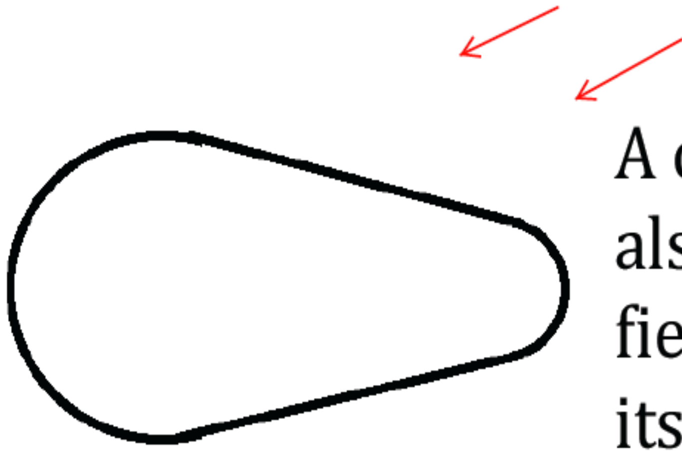
Conductors in electric field

Conductor	**dielectric**
Has many free charges (electrons in solids) easily moving inside it	**Does not have free charges, charges a tightly bond together and can by only slightly shifted or turned around**

When a conductor is immersed into an electric field, the Coulomb's force starts acting and electrons start moving until they reach equilibrium position (it happens very fast!).



A conductor is charged and also in an external electric field. What can we say about its electric field and potential inside the conductor?

1. Electric field is zero and potential is zero
2. Electric field is equal to some non-zero constant value and potential is zero
3. Electric field is zero and potential is equal to some non-zero constant value
4. Both are equal to some non-zero constant value

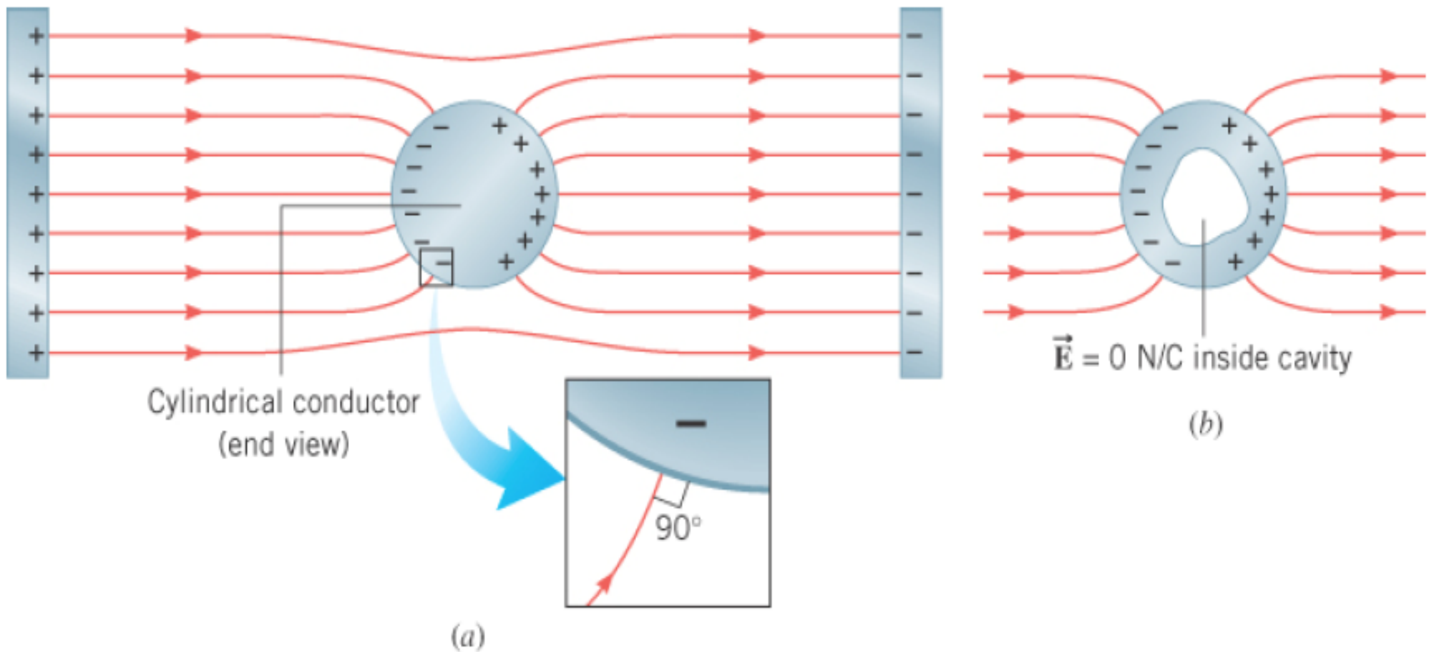
When a conductor is in equilibrium, what is the direction of the electric field at its surface?

1. Parallel to the surface

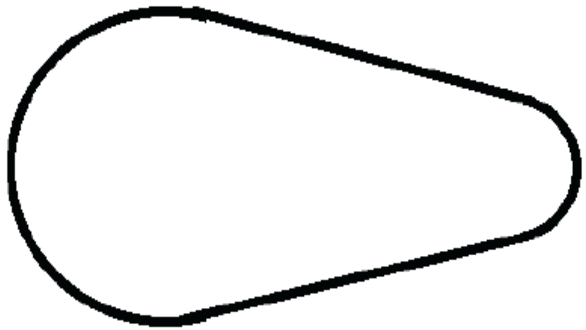
2. Perpendicular to the surface

3. The direction depends on the shape of the conductor

The Electric Field Inside a Conductor: Shielding



The electric field just outside the surface of a conductor is perpendicular to the surface at equilibrium under electrostatic conditions.



A conductor in the picture is charged and in equilibrium.

What can we say about its electric potential on the surface?

1. It is the highest at its wider end

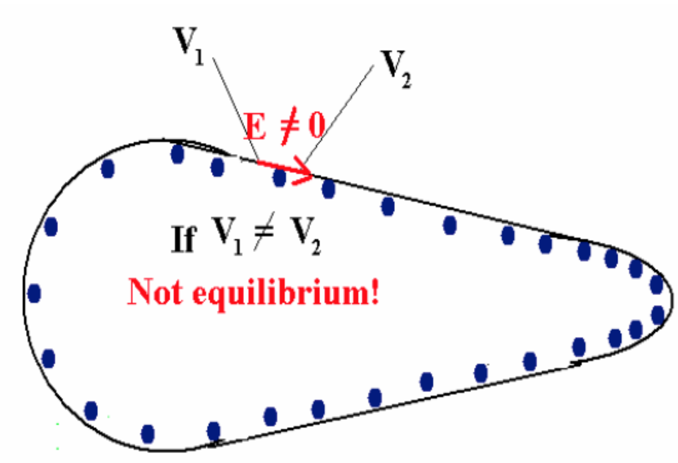
2. It is the highest at its sharper end

3. It is the same everywhere

a conductor in an electric field

The theorem: the outer surface of a conductor has the same potential at any point of it (i.e. it is an equipotential surface!)

The proof: if it were not true, electrons would be moving, but they are not (equilibrium!)



If potential were not the same,

$$V_1 \neq V_2$$

The potential difference would not be 0!

$$|\Delta V| \neq 0$$

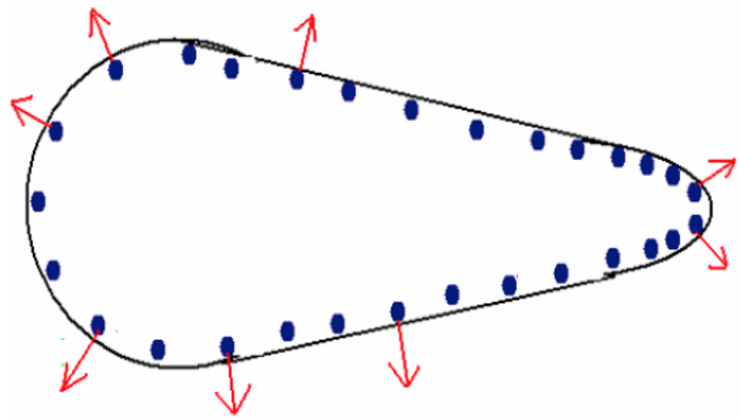
The electric field would not be 0!

$$E = |\Delta V|/d \neq 0$$

The force on electrons would not be 0! $F = eE \neq 0$,

Electrons would be moving.

Charging a conductor



If a conductor is charged with the charge |Q|, it has some potential |V|.

The voltage on the conductor is directly proportional to the charge: $|V| = const_*|Q|$

but usually this relationship between the charge and the potential is written as:

$$|Q| = C_*|V|$$

The coefficient C depends on the shape and size of the conductor and is called *a capacitance* (*the unit* of capacitance is *farad* (F))

Two different metal balls carry charges of +3 μC and -12 μC. They are 3 m apart. We brought the charges together and then placed again at the same distance. We see that a bigger sphere has the radius four times of the radius of the smaller sphere. (a) Calculate the charge on each ball after they has been brought in a contact. (b) If both charges are positive (or negative), calculate the new charges on each ball.

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$$Q_{1i} + Q_{2i} = Q_{1f} + Q_{2f}$$

$$Q_{1i} = C_1 V_{1i} \qquad Q_{1f} = C_1 V_{1f}$$

$$Q_{2i} = C_2 V_{2i} \qquad Q_{2f} = C_2 V_{2f}$$

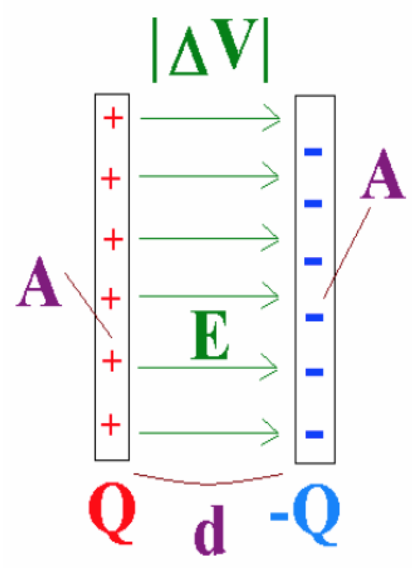
$$R_2 = 4R_1 \quad => \quad C_2 = 4C_1$$

$$=> \quad Q_{2f} = 4Q_{1f}$$

$$V_{2f} = V_{1f} \; (!one\, conductor!)$$

$$3\mu C + -12\mu C = Q_{1f} + 4Q_{1f}$$

A parallel-plate capacitor



For a capacitor storing charge Q, one conductor has a charge of $+Q$ and the other has a charge of $-Q$.

A parallel-plate capacitor is a pair of identical conducting plates, each of area A, placed parallel to one another and separated by a distance d.

With nothing between the plates, the capacitance is

$$C = \frac{\varepsilon_0 A}{d}$$

$$\varepsilon_0 = \frac{1}{4\pi k} = 8.85\times 10^{-12}\ \mathrm{C^2/(N\ m^2)}$$

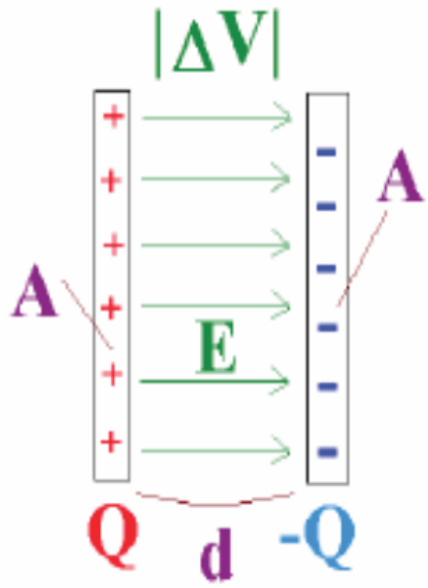
ε_0 is called *the permittivity of free space.*

For a capacitor with a charge of $+Q$ on one plate and $-Q$ on the other:

$$|Q| = C\,|\Delta V|$$

$$E_{CAP\_VAC} = \frac{Q}{\varepsilon_0 A}$$

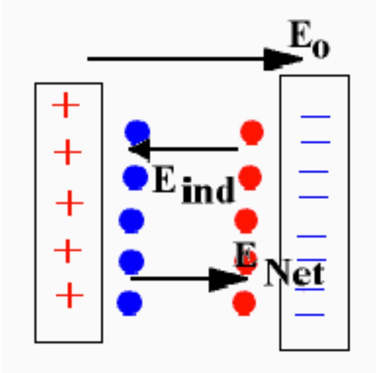
And, we can add another equation: $|\Delta V| = E * d$



When we insert a dielectric between the plates, the electric field inside the capacitor

1. does not change
2. increases
3. decreases

A capacitor with a dialectric



Completely filling the space between capacitor plates with a dielectric increases the capacitance by a factor of the dielectric constant:

$$C = \kappa C_o$$

where C_o is the capacitance with no dielectric between the plates.

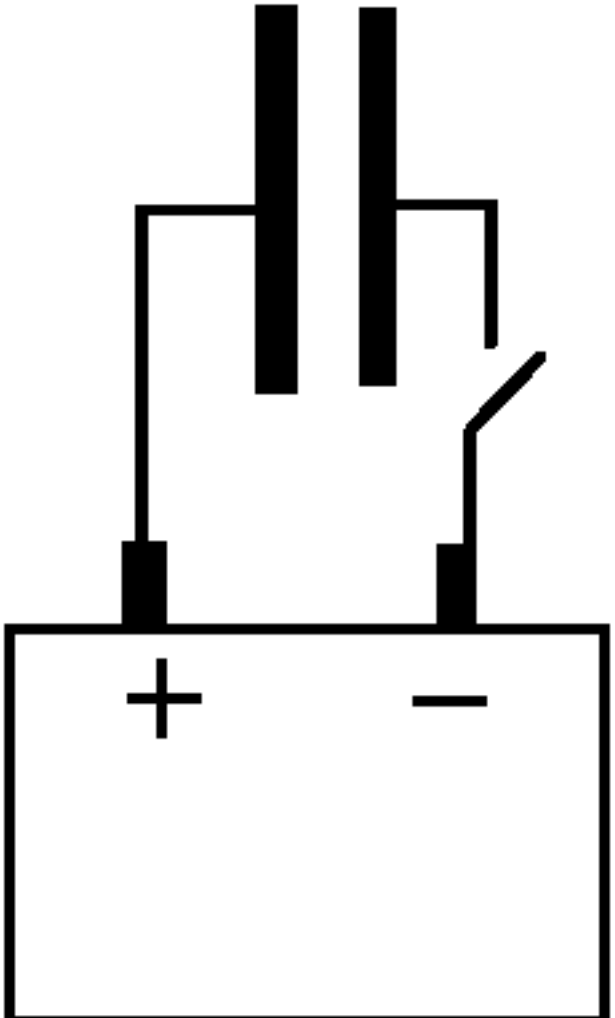
For a parallel-plate capacitor containing a dielectric that completely fills the space between the plates, the capacitance is given by:

$$\kappa = \frac{E_o}{E_{Net}} > 1$$

$$C = \frac{\kappa \varepsilon_o A}{d}$$

$$E_{CAP} = \frac{Q}{\kappa \varepsilon_0 A}$$

Capacitance increases as the dielectric constant and the plate area increase, and as the distance between the plates decreases.



After we close the switch the capacitor is getting charged.

Does the battery do any work?

1. Yes
2. No

Energy in a capacitor

Let's charge a capacitor (see SIM)

When we move a single charge q through a potential difference Δv, its potential energy changes by $q\,\Delta v$.

To charge a capacitor we move a *large number* of charges from one plate to another. With every new charge moved over, the potential between the plates changes!

If ΔV is the *final* potential difference on the capacitor, and Q is the *magnitude* of the *final* charge on each plate, the *energy* stored in the capacitor is:

$$U = \frac{1}{2} Q\,\Delta V$$

Using $Q = C\,\Delta V$, the energy stored in a capacitor can be written as:

$$U = \frac{1}{2} Q\,\Delta V = \frac{1}{2} C (\Delta V)^2 = \frac{Q^2}{2C}$$

These expressions *do not depend* on the construction of a capacitor, they are true no matter if there is a dielectric in it or not.

Calculate the capacitance of a capacitor consisting of two parallel plates separated by a layer of paraffin wax 1 cm thick, the area of each plate being 320 cm^2. The dielectric constant for the wax is 3.0. If the capacitor is connected to a 100-V source, calculate the charge on the capacitor and the energy stored in the capacitor.

Capacitance: 1. 14 pF 2. 28 pF 3. 56 pF 4. 84 pF

Charge: 1. 1.4 nC 2. 2.8 nC 3. 5.6 nC 4. 8.4 nC

Energy: 1. 0.14 µJ 2. 0.42 µJ 3. 0.84 µJ 4. 1.26 µJ

Our basic capacitor equations are: $|Q| = C\,|\Delta V|$

and, for a parallel-plate capacitor, $C = \frac{\varepsilon_0 A}{d}$

The parallel-plate equation applies to a capacitor with vacuum (air is close enough) between the plates.

Double the charge on each plate. The capacitance ...

1. Increases
2. Decreases
3. Stays the same

Based on $|Q| = C\,|\Delta V|$,

what happens to C when $|Q|$ increases?

Nothing!!! C does not depend on the charge; the value of C is based on the construction of the capacitor only!

(no |Q| in this equation!)

$$C = \frac{\varepsilon_0 A}{d}$$

changing Q with no changing A and d means changing ΔV

Our basic capacitor equations are: $|Q| = C\,|\Delta V|$

and, for a parallel-plate capacitor, $C = \dfrac{\varepsilon_0 A}{d}$

The parallel-plate equation applies to a capacitor with vacuum (air is close enough) between the plates.

Increase the area of each plate. The capacitance ...

1. Increases
2. Decreases
3. Stays the same

Change?

Capacitance is proportional to area, so increasing area increases capacitance.

$$\nearrow C = \frac{\varepsilon_0 A \nearrow}{d}$$

$$C = \frac{\kappa \varepsilon_0 A}{d}$$

A capacitor with wax between its plates has a capacitance 8.4 pF. What is its capacitance when wax is taken out from the capacitor? The dielectric constant for wax is 2.8.

1. 1.0 pF **2. 2.0 pF** **3. 3.0 pF** **4. 4.0 pF** **5. 5.0 pF**

Take a parallel-plate capacitor and shorten its plates by a wire.

Remove the wire, then connect the capacitor to a power supply.

A) Right after you remove the wire …

B) Right after you connect the power supply …

1. *C* decreases, *Q* decreases, and *V* stays the same
2. *C* decreases, *Q* increases, and *V* increases
3. *C* decreases, *Q* stays the same, and *V* increases
4. All three decrease
5. None of the above

Take a parallel-plate capacitor and connect it to a power supply.

Then disconnect the capacitor from the power supply.

Immediately after you remove the wires …

1. C decreases, Q decreases, and V stays the same

2. C decreases, Q increases, and V increases

3. C decreases, Q stays the same, and V increases

4. All three decrease

5. None of the above

Take a parallel-plate capacitor and connect it to a power supply. Then disconnect the capacitor from the power supply. After this, the distance between the plates is increased.

When this occurs, what happens to C, Q, and ΔV?

1. C decreases, Q decreases, and ΔV stays the same
2. C decreases, Q increases, and ΔV increases
3. C decreases, Q stays the same, and ΔV increases
4. All three decrease
5. None of the above

$$|Q| = |C|\,|\Delta V|$$

$$C = \frac{\varepsilon_0 A}{d}$$

Because the charge is stranded on the capacitor plates, the charge *cannot* change. $|Q| = \text{const}$

Moving the plates further apart decreases the capacitance, because: $$\downarrow C = \frac{\varepsilon_0 A}{d \uparrow}$$

To see what happens to the potential difference, look at

$$|Q| = C\,|\Delta V| \quad \text{or} \quad |\Delta V| = |Q|/C$$

Decreasing C while keeping the charge the same means that the potential difference $|\Delta V|$ *increases*.

We can also get that from $|\Delta V| = Ed$. The field E stays the same, d increases, so does $|\Delta V|$.

Take a parallel-plate capacitor and connect it to a power supply. Then *disconnect* the capacitor from the power supply.

After this, the area of the plates is doubled.

When this occurs, what happens to *C*, *Q*, and *V*?

1. C halves, Q halves, and V stays the same
2. C halves, Q doubles, and V doubles
3. C halves, Q stays the same, and V doubles
4. All three halves
5. None of the above

$$|Q| = |C||\Delta V|$$

$$C = \frac{\varepsilon_0 A}{d}$$

Playing with a capacitor

Let's Take a parallel-plate capacitor and connect it to a power supply. The power supply sets the potential difference between the plates of the capacitor.

While the capacitor is still connected to the power supply, the distance between the plates is *increased*.

When this occurs, what happens to C, $|Q|$, and $|\Delta V|$?

1. C decreases, $|Q|$ decreases, and $|\Delta V|$ stays the same
2. C decreases, $|Q|$ increases, and $|\Delta V|$ increases
3. C decreases, $|Q|$ stays the same, and $|\Delta V|$ increases
4. All three decrease
5. None of the above

$$|Q| = C\,|\Delta V|$$

$$C = \frac{\varepsilon_0 A}{d}$$

Playing with a capacitor

Does anything stay the same?

Because the capacitor is still connected to the power supply, the potential difference can't change.

$$|\Delta V| = \text{constant}$$

Moving the plates *further* apart *decreases* the capacitance, because:

$$\downarrow C = \frac{\varepsilon_0 A}{d \uparrow}$$

To see what happens to the charge, look at

$$|Q| = C\,|\Delta V|\ .$$

Decreasing C *decreases* the charge stored on the capacitor (with the capacitor *connected*!).

Playing with a Dielectric

Consider a parallel plate capacitor that is *connected* to a power supply.

When a piece of dielectric is inserted between the plates

What happens to the charge on the plates? (The dielectric constant k is larger than 1)

1 The charge increases

2 The charge decreases

3 The charge stays the same

Playing with a Dielectric

When a capacitor is connected to a battery *nothing happens to the potential difference* across the plates (the potential difference is defined by the battery the capacitor is connected to no matter what we do!).
Inserting the piece of dielectric increases the capacitance, so with V = const the charge increases.

$\kappa\uparrow \quad \Rightarrow \quad C\uparrow \quad$ (but $|\Delta V| = \text{const}$) $\quad Q = C\,|\Delta V| \quad \Rightarrow \quad Q\uparrow$

A. The charge increases

This is one reason why most capacitors have dielectric material between the plates, so they can store more charge.

Playing with a Dielectric (again)

Consider now a capacitor that is charged by connecting it to a power supply.

The connections are *removed*, and then a piece of dielectric is inserted between the plates. Which of the following is true?

1. The charge on the plates increases, as does the potential difference.
2. The charge on the plates increases, while the potential difference stays constant.
3. The charge on the plates stays the same, while the potential difference increases.
4. The charge on the plates stays the same, while the potential difference decreases.
5. Neither the charge nor the potential difference changes because the capacitor has been disconnected from the power supply.

Playing with a Dielectric (again)

The charge *can't* change, because it is stranded on the plates when we remove the connections to the power supply.

The capacitance has increased, so the potential difference must decrease.

$$\kappa\uparrow \quad \Rightarrow \quad C\uparrow \quad (\text{but } Q = \text{const}) \quad |\Delta V| = Q/C \quad \Rightarrow \quad |\Delta V|\downarrow$$

D. The charge on the plates stays the same, while the potential difference decreases.

Another argument is that the potential difference is proportional to the field between the plates.

$$|\Delta V| = E_{Net*}d$$

$$Q = \text{const},\ d = \text{const}, \quad \kappa\uparrow \quad \Rightarrow \quad E_{Net}\downarrow \Rightarrow \quad |\Delta V|\downarrow$$

Inserting a dielectric decreases the field, so the potential difference decreases.

A capacitor with wax between its plates has a capacitance 8.4 pF. What is its capacitance when wax is taken out from the capacitor? The dielectric constant for wax is 2.8.

1. 1.0 pF   2. 2.0 pF   3. 3.0 pF   4. 4.0 pF   5. 5.0 pF

Energy and dielectrics

The energy stored in a capacitor is still given by:

$$U = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2 = \frac{Q^2}{2C}$$

Consider a capacitor with nothing between the plates. The capacitor is charged by connecting it to a battery, but the connections to the battery are then removed.

When a dielectric is added to the capacitor, what happens to the stored energy?

1. The energy increases
2. The energy decreases
3. Energy is conserved! The energy stays the same.

Energy and dielectrics

Consider a capacitor with nothing between the plates. The capacitor is charged, and isolated so the charge on the plates is constant.

$$Q = \text{const}$$

Inserting a dielectric increases the capacitance, reducing the energy stored in the capacitor.

$$U = \frac{Q^2}{2C}$$

$$\kappa\uparrow \quad \Rightarrow \quad C\uparrow \quad \Rightarrow \quad U\downarrow$$

Where does the energy go?

A capacitor with wax between its plates has a capacitance 8.4 pF. It is connected to a battery. What happens to the electric field (or C, V, Q, U) between the plates when wax is taken out from the capacitor? (a) The capacitor remains connected to the battery. (b) The capacitor gets disconnected from the battery before the wax is pulled out. The dielectric constant for wax is 2.

1. E halves **2. E does not change** **3. E doubles**

$$C = \frac{\kappa \varepsilon_0 A}{d}$$

$$|\Delta V| = E * d$$

problem solving tips

Know the connections: $Q = C\,|\Delta V|$ $\quad |\Delta V| = E_{Net} * d$

$C = \kappa\, C_o$ (for vacuum or air k = 1) $\quad \varepsilon_0 = \dfrac{1}{4\pi k} = 8.85\times 10^{-12}\ C^2/(N\ m^2)$

$$C = \frac{\kappa\, \varepsilon_0\, A}{d} \qquad U = \frac{Q\,|\Delta V|}{2} = \frac{C\,\Delta V^2}{2} = \frac{Q^2}{2C} \qquad \kappa = \frac{E_o}{E_{Net}}$$

Know what is *not* changing: $|\Delta V|$ = const when the plates of a capacitor are *connected* from the battery; Q = const when the plates of a capacitor are *disconnected* from the battery (no matter what else is happening).

Know what is changing: if d ↓; or A ↑; or k ↑ => C ↑

Set variables before and after:

Before	After
Q, C, $\lvert\Delta V\rvert$, E_0, E_{Net}, d, A, C, k, U	Q, C, $\lvert\Delta V\rvert$, E_0, E_{Net}, d, A, C, k, U

Use the equations with a fewer changing variables: for example, if you know that Q = const, use $U = Q^2/(2C)$

Analyzing a capacitor

	d										A										κ									
	↑ (increasing)					↓ (decreasing)					↑ (increasing)					↓ (decreasing)					↑ (increasing/ insert)					↓ (decreasing/out)				
	Q	V	C	E	U	Q	V	C	E	U	Q	V	C	E	U	Q	V	C	E	U	Q	V	C	E	U	Q	V	C	E	U
battery remains connected																														
Battery is disconnected																														

$$Q = CV \qquad E = \frac{Q}{\kappa\varepsilon_0 A} \qquad E = \frac{V}{d} \qquad C = \frac{\kappa\varepsilon_0 A}{d} \qquad U = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C}$$

When analyzing a capacitor:

1. State, what definitely does NOT change.
2. Use an expression with only TWO changing variables, one which change is already known, another, which change you are trying to figure out.

Analyzing a capacitor

	d										A										κ									
	↑ (increasing)					↓ (decreasing)					↑ (increasing)					↓ (decreasing)					↑ (increasing/ insert)					↓ (decreasing/out)				
	Q	V	C	E	U	Q	V	C	E	U	Q	V	C	E	U	Q	V	C	E	U	Q	V	C	E	U	Q	V	C	E	U
battery remains connected	↓	=	↓	↓	↓	↑	=	↑	↑	↑	↑	=	↑	=	↑	↓	=	↓	=	↓	↑	=	↑	=	↑	↓	=	↓	=	↓
Battery is disconnected	=	↑	↓	=	↑	=	↓	↑	=	↓	=	↓	↑	↓	↓	=	↑	↓	↑	↑	=	↓	↑	↓	↓	=	↑	↓	↑	↑

$$Q = CV \qquad E = \frac{Q}{\kappa \varepsilon_0 A} \qquad E = \frac{V}{d} \qquad C = \frac{\kappa \varepsilon_0 A}{d} \qquad U = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C}$$

What is happening if you make several different steps in a row?

Example

	Q	V	C	E	U
Connect	Q_i	V_i	C_i	E_i	U_i
Insert					
Disconnect					
Take out					
Double *d*					?

$Q = CV$ $\quad E = \dfrac{Q}{\kappa\varepsilon_0 A}$ $\quad E = \dfrac{V}{d}$ $\quad C = \dfrac{\kappa\varepsilon_0 A}{d}$ $\quad U = \dfrac{QV}{2} = \dfrac{CV^2}{2} = \dfrac{Q^2}{2C}$

?

$1.\ \varepsilon U_i \quad 2.\ 2\varepsilon U_i \quad 3.\ \varepsilon^2 U_i \quad 4.\ 2\varepsilon^2 U_i \quad 5.\ \varepsilon U_i^2 \quad 6.\ 2\varepsilon U_i^2$

Example

	Q	V	C	E	U
Connect	Q_i	V_i	C_i	E_i	U_i
Insert	$\varepsilon\ Q_i$	V_i	$\varepsilon\ C_i$	E_i	$\varepsilon\ U_i$
Disconnect	$\varepsilon\ Q_i$	V_i	$\varepsilon\ C_i$	E_i	$\varepsilon\ U_i$
Take out	$\varepsilon\ Q_i$	$\varepsilon\ V_i$	C_i	$\varepsilon\ E_i$	$\varepsilon^2\ U_i$
Double *d*	$\varepsilon\ Q_i$	$2\varepsilon\ V_i$	$\frac{1}{2}\ C_i$	$\varepsilon\ E_i$	$2\varepsilon^2\ U_i$

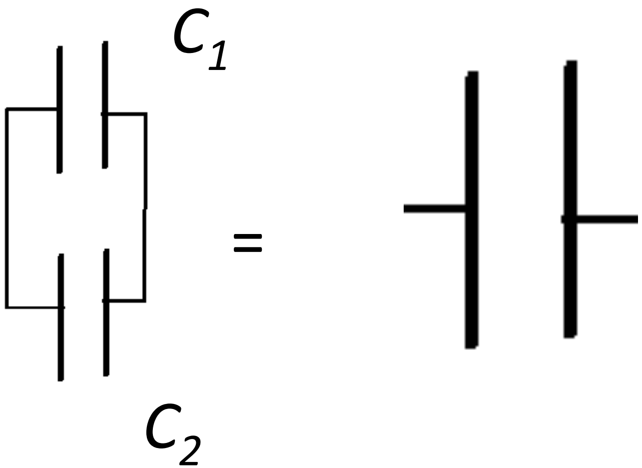
$$Q = CV \qquad E = \frac{Q}{\kappa\varepsilon_0 A} \qquad E = \frac{V}{d} \qquad C = \frac{\kappa\varepsilon_0 A}{d} \qquad U = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C}$$

1. $C_{eq} = C_1 + C_2 + \dots$ 2. $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$

When capacitors are connected in series we have to use expression … to calculate the equivalent capacitance.

Three capacitors (2 pF, 5 pF, and 7 pF) are connected in series or in parallel. What is their equivalent capacitance?

1. 1.2 pF 2. 8 pF 3. 12 pF 4. 14 pF 5. 15.4 pF



$$C_{eq} = C_1 + C_2 + \ldots$$

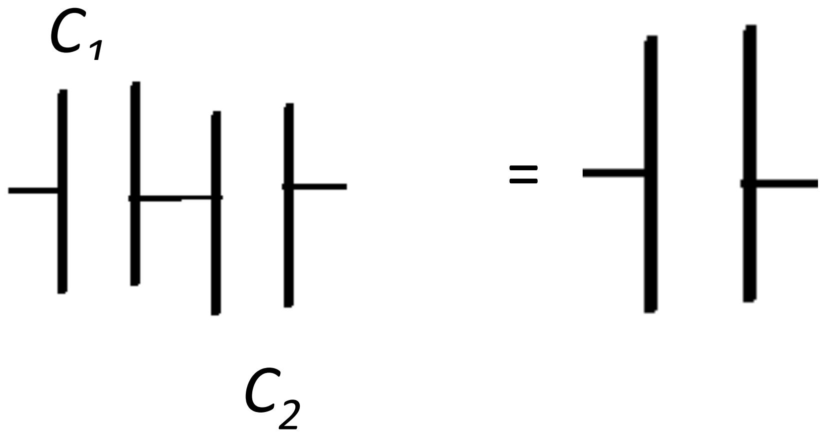
$Q_{eq} = Q_2 + Q_1$

$$Q_1 = C_1 V_1$$

$V_{eq} = V_2 = V_1$

$$Q_2 = C_2 V_2$$

$$Q_{eq} = C_{eq} V_{eq}$$



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$$

$Q_{eq} = Q_2 = Q_1$

$$Q_1 = C_1 V_1$$

$V_{eq} = V_2 + V_1$

$$Q_2 = C_2 V_2$$

$$Q_{eq} = C_{eq} V_{eq}$$

A 2.0 pF capacitor is charged to 100 V and then connected in parallel (positive plate to positive plate) with a 4.0 pF capacitor charged to 200 V. (a) What is the potential difference across each capacitor after the equilibrium is reached? (b) What are the final charges on the capacitors? (c) Calculate the values if the positive plate of one capacitor is connected to the negative plate of the other.

1. 41.2 V **2. 83 V** **3. 167 V** **4. 250 V**

Discharging a capacitor

Let's try discharging a capacitor, after reading the label on the side:
"WARNING: the energy stored in this capacitor is lethal."

How much energy do you think is enough to kill you?
1000 J? A million joules?

Let's work out how much our 8 μF capacitor has when it has a potential difference of 4000 V.

Then we'll discharge it with a well-insulated screwdriver (don't try this at home!!!).

Discharging a capacitor

"WARNING: the energy stored in this capacitor is lethal."

$$U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(8 \times 10^{-6}\ \text{F})(4000\ \text{V})^2$$

The factor of 10^{-6} in the capacitance cancels the factor of 1000^2, so we get:

$$U = \frac{1}{2}(8\ \text{F})(4\ \text{V})^2 = 64\ \text{J}$$

That doesn't sound like enough to kill you, but I would not want to discharge the capacitor with my hand!

Discharging a capacitor

"WARNING: the energy stored in this capacitor is lethal."

$$U = \frac{1}{2}(8\text{ F})(4\text{ V})^2 = 64\text{ J}$$

That doesn't sound like enough to kill you, but I would not want to discharge the capacitor with my hand!

Why is it dangerous?

1 64 J is a huge energy
2 The spark is very powerful
3 The capacitor is very heavy
4 The screwdriver is very sharp

Discharging a capacitor

"WARNING: the energy stored in this capacitor is lethal."

$$U = \frac{1}{2}(8\ \mathrm{F})(4\ \mathrm{V})^2 = 64\ \mathrm{J}$$

Why is it dangerous?

B. The spark is very powerful

The spark takes so tiny time, so the power released during the discharging is huge:

$$P = U/t = 64\mathrm{J}/0.01\mathrm{s} = 6400\ \mathrm{W}\ !$$

The electric impulse is strong enough to kill a man.