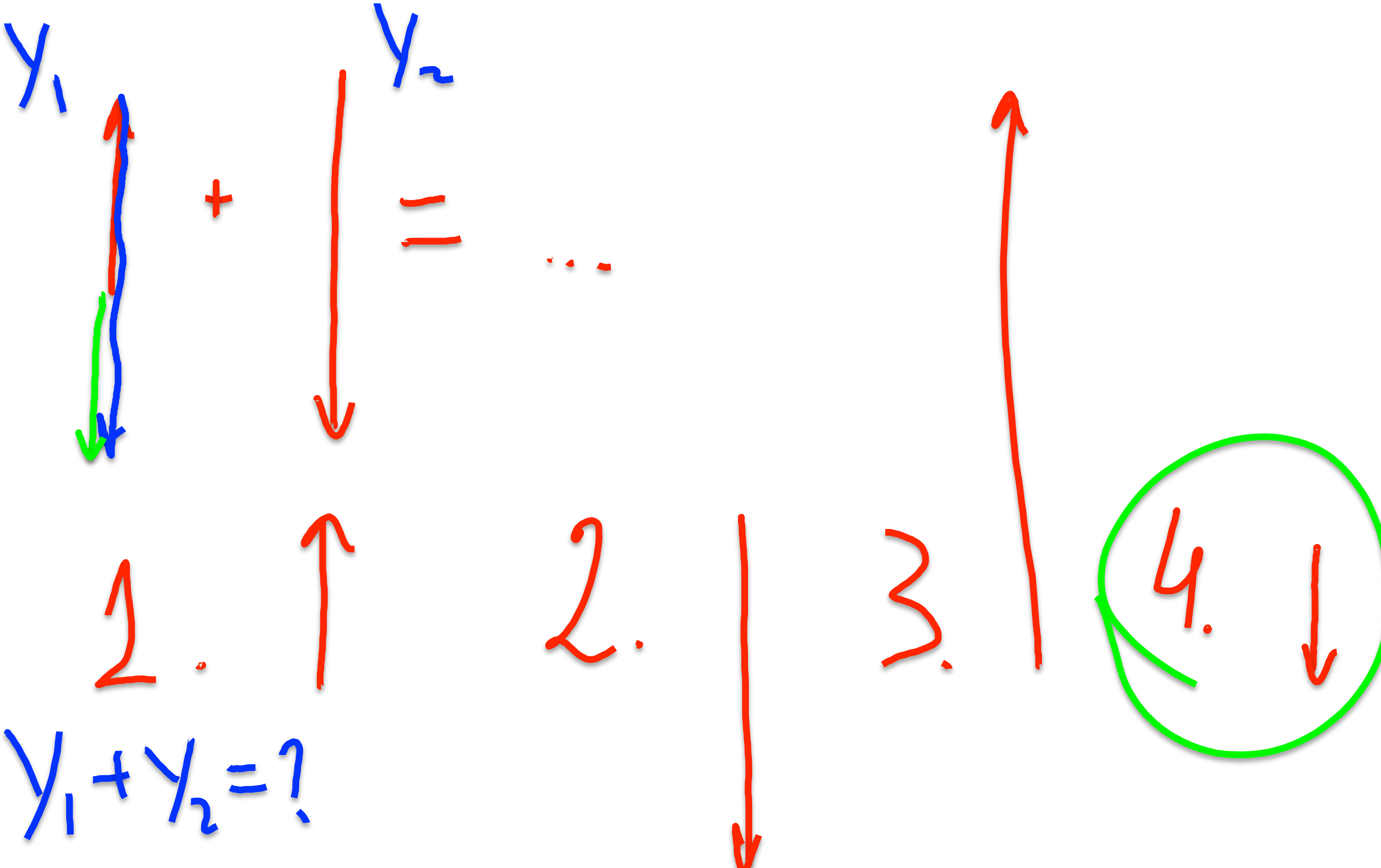


y_1
+
y_2
= ...
1.
2.
3.
4.
$y_1 + y_2 = ?$

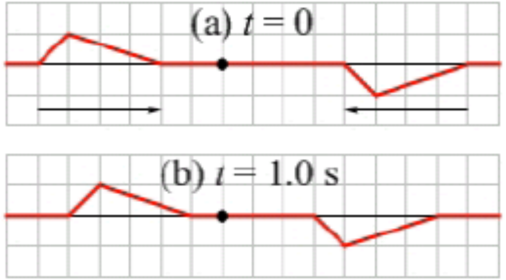
Adding waves: the principle of superposition

When more than one wave is traveling in a medium the waves simply add.

The principle of superposition:

the net displacement of any point in the medium is the sum of the displacements at that point due to each wave individually.

$$y_1(x,t) + y_2(x,t) = y_{actual}(x,t)$$



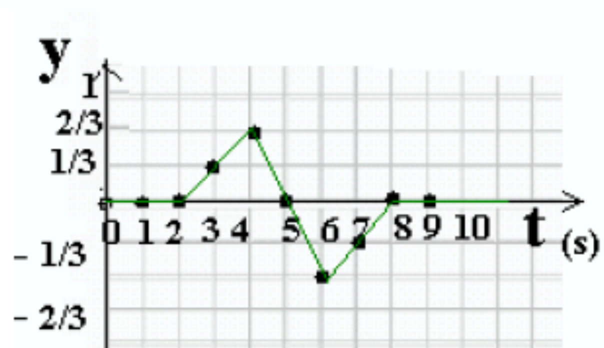
Two pulses are traveling along a string, as shown in the Figure. A particular point on the string is marked with a black dot. Plot the displacement of that point as a function of time, over the time interval $t = 0$ to $t = 8.0$ s.

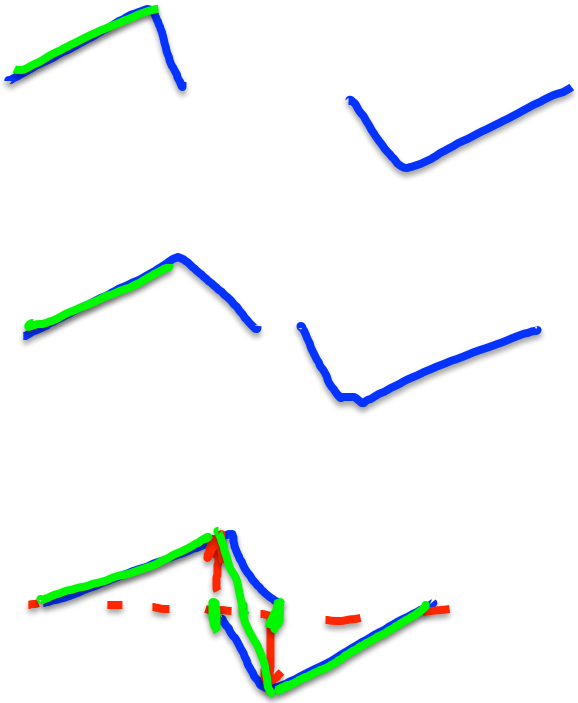
Let's the sides of the squares are equal to 1. Than at each moment we can mark all the displacements (y, "heights) for each pulse.

When the pulse overlap, the total displacement is the sum of the individual ones.

So, we can set up the table for the displacement At the black dot at different times

time	y_1	y_2	$y_{total} = y_1 + y_2$
0	0	0	0
1	0	0	0
2	0	0	0
3	1/3	0	1/3
4	2/3	0	2/3
5	1	-1	0
6	0	-2/3	-2/3
7	0	-1/3	-1/3
8	0	0	0
9	0	0	0

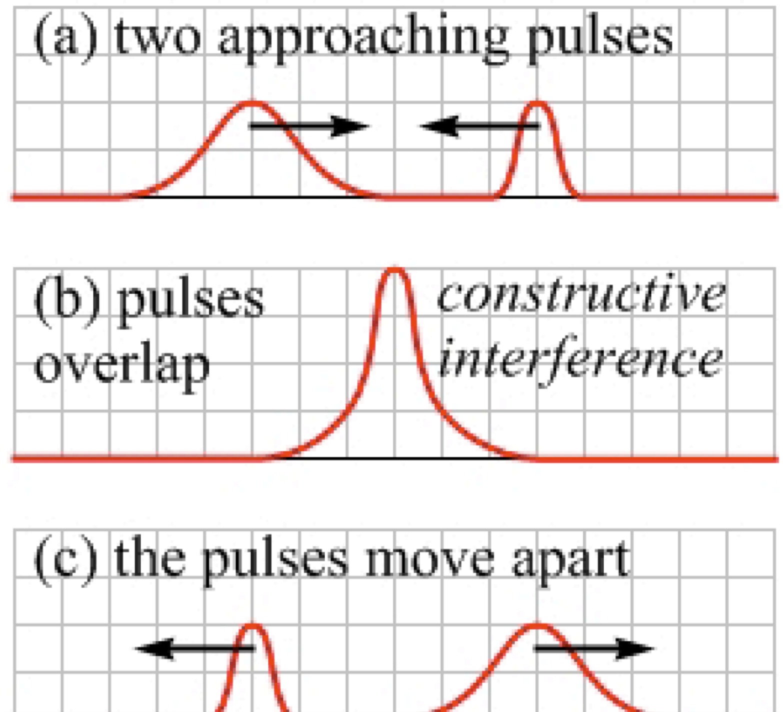




Superposition - Constructive interference

When the displacements of individual waves go in the same direction at a point, the result is a large amplitude there, because the displacements add. This is known as constructive interference. Simulation.

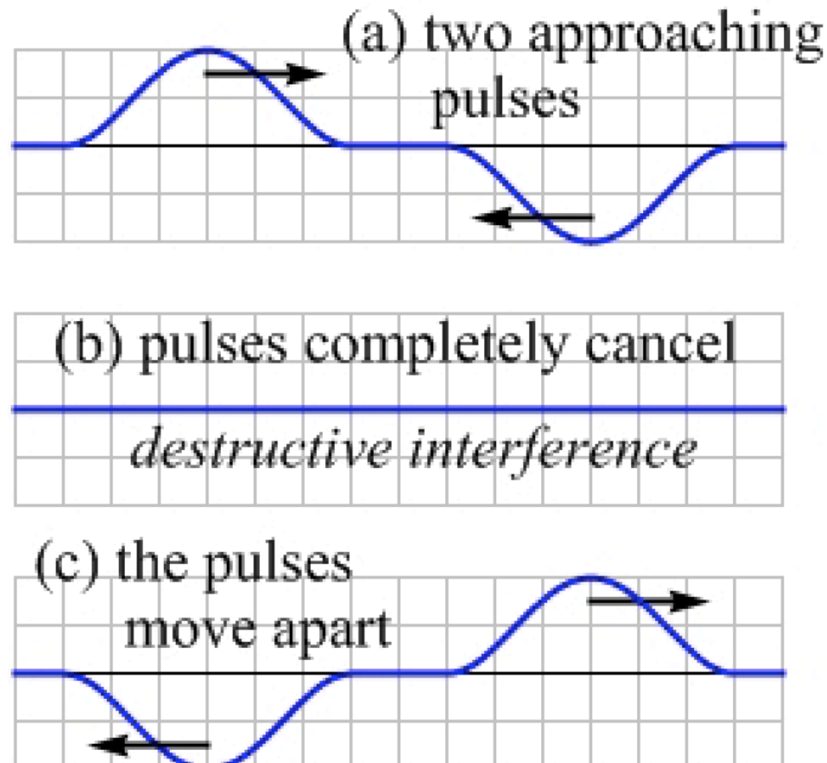
A neat feature of waves is that, after passing through one another, waves (or pulses) travel as if they had never met.

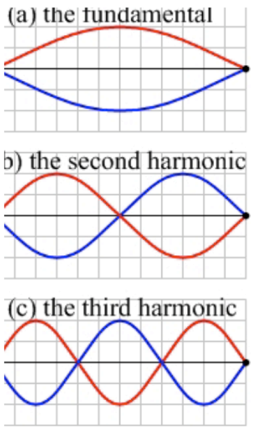


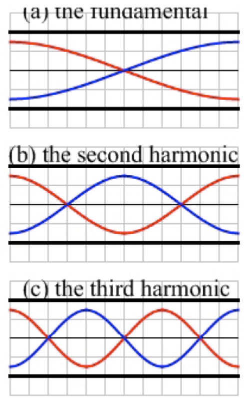
Superposition - Destructive interference

When the displacements of individual waves are in opposite directions at a point, the waves cancel (at least partly). This is known as destructive interference. Simulation.

How is it possible for the two pulses to re-emerge from the flat string? Where is the energy to do this?







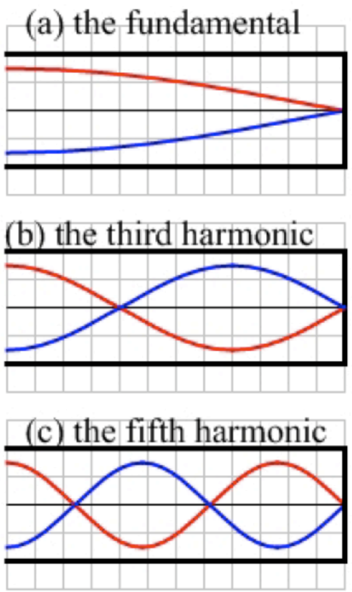
a pipe open at both ends

<u>The string fixed at both ends</u> (*or* the flexible rod with free ends; *or the pipe open at both ends*) of the length L has:

The fundamental (lowest) frequency: $f_1 = \dfrac{v}{2L}$

The fundamental (longest) wavelength: $\lambda_1 = 2L$

The harmonics: $f_n = \dfrac{nv}{2L}$ $\lambda_n = 2L/n$ $n = 2, 3, \ldots$



a pipe closed at one end

<u>The string fixed at one end</u> (*or* the flexible rod with one free end; *or the pipe open at one end only*) of the length L has:

The fundamental (lowest) frequency: $f_1 = \dfrac{v}{4L}$

The fundamental (longest) wavelength: $\lambda_1 = 4L$

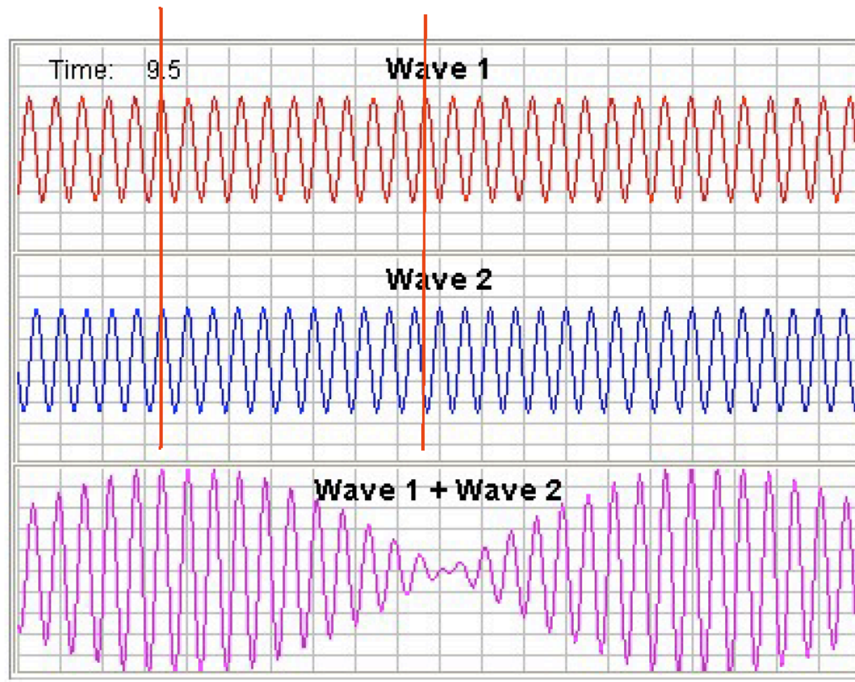
The harmonics: $f_n = \dfrac{nv}{4L}$ $\lambda_n = 4L/n$ $n = 3, 5, 7, \ldots$

Superposition – Beats

When you listen to two sound waves of similar frequency, you hear beats - the intensity of the sound rising and falling.

When the waves are exactly in phase with one another, constructive interference produces a loud sound.

Waves of different frequencies drift out of phase until completely destructive interference takes place and you hear nothing. The phase difference continues to grow and, the closer it gets to a full wavelength shift, the higher the intensity.



Beats

The intensity of the sound oscillates from maximum to zero and back again continually. The closer the waves are in frequency, the slower the cycle of rising and falling intensity.

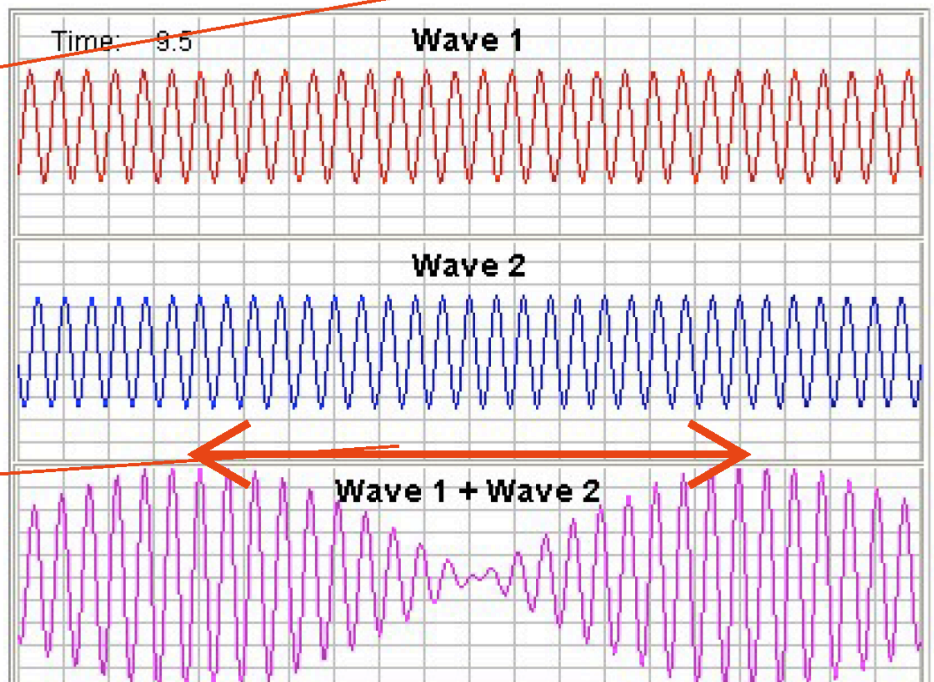
The frequency of the rising and falling is known as the beat frequency, which equals the difference in frequency between the two waves.

$\omega_{beat} = |\omega_1 - \omega_2|$

or

$f_{beat} = |f_1 - f_2|$

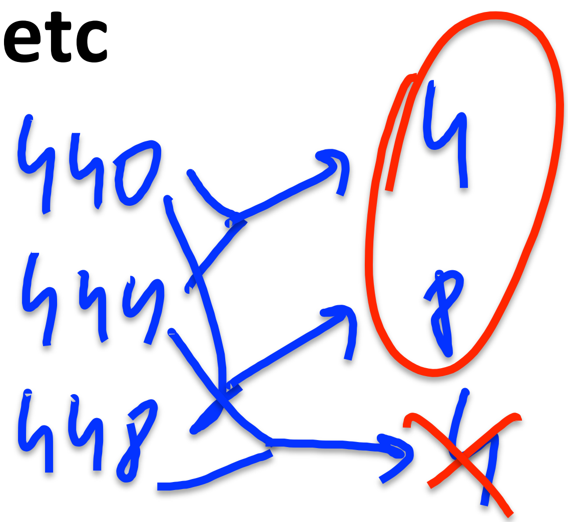
$T_{beat} = 2\pi/\omega_{beat}$



Home many different beat frequencies can you generate using three tuning forks (two at a time), with the natural frequencies of 440 Hz, 444 Hz, and 448 Hz?

$$f_{beat} = |f_1 - f_2|$$

1   2   3   4   etc



Standing waves: a string fixed at both ends

$$v = \sqrt{\frac{F_T}{m / L}}$$

All stringed musical instruments have strings fixed at both ends. When they are played, the sound you hear is some combination of the fundamental frequency and the different harmonics - it's because the harmonics are included that the sound sounds musical. A pure sine wave does not sound nearly so nice.

Standing waves

When identical waves travel in opposite directions in a medium, the result is a standing wave - a wave that does not travel one way or the other.

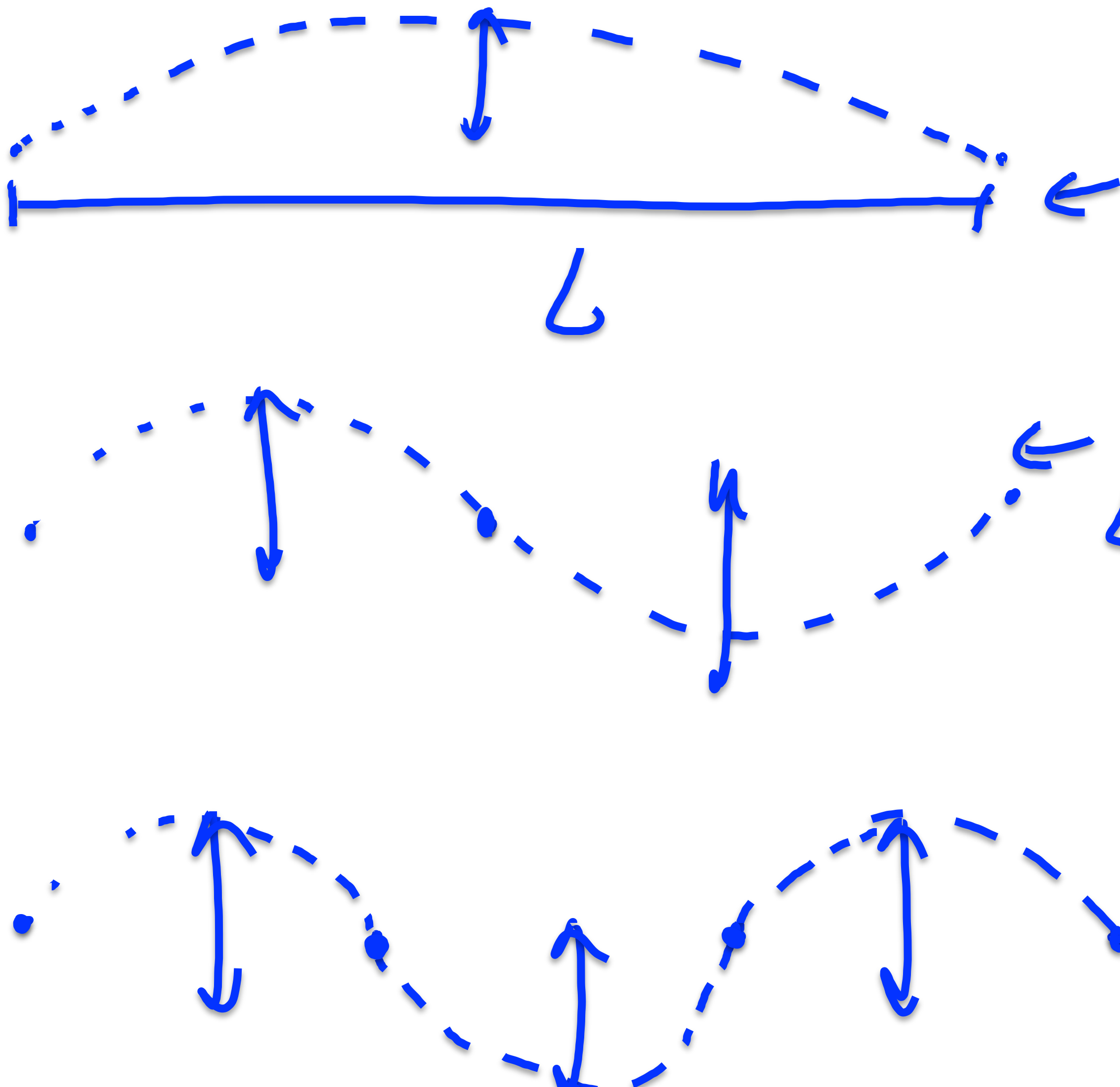
The equations for the waves are:

$y_1 = A \sin(kx - \omega t)$ and $y_2 = A \sin(kx + \omega t)$

The sum can be written as:

$y = 2A \sin(kx) \cos(\omega t)$

The picture shows an instant when the string is in equilibrium



$L = 1 \cdot \frac{\lambda}{2}$ $\quad \frac{\lambda_1}{2}$

$L = 2 \cdot \frac{\lambda_2}{2}$

$L = 3 \cdot \frac{\lambda_3}{2}$

$$L = n \cdot \frac{\lambda_n}{2}$$

$$\lambda_n \qquad n, L \; ; \qquad v = f_n \cdot \lambda_n \; ; \quad v = \sqrt{\frac{T}{\mu}}$$

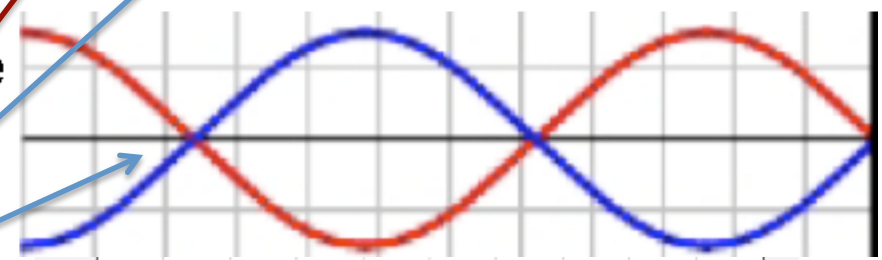
$$f_n = \frac{v}{\lambda_n} = \frac{v}{2L} \cdot n = f_1 \cdot n$$

Standing waves

$y = 2A \sin(kx) \cos(\omega t)$

This is not a traveling wave, because the spatial part is separated from the time part. The string is totally flat at certain instants in time, and there are certain positions where the amplitude is always zero - these points are called nodes.

There are other points halfway between the nodes where the amplitude is maximum - these are the anti-nodes.



Standing waves: a string fixed at both ends

constructive interference takes place only when the wavelength is related to the length L of the string by:

$$n\frac{\lambda}{2} = L \quad \text{where } n = 1, 2, 3, \ldots$$

Using $f = \dfrac{v}{\lambda}$, the corresponding frequencies are:

$$f_n = \frac{nv}{2L} \text{, where } n = 1, 2, 3, \ldots$$

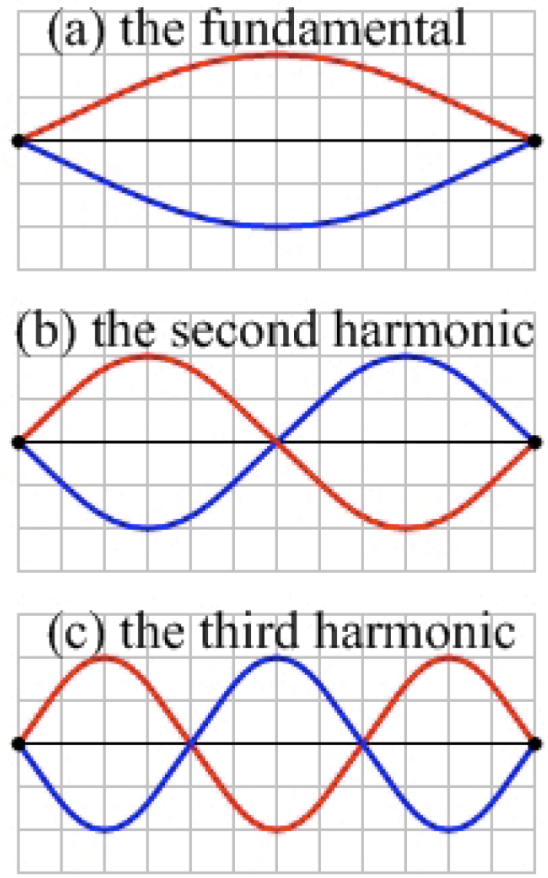
$$f_n = nf_1; \quad f_1 = \frac{v}{2L}$$

Standing waves: a string fixed at both ends

or a rod open

$$n\frac{\lambda}{2} = L \qquad f_n = \frac{nv}{2L}$$

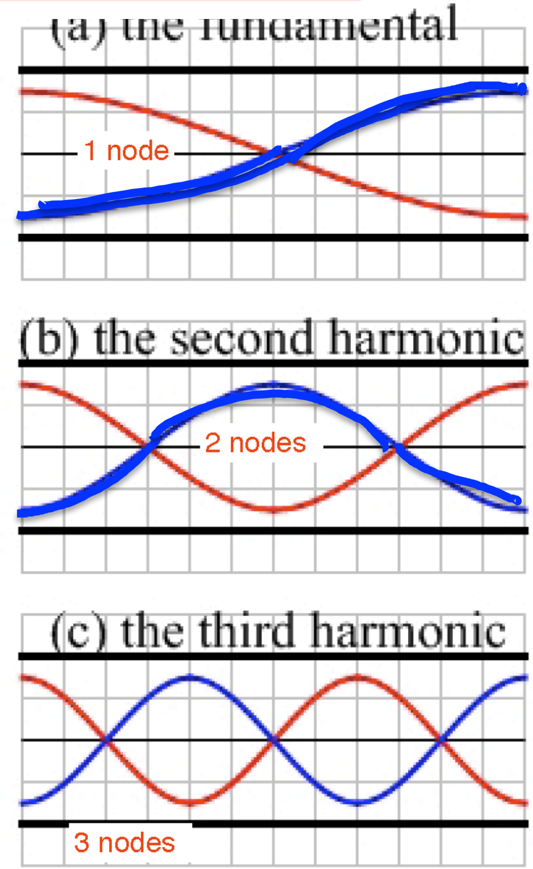
The lowest resonance frequency (n = 1) is known as the fundamental frequency for the string. All the higher frequencies are known as harmonics - these are integer multiples of the fundamental frequency.



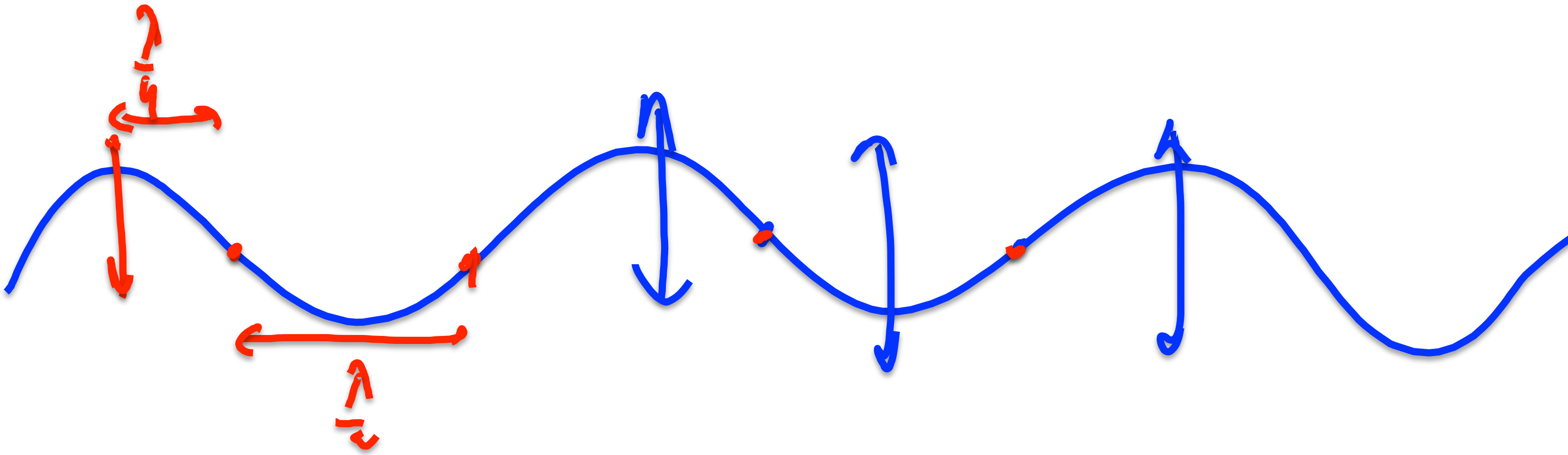
Standing waves: a tube open at both ends

or a rod open

For a tube open at both ends, reflections of the sound at both ends produce a large-amplitude wave for particular resonance frequencies. For the standing waves, an open end is an anti-node (maximum amplitude point) for displacement.



a pipe open at both ends


λ/4
λ/2

Standing waves: a tube closed at one end only

For a tube closed at one end, the closed end is a node (zero displacement) while the open end is an anti-node (maximum displacement). This leads to a different equation for the resonance frequencies.

odd!

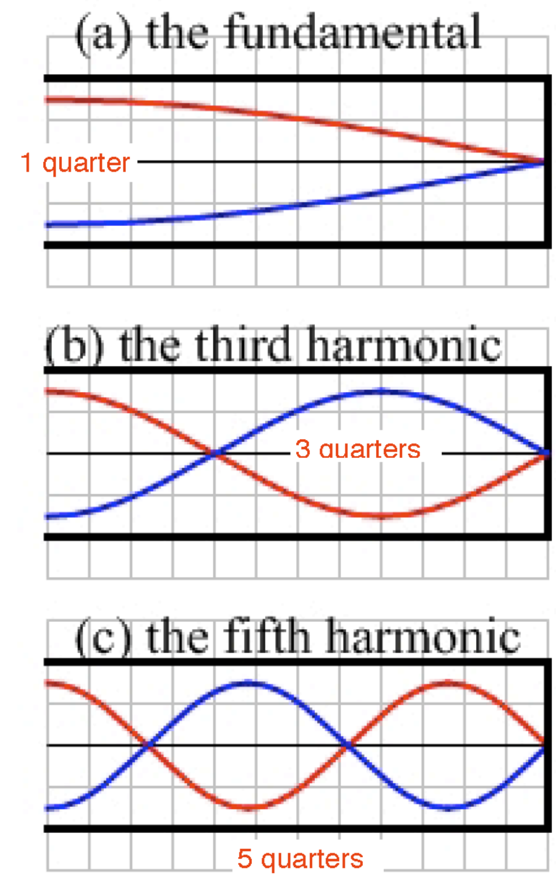
$$f_n = \frac{nv}{4L} = nf_1$$

<=> the string with one fixed end !

only be odd integers

Simulation: transverse

Simulation: longitudinal

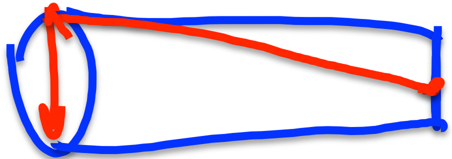


a pipe closed at one end

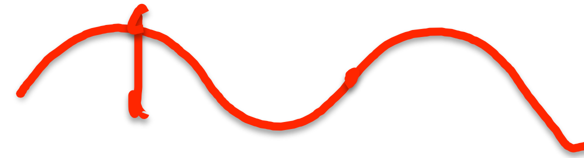
$1 \cdot \frac{\lambda_1}{4}$

$3 \cdot \frac{\lambda_3}{4}$

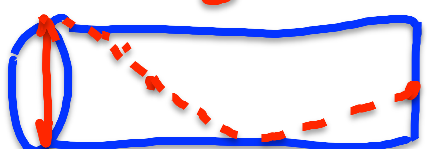
$5 \cdot \frac{\lambda_5}{4}$



$$L = 1 \cdot \frac{\lambda_1}{4}$$



L

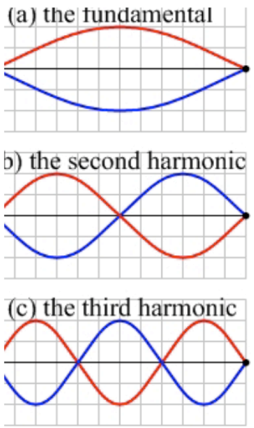


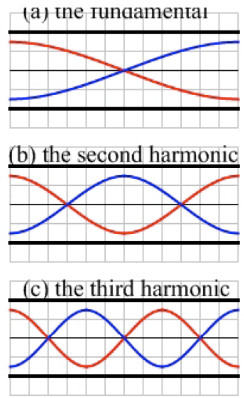
$$L = \frac{\lambda_3}{4} + \frac{\lambda_3}{2} = 3 \cdot \frac{\lambda_3}{4}$$

$$L = n \cdot \frac{\lambda_n}{4} \; ; \quad \lambda_n = \frac{L \cdot 4}{n} \; ; \quad v = f_n \cdot \lambda_n$$

$n \to$ odd

$$f_n = \frac{v}{4L} \cdot n \; ; \quad n \to \text{odd}$$





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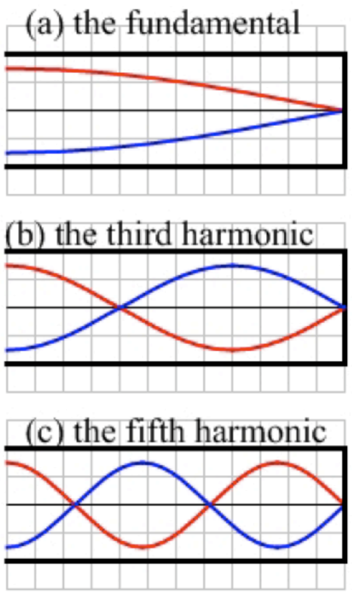
The harmonics: $f_n = \frac{nv}{2L}$ $\lambda_n = 2L/n$ $n = 2, 3, \ldots$



a pipe closed at one end

The string fixed at one end (*or* the flexible rod with one free end; *or the pipe open at one end only*) of the length L has:

The fundamental (lowest) frequency: $f_1 = \frac{v}{4L}$

The fundamental (longest) wavelength: $\lambda_1 = 4L$

The harmonics: $f_n = \frac{nv}{4L}$ $\lambda_n = 4L/n$ $n = 3, 5, 7, \ldots$

Draw below a pictorial representation for several harmonics of standing waves excited in a pipe opened *at both ends*.

You have an open pipe and generate several consecutive harmonics with frequencies 210 Hz, 280 Hz, and 350 Hz. Find the 9th harmonic. If the speed of the sound is 340 m/s, find the length of the pipe. If you take a twice shorter pipe, what will be its fundamental?

$$f_n = \frac{nv}{2L} = nf_1$$

$f_9 = 9 \cdot f_1$

n $\quad$ $n+1$ $\quad$ $n+2$

$n \cdot f_1 = 210$ $\quad$ $(n+1) f_1 = 280$

1. 210 $\quad$ 2. 280 $\quad$ 3. 490 $\quad$ 4. 70 $\quad$ Hz

$$n \cdot f_1 = 210$$

$$(n+1) f_n = 280$$

$$n \cdot f_1 + 1 \cdot f_1 = 280$$

$$210 + f_1 = 280 \; ; \; f_1 = 280 - 210 = 70 \text{ Hz}$$

$$f_n = 9 \cdot 70 = 630 \text{ Hz}$$

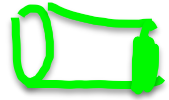
Draw below a pictorial representation for several harmonics of standing waves excited in a pipe closed *on one end.*

You have a pipe with one end closed and generate several consecutive harmonics with frequencies 210 Hz, 280 Hz . d

Find the 9th harmonic. If the speed of the sound is 340 m/s, find the length of the pipe. If you take a twice shorter pipe, what will be its fundamental?

$$f_n = \frac{nv}{4L} = nf_1$$

f_9 1. 315 2. 1.214 3. 140 4. 630



$f_n = f_1 \cdot n$ → odd

1. yes

2. No

210 Hz → ej

280 Hz → ej+2

$210 = f_{ej} = f_1 \cdot ej$; $f_{ej+2} = f_1 \cdot (ej+2) = 280 =$

$= f_1 \cdot ej + f_1 \cdot 2 = 210 + 2 \cdot f_1$

$f_1 = 35$

$f_9 = 315\ Hz$