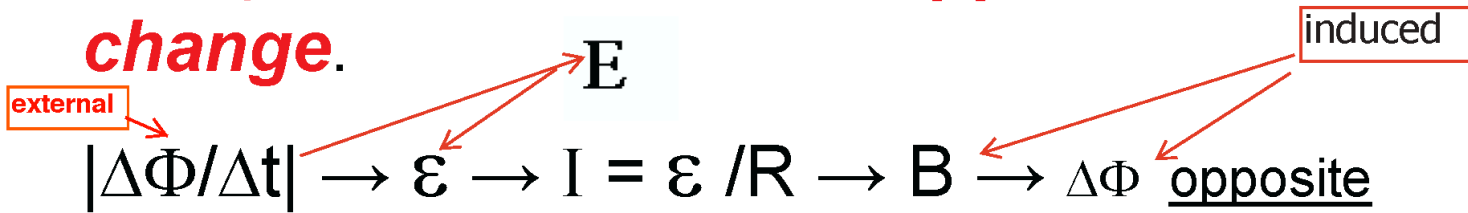
1. there is more flux in the loop on the left
2. there is less flux in the loop on the left



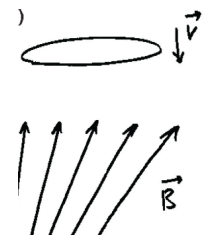
$$\Phi_B = BA \cos \theta$$

Lenz's Law

Lenz's Law: A changing magnetic flux induces a response that *tends to oppose the change*.

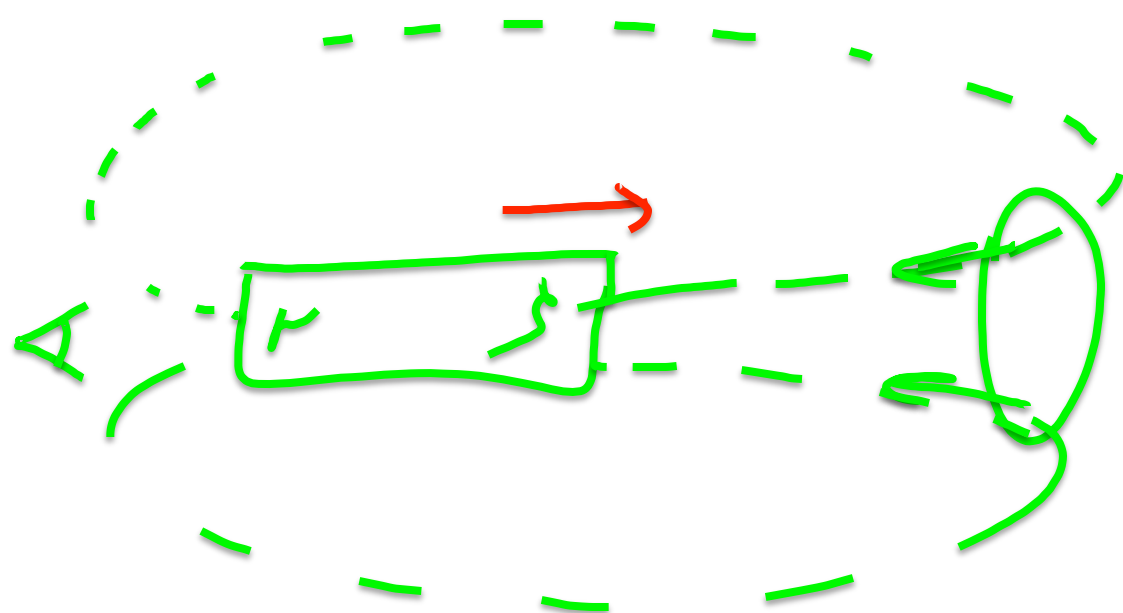


The change can be made, but the coil or loop tries to oppose the change while the change is taking place. This tendency to oppose is why there is a minus sign in Faraday's Law.



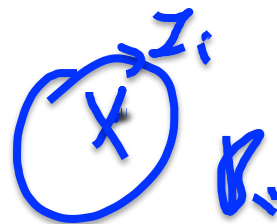
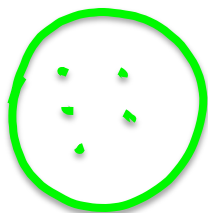
$$\epsilon = -N \frac{\Delta\Phi}{\Delta t}$$

$$|\epsilon| = \left| N \frac{\Delta\Phi}{\Delta t} \right|$$



$$1. I_i \rightarrow B_i$$

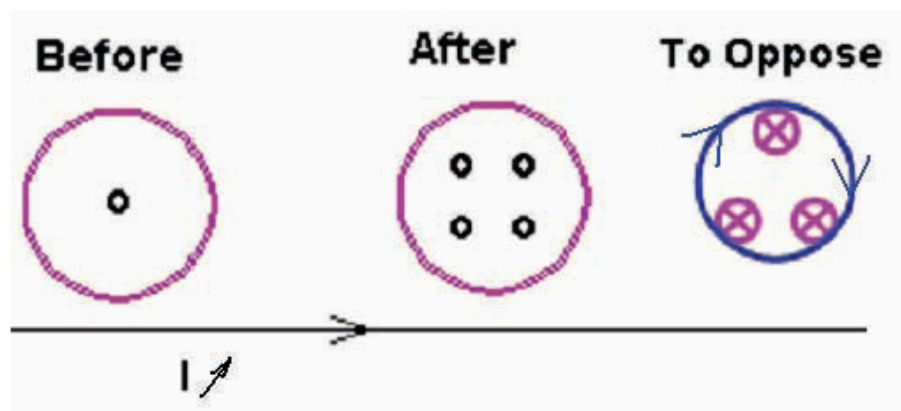
$$2. B_i \rightarrow I_i$$



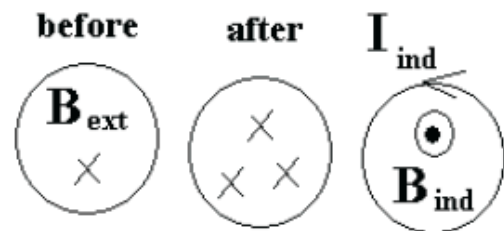
Lenz's Law example

With the current in the long straight wire, the loop is parallel to the wire. If **the current in the wire is increasing**, in what direction is the induced current in the loop?

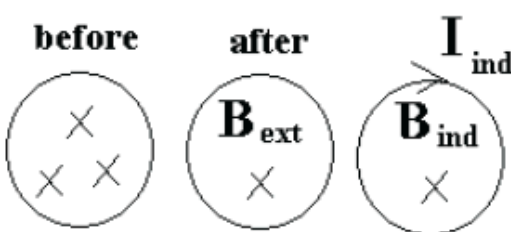
1. The induced current is clockwise.
2. The induced current is counter-clockwise.
3. There is no induced current.



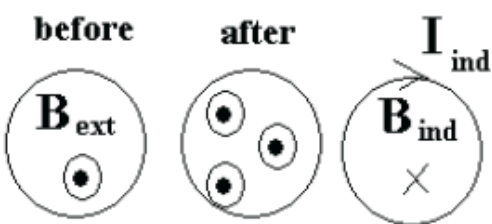
Four options



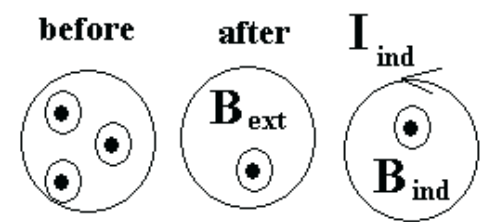
An external B field is *in* the page and *increasing* in its magnitude. The induce B field opposes changes and *opposes* the external field and has the direction *opposite* to the external B field.



An external B field is *in* the page and *decreasing* in its magnitude. The induce B field opposes changes and *supports* the external field and has *the same* direction as the external B field.

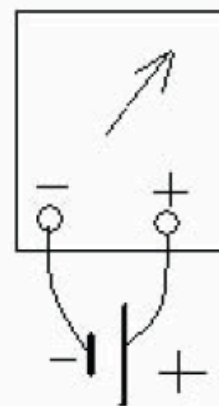


An external B field is *out of* the page and *increasing* in its magnitude. The induce B field opposes changes and *opposes* the external field and has the direction *opposite* to the external B field.



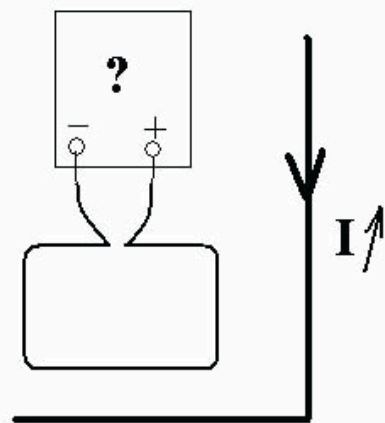
An external B field is *out of* the page and *decreasing* in its magnitude. The induce B field opposes changes and *supports* the external field and has *the same* direction as the external B field.

When a galvanometer is connected as shown in picture 1, the needle deflects to the right. Note that the galvanometer needle points straight up when no current passes through the galvanometer, and it would deflect to the left if we reversed the battery in picture 1.



pic. 1

Picture 2 shows a wire with an increasing current in it and a loop of wire connected to the galvanometer.

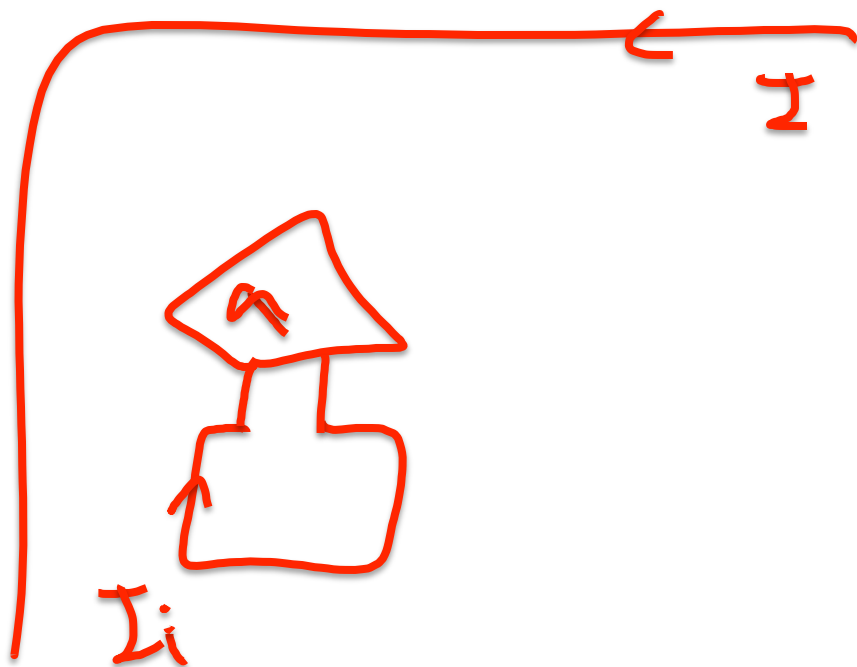


pic. 2

In which direction is the needle of the galvanometer deflected in picture 2?

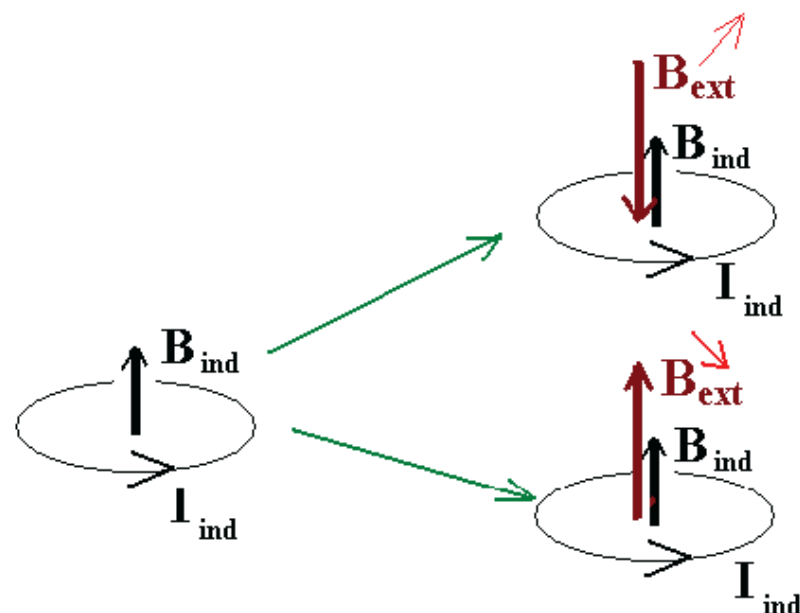
1. to the left

2. to the right



Some combinations

(working backwards)



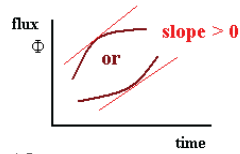
Let's say we know the direction of the induced current $\mathbf{I}_{\text{ind}} \Rightarrow$ we know the direction of the induced magnetic field \mathbf{B}_{ind} (as a current in the loop and the field in it).

There are two situations which can lead to the known direction of the induced field.

A). An external magnetic field is anti-parallel to the induced one and its magnitude increases.

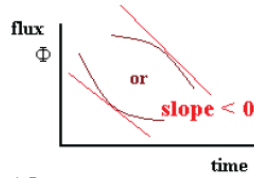
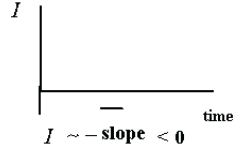
B). An external magnetic field is parallel to the induced one and its magnitude decreases.

Examples



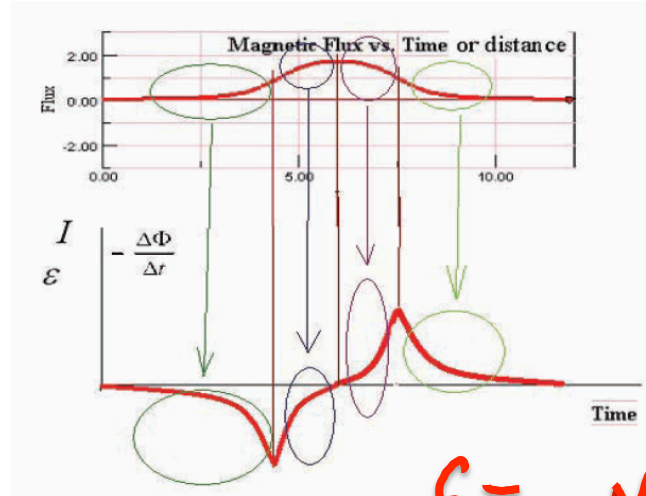
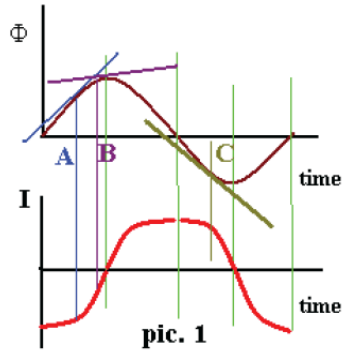
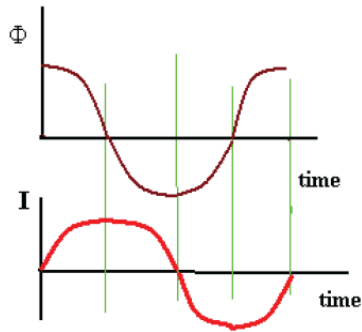
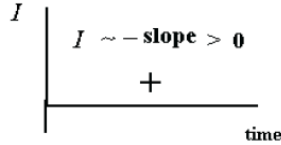
$$\frac{\Delta \Phi}{\Delta t} = \text{slope} > 0$$

$$I = \varepsilon / R = (-N \frac{\Delta \Phi}{\Delta t}) / R \sim -\text{slope}$$



$$\frac{\Delta \Phi}{\Delta t} = \text{slope} < 0$$

$$I = \varepsilon / R = (-N \frac{\Delta \Phi}{\Delta t}) / R \sim -\text{slope}$$



$$\varepsilon = -N \cdot \text{slope}(\Phi)$$

In the picture 1:

$$|\text{slope A}| > |\text{slope B}| \Rightarrow |I_A| > |I_B|$$

$$|\text{slope C}| > |\text{slope B}| \Rightarrow |I_C| > |I_B|$$

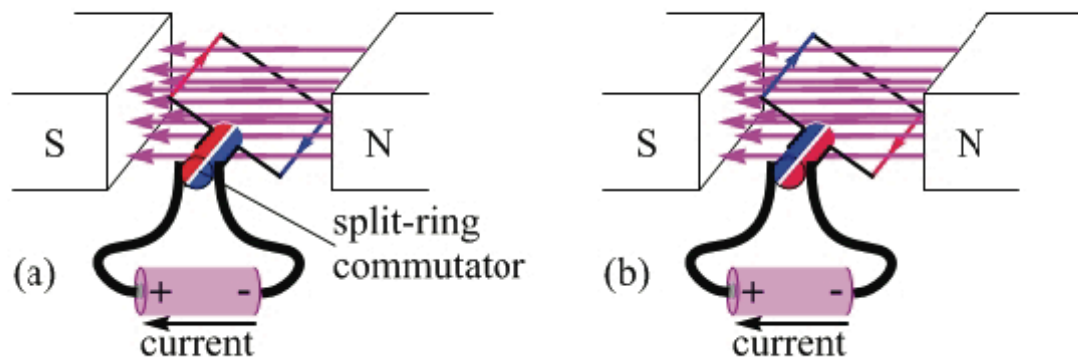
$$|\text{slope A}| = |\text{slope C}| \Rightarrow |I_A| > |I_C|$$

I_A is negative and I_C is positive \Rightarrow for the induced B field: $B_A \uparrow \downarrow B_C$

Electric generators

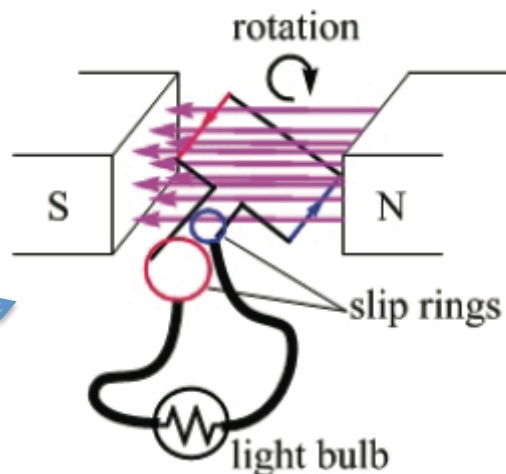
$$\Phi_B = BA \cos \theta$$

If a current is passed through the coil, the interaction of the magnetic field with the current causes the coil to spin – that's a motor.



$$|EMF_{\max}| = N \cdot BA \omega$$

If we spin the coil, the changing flux through the coil induces a current – now it's a generator.

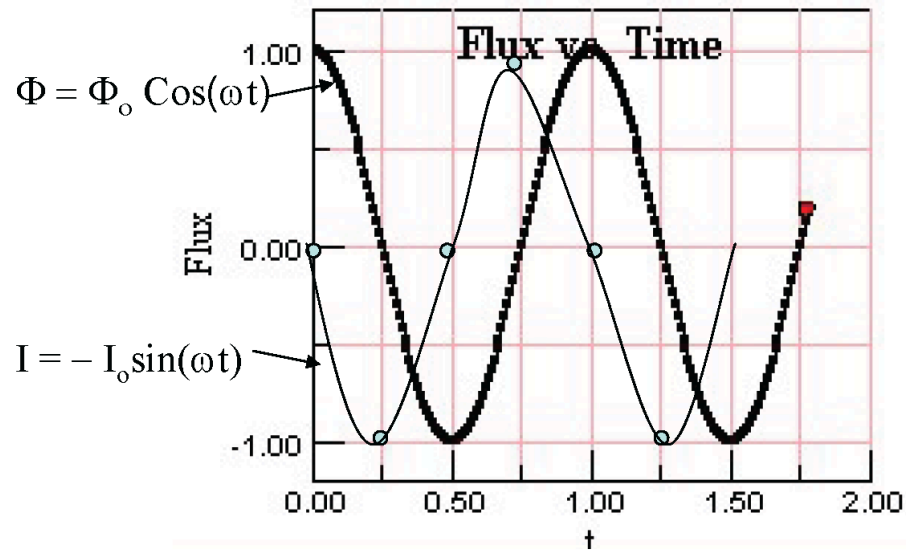


Maximum generator current

The magnitude of the current is proportional to the slope of the flux graph, so let's draw the flux graph.

When is the current maximum?

When the flux is zero !



Transformers

Both coils are exposed to the same changing flux, so:

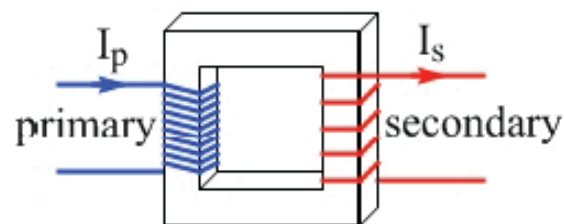
$$-\frac{\Delta\Phi}{\Delta t} = \frac{\Delta V_1}{N_1} = \frac{\Delta V_2}{N_2}$$

Energy (or, equivalently, power) has to be conserved, so:

$$P = \Delta V_1 I_1 = \Delta V_2 I_2 \quad \text{so}$$

$$\frac{N_1}{N_2} = \frac{\Delta V_1}{\Delta V_2} = \frac{I_2}{I_1}$$

V is proportional to N
I is inversely proportional to N.



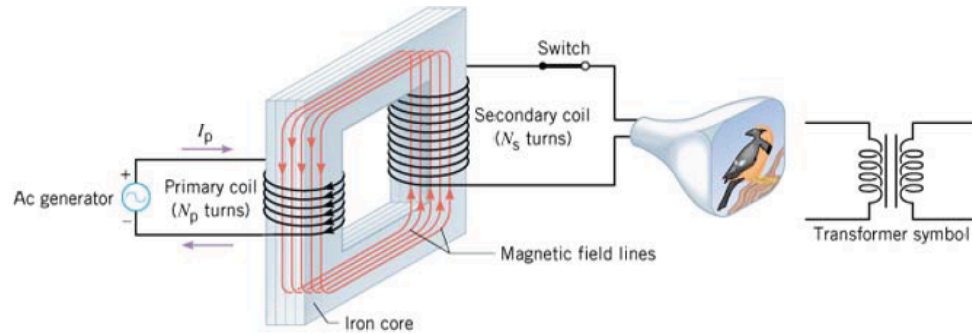
10 : 5 turns ratio (Primary : secondary)

Voltage “stepped down” by factor of 2,

Current “stepped up” by factor of 2,

Power in = power out [ideal transformer]

A **transformer** is a device for increasing or decreasing an ac voltage.



$$EMF_p = -N_p \frac{\Delta\Phi}{\Delta t}$$

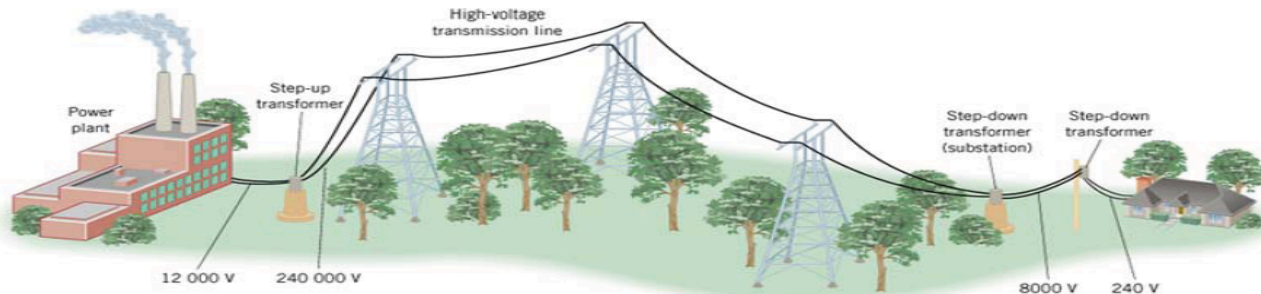
$$EMF_s = -N_s \frac{\Delta\Phi}{\Delta t}$$

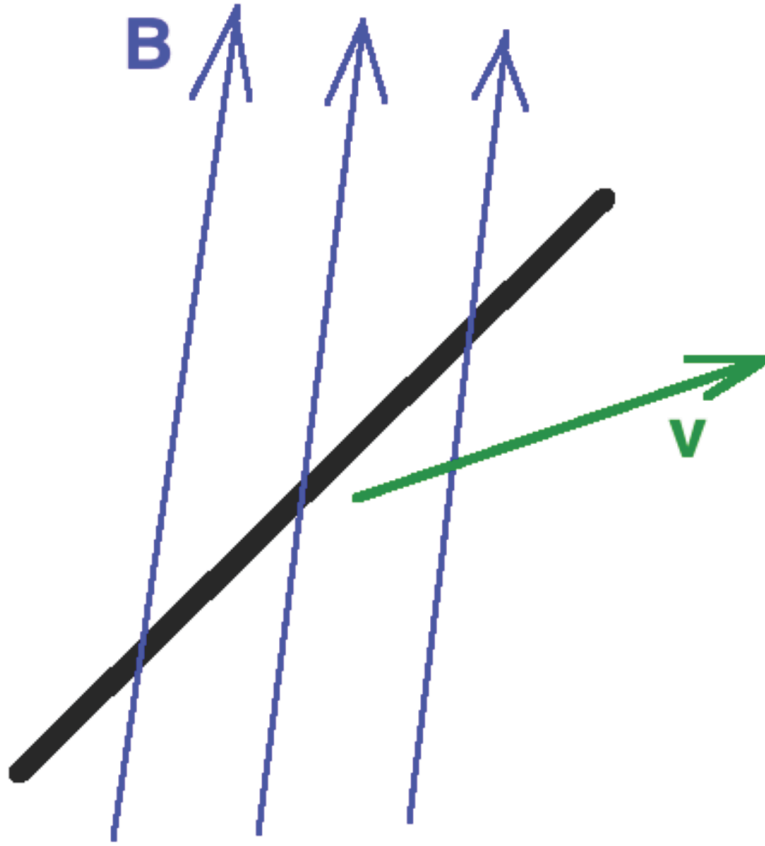
$$\frac{EMF_s}{EMF_p} = \frac{N_s}{N_p}$$

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

$$\frac{I_s}{I_p} = \frac{V_p}{V_s} = \frac{N_p}{N_s}$$

A transformer that steps up the voltage simultaneously steps down the current, and a transformer that steps down the voltage steps up the current.



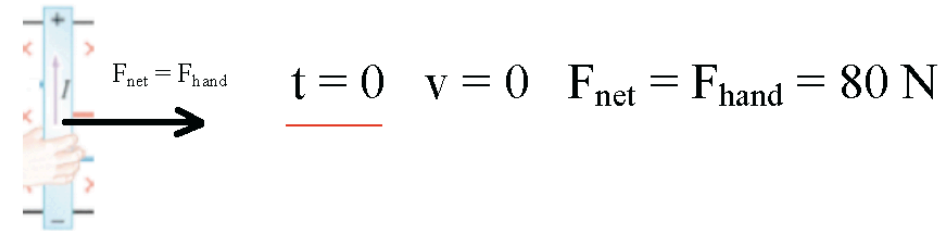
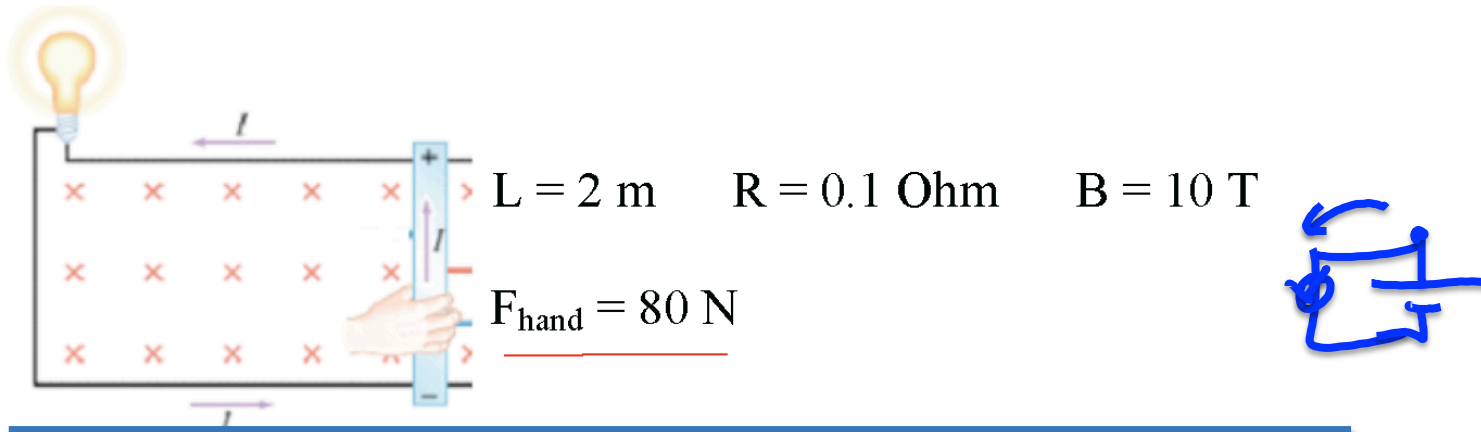


No right angles!

What now?

Use only the
components
perpendicular to
the rod!

$$\mathcal{E} = Lv_{\perp} B_{\perp}$$



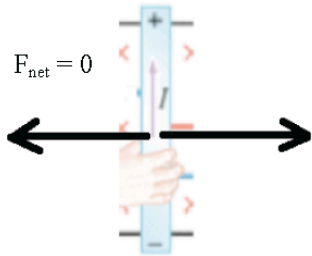
$t \gg 0$ $v = v_{\text{terminal}} = \text{const}$ $F_{\text{net}} = 0$

$F_{\text{hand}} = F_{\text{magn}} = IBL \Rightarrow I = F_{\text{hand}}/BL = 80/(10 \cdot 2) = 4 \text{ A}$

$\text{EMF} = I \cdot R = 0.4 \text{ V}$

$\text{EMF} = LvB$

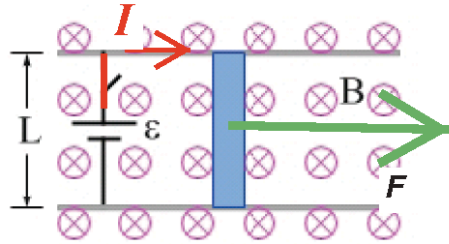
$\Rightarrow v = v_{\text{terminal}} = \text{EMF}/LB = 0.4/(2 \cdot 10) = 0.02 \text{ m/s}$



$$t = 0$$

The Figure shows a conducting bar of 2 m length that can move without friction on a pair of conducting rails. The rails are joined at the by a battery of 36 V emf and a switch that is initially open. The bar, which is initially at rest, has a resistance 2 Ohm , and we will assume the resistance of all other parts of the circuit is negligible. The whole apparatus is in a uniform magnetic field, directed into the page, of magnitude 0.5 T . Find the magnitude and direction of: (a) the current in the circuit immediately after the switch is closed, (b) the net force on the bar immediately after the switch is closed.

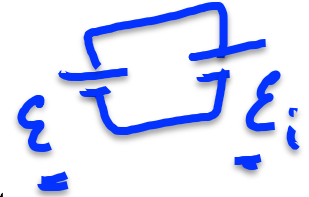
$$|\mathcal{E}| = |vLB|$$



$$V_i = 0$$

$$I = \mathcal{E}/R$$

$$F = ILB$$



$$F_{net_i} = 1.18\text{ N} \quad 2.19\text{ N} \quad 3.20\text{ N} \quad 4.21\text{ N} \quad 5.?$$

at $t \gg 0$ (when the terminal velocity is reached)

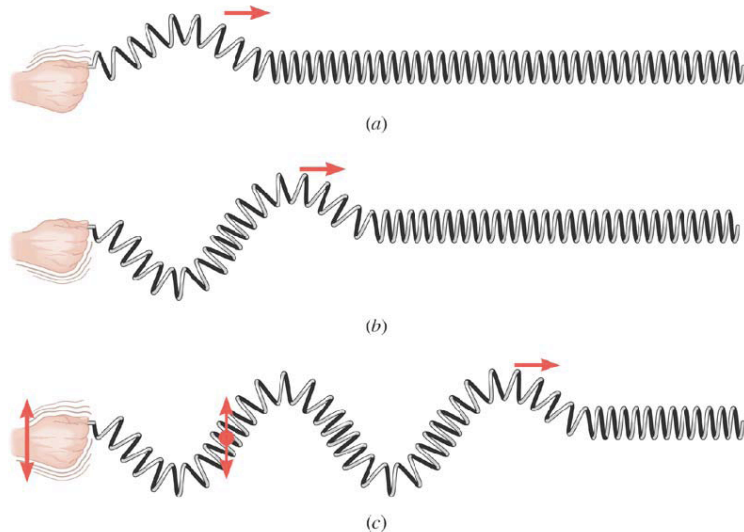
$$F_{net} = 1.18\text{ N} \quad 2.19\text{ N} \quad 3.20\text{ N} \quad 4.21\text{ N} \quad 5.?$$

$$v = 1.18\text{ m/s} \quad 2.36\text{ m/s} \quad 3.72\text{ m/s} \quad 4.?$$

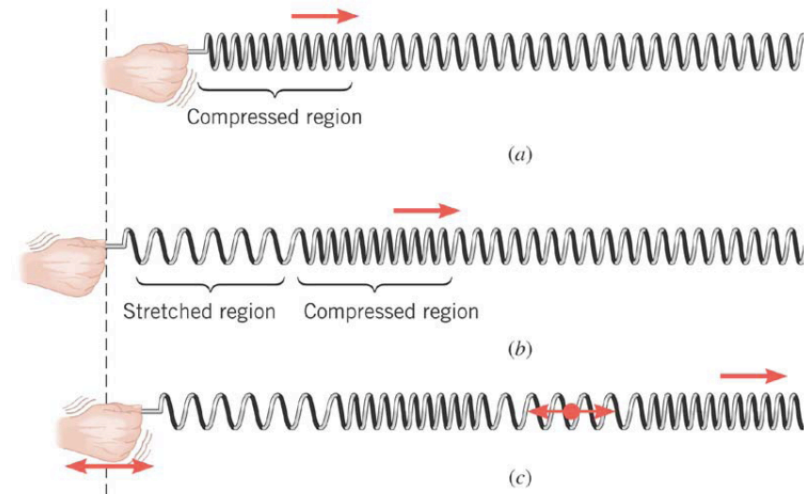
Periodic waves consist of cycles or patterns that are produced over and over again by the source.

In the figures, every segment of the slinky vibrates in simple harmonic motion, provided the end of the slinky is moved in simple harmonic motion.

Transverse Waves

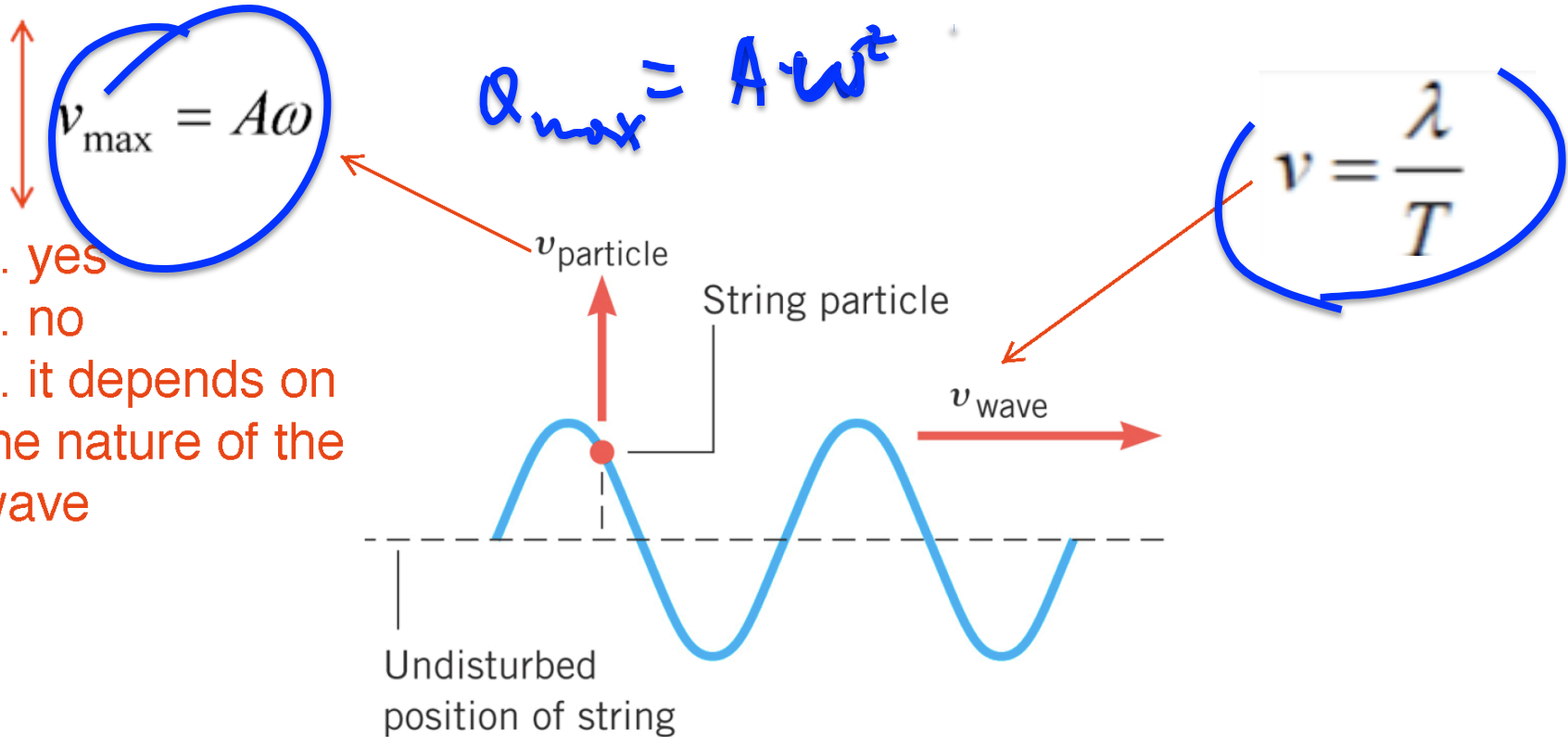


Longitudinal Waves



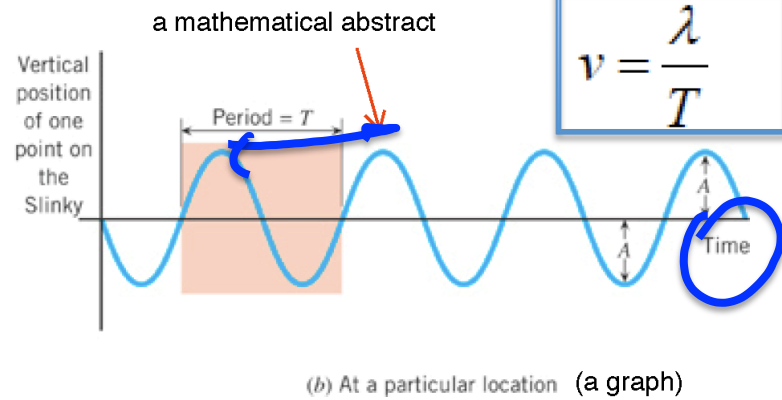
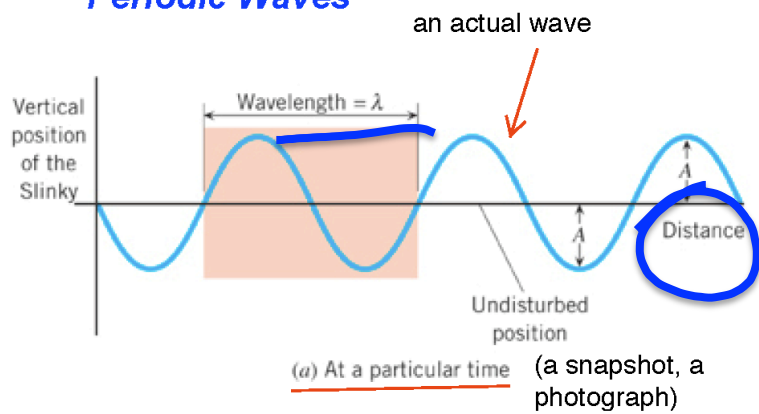
Wave Speed Versus Particle Speed

Is the speed of a transverse wave on a string the same as the speed at which a particle on the string moves?



1. yes
2. no
3. it depends on the nature of the wave

Periodic Waves



In the drawing, one **cycle** is shaded in color.

The **amplitude** A is the maximum excursion of a particle of the medium from the particles undisturbed position.

The **wavelength** is the horizontal length of one cycle of the wave.

The **period** is the time required for one complete cycle.

$$f = \frac{1}{T}$$

The **frequency** is related to the period and has units of Hz, or s^{-1} .

If a certain point on a wave makes 30 oscillations in 1 minute, what are T and f ?

1. 0 2. 0.5 3. 1 4. 1.5 5. 2 6. 2.5 7. 3

What distance does a single point on the wave travel if the amplitude is 1 mm?

1. 30 2. 60 3. 90 4. 120 5. 150 6. 180 7. 210

Making use of the mathematical description

Our equation describing a transverse wave moving in one dimension is:

$$y(x,t) = A \sin(\omega t - kx)$$



Sometimes a cosine is appropriate, rather than a sine.

The above equation works if the wave is traveling in the positive x-direction. If it goes in the negative x-direction, we use:

$$y(x,t) = A \sin(\omega t + kx)$$



$$k = \frac{2\pi}{\lambda}$$

SIN or COS – does it affect the direction of the propagation? 1. Yes 2. No

Making use of the mathematical description

$$y(x,t) = (0.9 \text{ cm}) \sin \left[(5.0 \text{ s}^{-1})t - (1.2 \text{ m}^{-1})x \right]$$

- 1) The amplitude is whatever is multiplying the sine.

$$A = 0.9 \text{ cm}$$

- 2) The wavenumber k is whatever is multiplying the x :

$k = 1.2 \text{ m}^{-1}$. The wavelength is:

$$\lambda = \frac{2\pi}{k} = 5.2 \text{ m}$$

3)

The angular frequency ω is whatever is multiplying the t .

$\omega = 5.0 \text{ rad/s}$. The frequency is:

$$f = \frac{\omega}{2\pi} = 0.80 \text{ Hz}$$

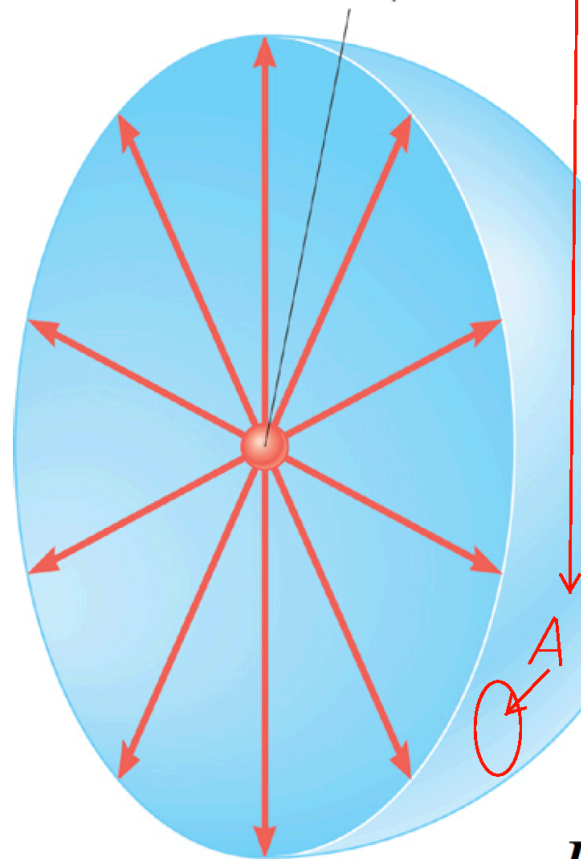
Sound Intensity

$$I_A = \frac{P_A}{A} = \frac{E_A/t}{A} = \frac{E_A}{At}$$

For a whole sphere:

power of sound source

Sound source at
center of sphere



$$I = \frac{P}{4\pi r^2}$$

the same
everywhere

$$I_A = I$$

area of sphere

$$E_A = I_A At = IAt = \frac{P}{4\pi r^2} At = \frac{PA t}{4\pi r^2}$$

For a 1000 Hz tone, the smallest sound intensity that the human ear can detect is about $1 \times 10^{-12} \text{ W/m}^2$. This intensity is called the **threshold of hearing**.

On the other extreme, continuous exposure to intensities greater than 1 W/m^2 can be painful.

The decibel scale

$$\beta = (10 \text{ dB}) \log_{10} \left(\frac{I}{I_0} \right) \quad I_0 = 1 \times 10^{-12} \text{ W/m}^2$$

where I is the intensity in W/m^2

Wave front

Wave front

General case!
Only 4 situations!!

Source

Observer

**Follow the
wave
fronts!!**

$$v_{SrelO} = \lambda_{SrelO} f_{SrelO}$$

$$v_{SrelO} = v_{SrelG} \pm v_{OrelG}$$

v
 λ

$$f_w = f_s \frac{1}{1 \mp \frac{v_s}{v}}$$

$-v_s \uparrow \uparrow v$

$+v_s \uparrow \downarrow v$

**Resting
observer.**

$$f_o = f_s \frac{1 \pm \frac{v_o}{v}}{1 \mp \frac{v_s}{v}}$$

General case!

$$f_o = f_w \frac{1 \pm \frac{v_o}{v}}{1}$$

$+v_o \uparrow \downarrow v$

$-v_o \uparrow \uparrow v$

Resting source.

Adding waves: the principle of superposition

When more than one wave is traveling in a medium the waves simply add.

The principle of superposition:

the net displacement of any point in the medium is the sum of the displacements at that point due to each wave individually.

$$y_1(x, t) + y_2(x, t) = y_{actual}(x, t)$$

Beats

The intensity of the sound oscillates from maximum to zero and back again continually. The closer the waves are in frequency, the slower the cycle of rising and falling intensity.

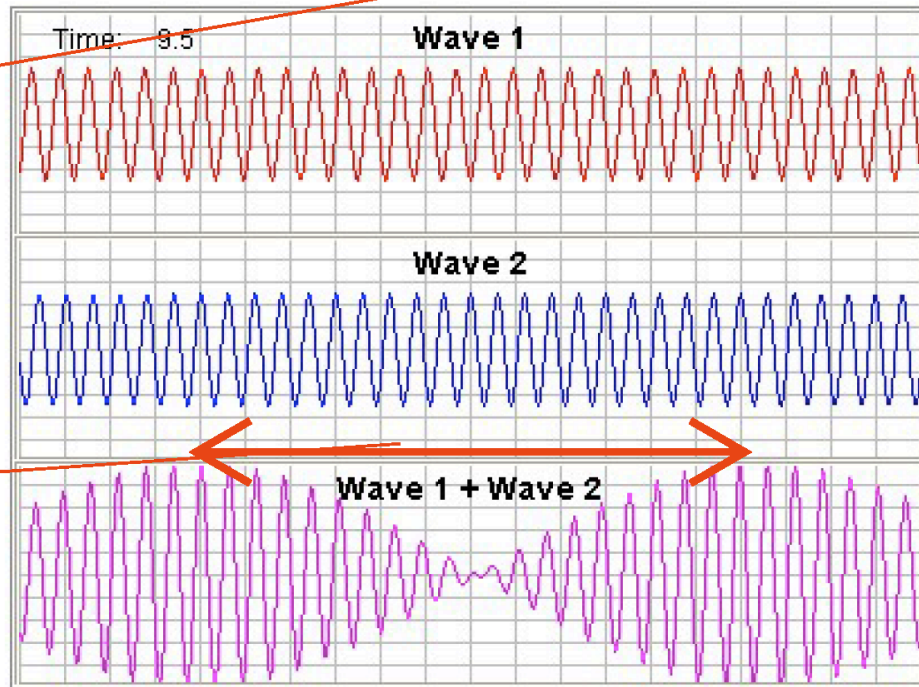
The frequency of the rising and falling is known as the **beat frequency**, which equals **the difference in frequency between the two waves**.

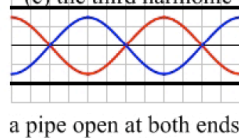
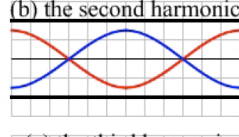
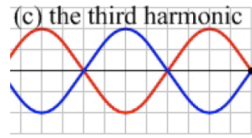
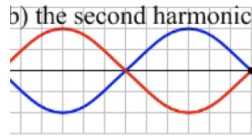
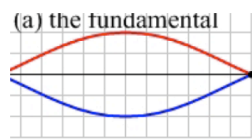
$$\omega_{\text{beat}} = |\omega_1 - \omega_2|$$

or

$$f_{\text{beat}} = |f_1 - f_2|$$

$$T_{\text{beat}} = 2\pi/\omega_{\text{beat}}$$





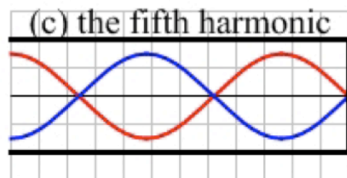
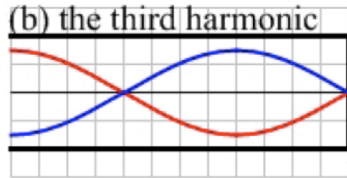
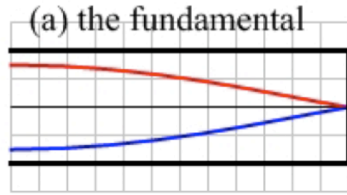
a pipe open at both ends

The string fixed at both ends (or the flexible rod with free ends; or the pipe open at both ends) of the length L has:

The fundamental (lowest) frequency: $f_1 = \frac{v}{2L}$

The fundamental (longest) wavelength: $\lambda_1 = 2L$

The harmonics: $f_n = \frac{nv}{2L}$ $\lambda_n = 2L/n$ $n = 2, 3, \dots$



a pipe closed at one end

The string fixed at one end (or the flexible rod with one free end; or the pipe open at one end only) of the length L has:

The fundamental (lowest) frequency: $f_1 = \frac{v}{4L}$

The fundamental (longest) wavelength: $\lambda_1 = 4L$

The harmonics: $f_n = \frac{nv}{4L}$ $\lambda_n = 4L/n$ $n = 3, 5, 7, \dots$