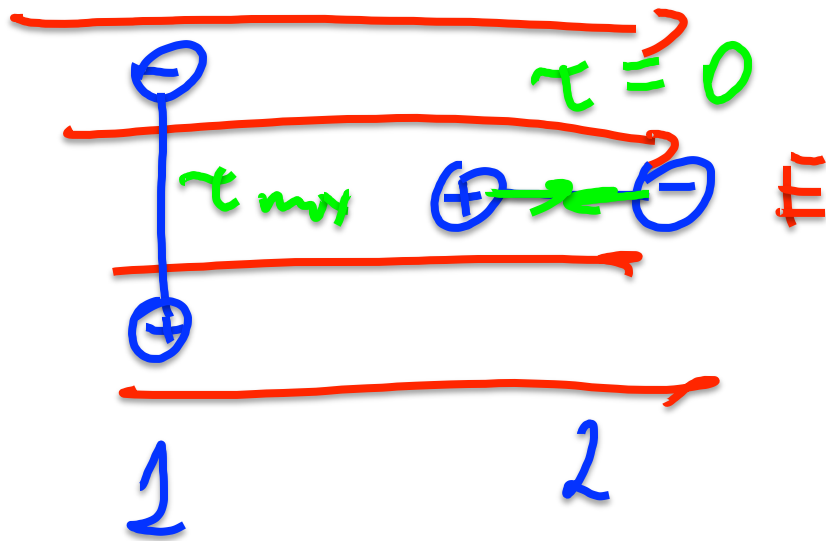


Arrows above represent uniform electric field acting on a dipole.  
If we release the dipole, it ...

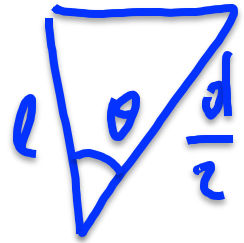
- |                             |                       |
|-----------------------------|-----------------------|
| 1. does not move            | 2. moves to the right |
| 3. moves to the left        | 4. rotates clockwise  |
| 5. rotates counterclockwise | 6. explodes           |



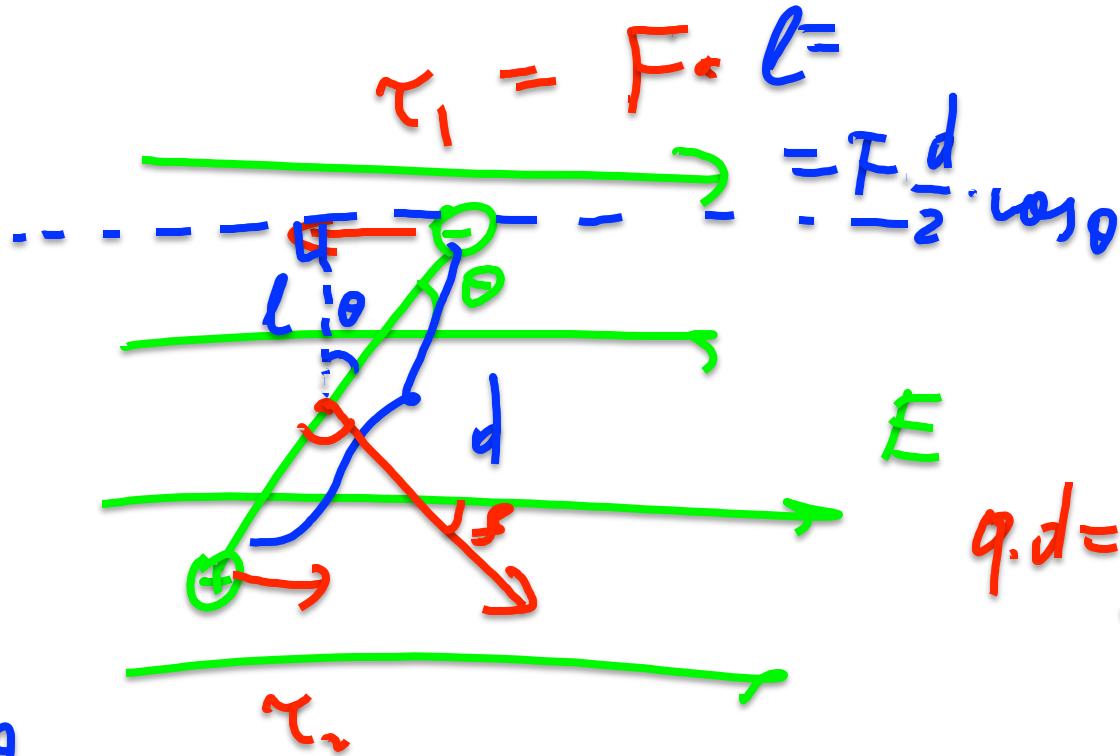
$$\tau_{max} = ?$$

1.

2.



$$l = \frac{d}{2} \cdot \csc \theta$$



$$\tau_{\text{net}} = 2 \cdot \tau_1 = \underline{F \cdot d \sin \theta} = q \cdot E \cdot d \cdot \sin \theta = \underline{q \cdot d} \cdot E \cdot \sin \theta = p E \sin \theta$$

Work of electric force:  $W_{El} = EPE_i - EPE_f$

$$W_{elf} = q_x(V_i - V_f)$$

We can say now that *electrostatic field has at every point*:

1. Electric field  $\mathbf{E}$  which creates a force  $\mathbf{F} = q\mathbf{E}$  on a charge  $q$  located at that point
2. Electric potential  $V$  which creates potential energy  $U = qV$

$$U_Q = k \frac{Q}{r} \quad E_Q = k \frac{|Q|}{r^2} \quad E = \frac{|Q|}{2\epsilon_0 A} \quad E = \text{const}$$
$$V_1 - V_2 = Ed$$



# Equipotential Surfaces and Their Relation to the Electric Field:

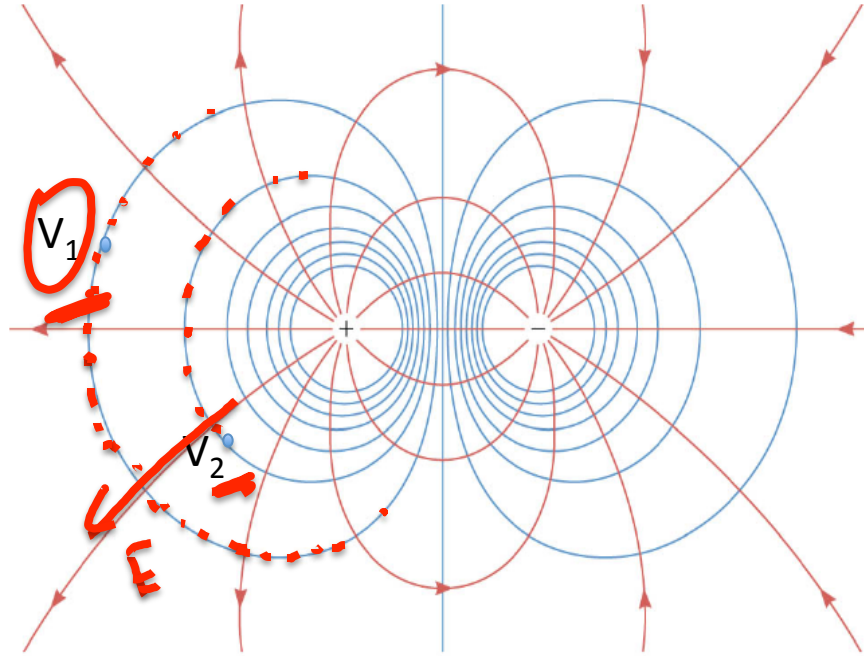
## A dipole

Handwritten notes on the left side of the diagram:

1.  $V = k \frac{q}{r}$

2.  $V = k \frac{q}{r}$

3.  $V = k \frac{q}{r}$



Handwritten notes on the right side of the diagram:

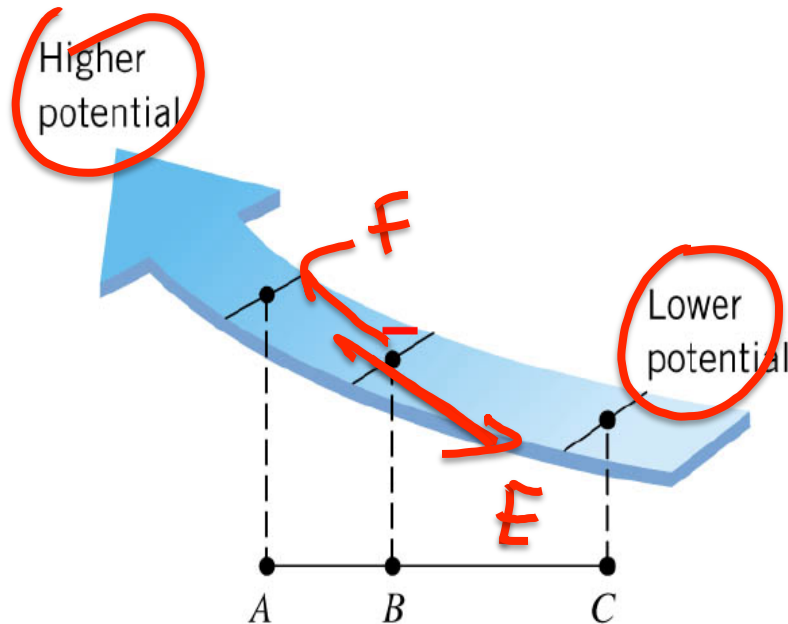
$V_1 > V_2$

In the picture above:

1.  $V_1 > V_2$

2.  $V_1 < V_2$

3.  $V_1 = V_2$



The blue arrow represents the region with electrostatic field in it. The electric potential increases in the direction of the arrow.

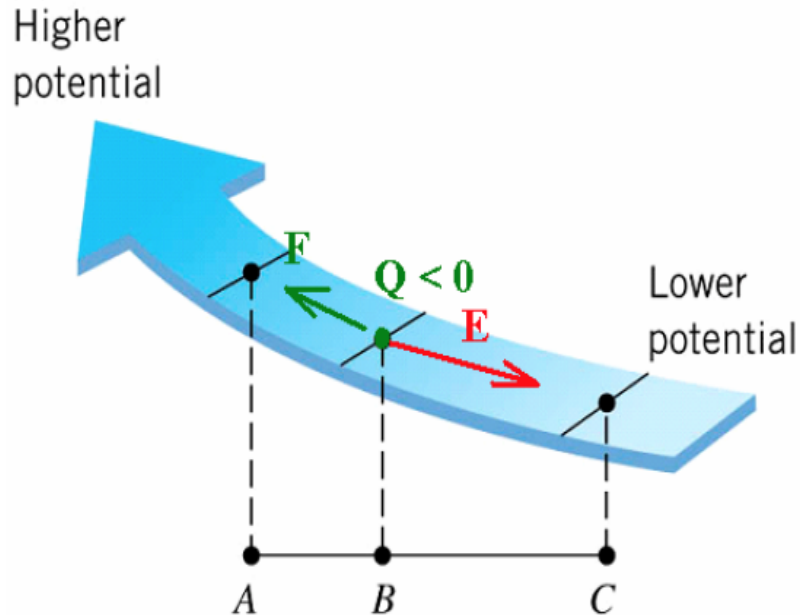
A negative test charge is released from the point B.

In what direction the charge will be moving?

1. Toward C
2. Toward A
3. Big blue arrow confuses me

A hint  $\vec{F} = Q\vec{E}$  and  $\vec{E}$  is directed to lower potential.

Draw  $\vec{E}$  and remember that  $Q < 0$ .



The blue arrow represents the region with electrostatic field in it. The electric potential increases in the direction of the arrow.

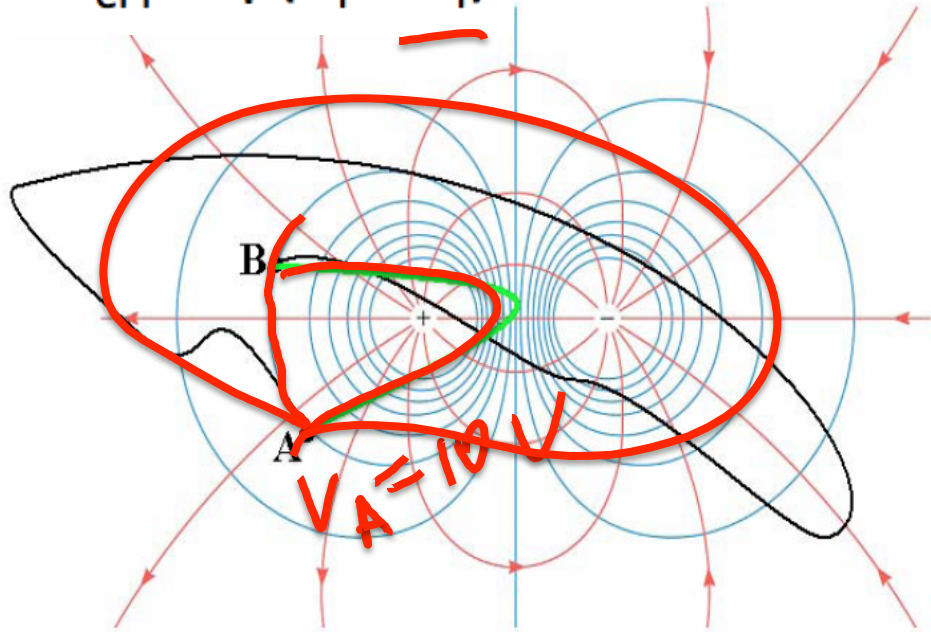
A negative test charge is released from the point B. In what direction the charge will be moving?

## B. Toward A

$\vec{E}$  is directed to lower potential, hence toward C.

$\vec{F} = Q\vec{E}$  but  $Q < 0$ , hence force is directed *opposite* to the field.

$$W_{\text{el f}} = q \times (V_i - V_f)$$

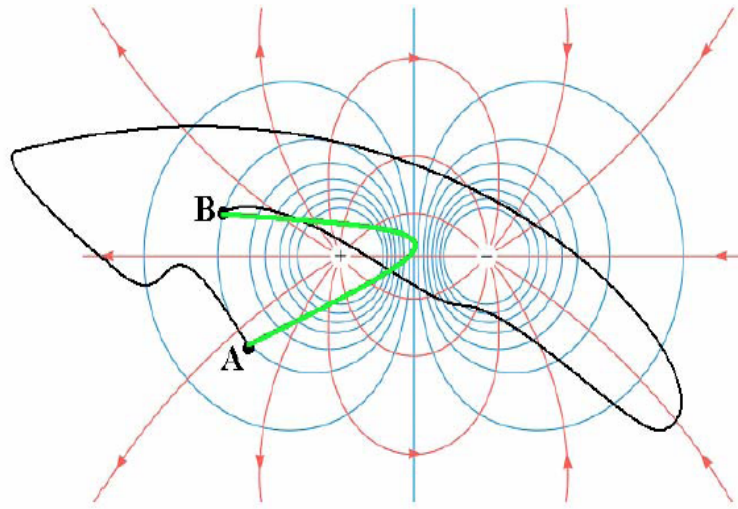


A charge  $Q$  was moved from the point A to the point B twice:

1. at first time it was moving along the path shown by a black curved line.
2. then it was moving along the green line.

When the electric field of a dipole did the greater work on the charge?

1. At first time (along the black trajectory)
2. At the second time (along the green trajectory)
3. The same work was done in both cases
4. Not enough information



A charge  $Q$  was moved from the point A to the point B twice:

1. at first time it was moving along the path shown by a black curved line.
2. then it was moving along the green line.

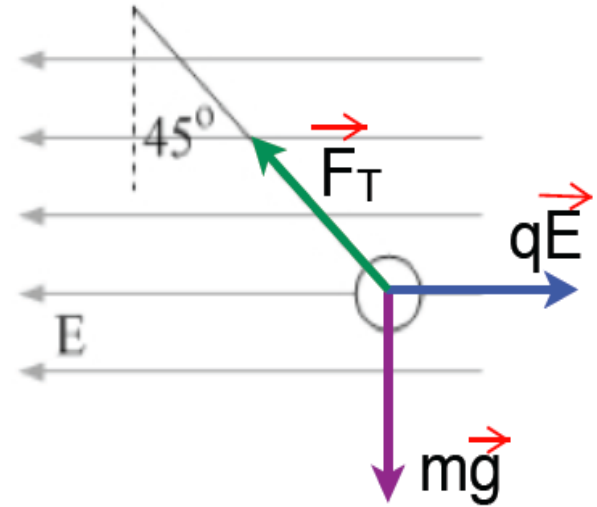
When the electric field of a dipole did the greater work on the charge?

**C. The same work was done in both cases**

$W_{El} = q(V_i - V_f) = q(V_A - V_B)$ . In both cases we deal to the same charge, initial potential and final potential.

It does not matter *how* the charge was moving;  
its initial and final locations *only* are important!!!

# A ball on a string - find the charge



$$\vec{F} = q \vec{E}$$

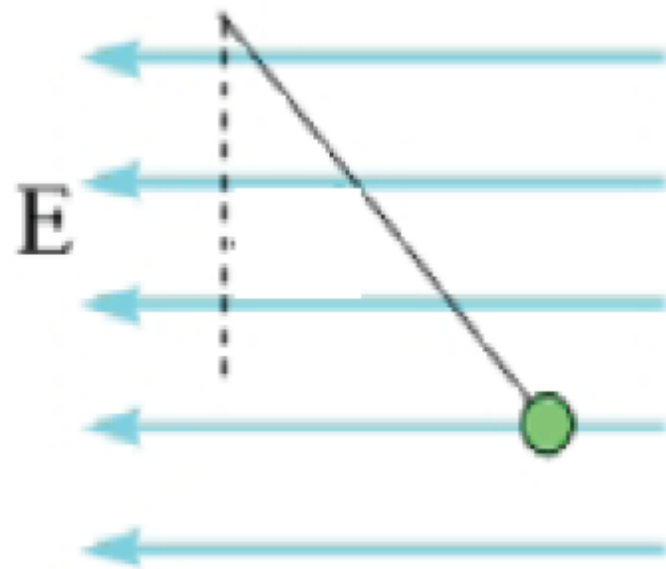
In this equation  $q$  represents ...

1. the magnitude of the charge
2. the actual value of the charge
3. could represent both
4. does not represent neither

# A ball on a string - find the charge

What's the sign of the charge?

1. positive
2. negative

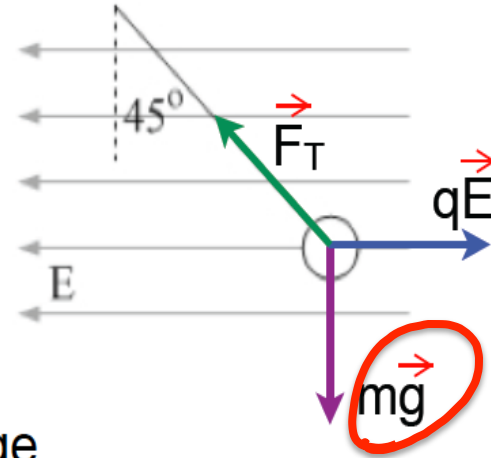


# A ball on a string - find the charge

What's the sign of the charge?

2. negative

Find the magnitude of the charge



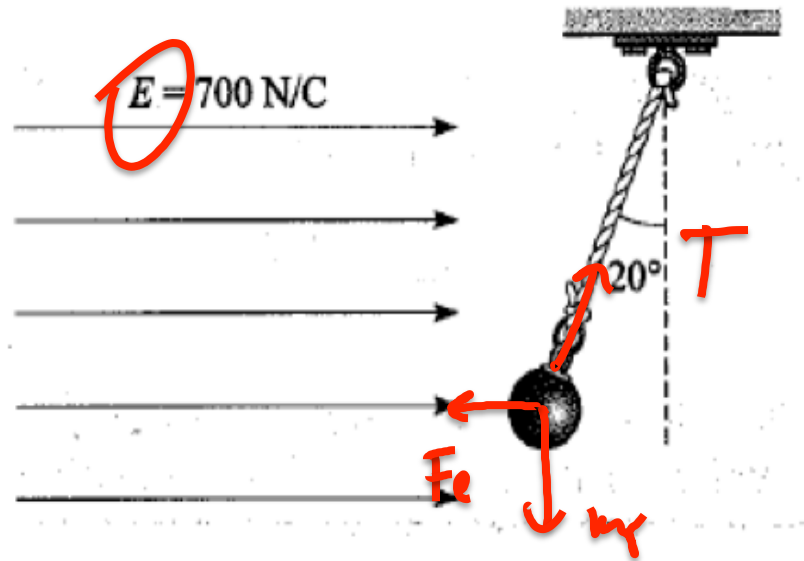
$$qE = mg$$

← In this equation  $q$  represents ...

1. the magnitude of the charge
2. the actual value of the charge
3. could represent both
4. does not represent neither



The tiny ball at the end of the thread shown in the picture has a mass of 0.30 g and is in a horizontal electric field of strength 700 N/C. It is in equilibrium in the position shown. What are the magnitude and sign of the charge on the ball?

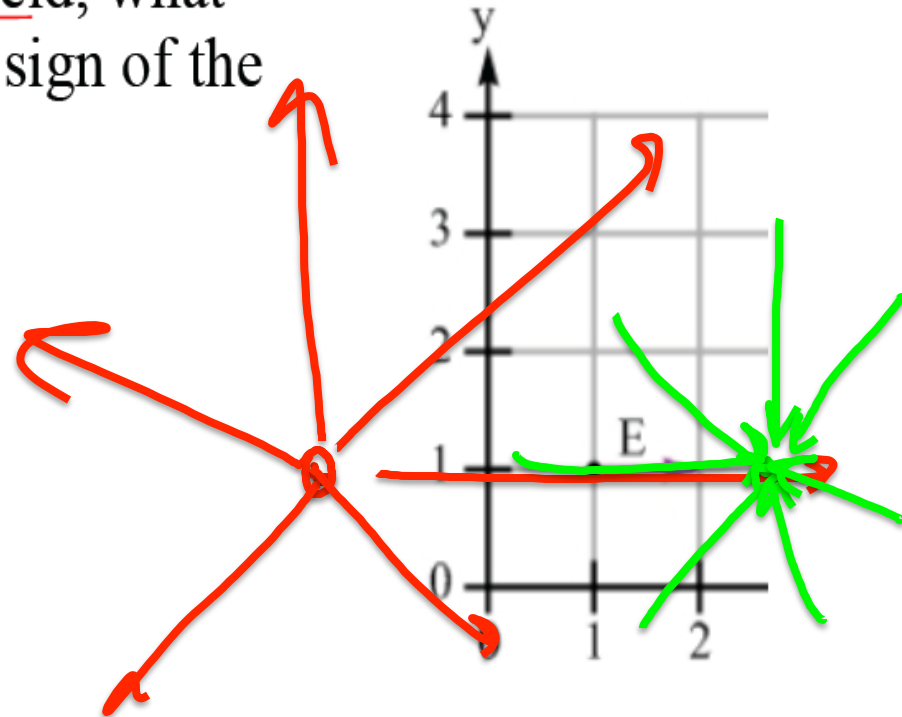


The charge is 1. positive 2. negative

The electric field in the region shown in Figure is produced by a single point charge, but the location of that point charge is unknown. At the point  $(x = 1, y = 1)$ , we know that the electric field is directed to the right.

If this is all we knew about the field, what could we say about the location and sign of the point charge?

1.  $q > 0$
2.  $q < 0$
3. ambiguous

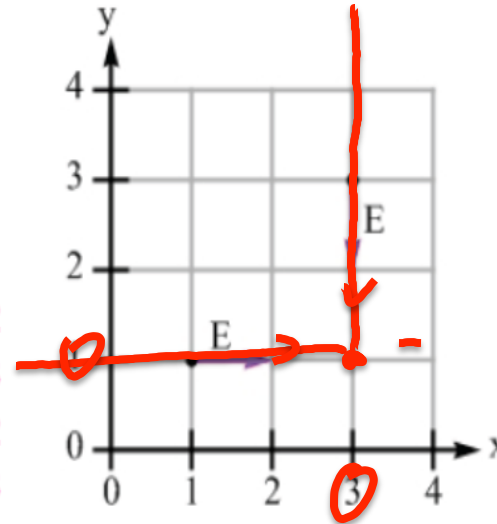




$$E = k \frac{|q|}{r^2}$$

The electric field in the region shown in Figure is produced by a single point charge, but the location of that point charge is unknown. At the point  $(x = 1, y = 1)$ , we know that the electric field is directed to the right.

We also know that, at the point  $(x = 3, y = 3)$ , the electric field is directed down. With this information, what can we say about the location and sign of the point charge?



1.  $X = 1, Y = 1$

2.  $X = 2, Y = 2$

3.  $X = 3, Y = 3$

4.  $X = 1, Y = 2$

5.  $X = 2, Y = 3$

6.  $X = 3, Y = 1$

7.  $X = 1, Y = 3$

8.  $X = 2, Y = 1$

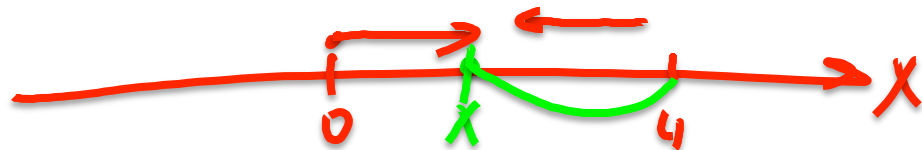
9.  $X = 3, Y = 2$

1.  $q > 0$

2.  $q < 0$

~~3. ambiguous~~

A single point charge is located at an unknown point on the  $x$ -axis. There are no other charged objects nearby. You measure the electric field at the origin to be 600 N/C in the positive  $x$ -direction, while the electric field on the  $x$ -axis at  $x = +4.0$  m is 5400 N/C in the negative  $x$ -direction. What is the sign and magnitude of the point charge, and where is it located?



$$E = k \frac{|q|}{r^2}$$

**$E$  represents 1. actual value 2. the magnitude**

$$E_1 = k \frac{|q|}{x^2} = 600$$

$$E_2 = \frac{k |q|}{(4-x)^2} = 5400$$

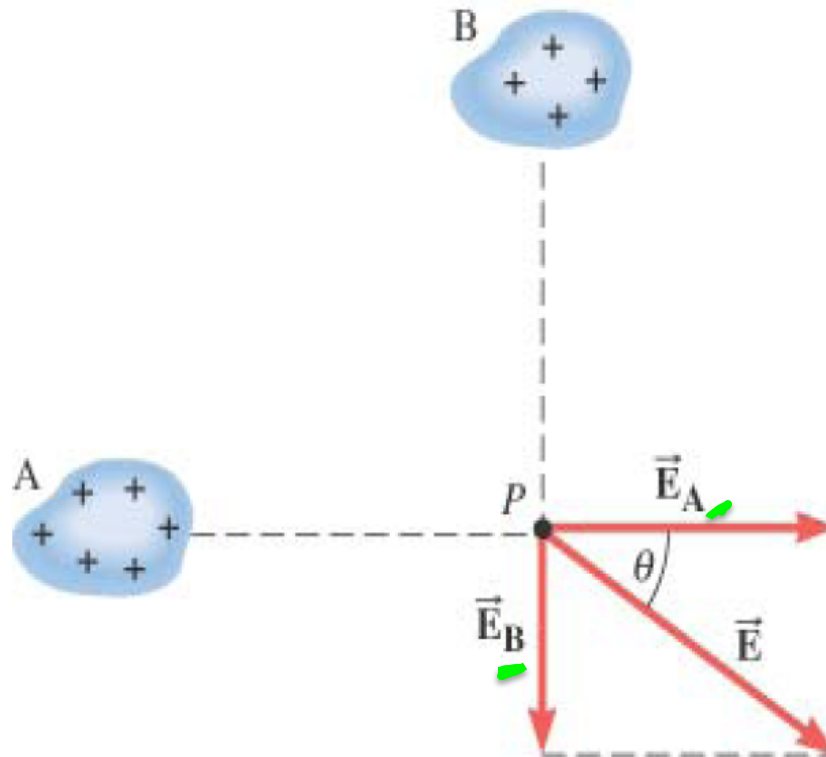
**1. the charge is positive**

**2. the charge is negative**

**1.  $x > 0$**

**2.  $x < 0$**

2D



Electric fields from different sources add as vectors.

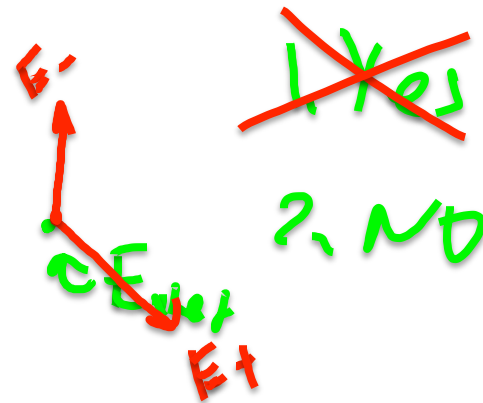
**Remember (!) Electric field is just a net force acting on a +1 C charge placed at the location of your interest!**

# Where is the net field equal to zero?

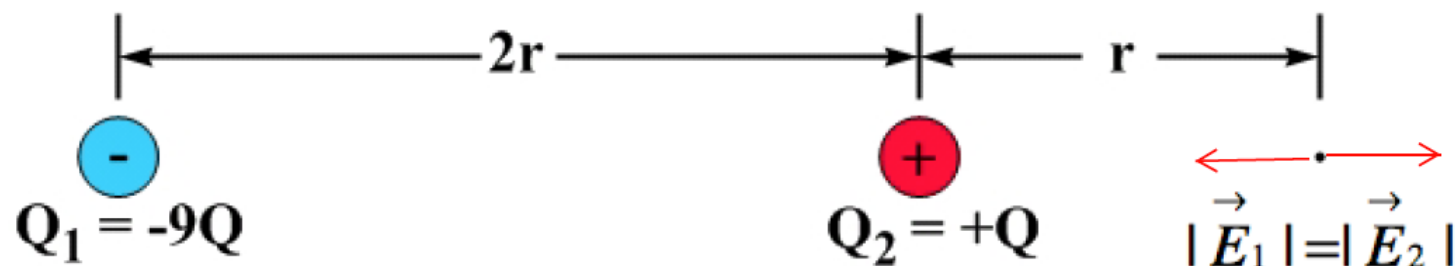


The field is zero at a point in:

1. Region I
2. Region II
3. Region III
4. two of the above
5. all of the above



## Two charges in a line



$$|k \frac{Q_1}{(3r)^2}| = |k \frac{Q_2}{r^2}|$$

The electric field from ball 1 and the electric field from ball 2 cancel out at the location where ball 3 was.

What is the value of net E field at the location of the red charge?

1. 0

2. between 0 and  $kQ/r^2$

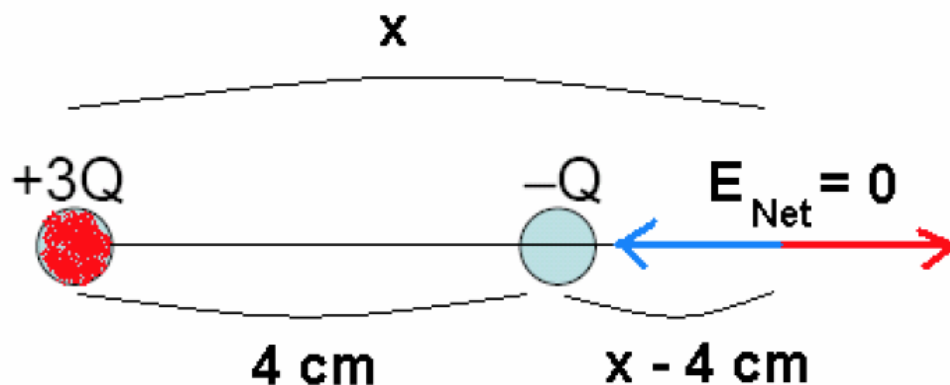
3. between  $-kQ/r^2$  and 0

4. none of the above



For the situation pictured:

A similar problem



The distance between the charges is  $4\text{ cm}$ .

The point at which the net electric field is 0 is located at the unknown distance  $x$  from the red charge.

In this case the distance from the blue charge to the point is  $x - 4\text{ cm}$ .

Notice: we do not know the values of the charges; we know the sign and the relative values only, but this is enough to solve the problem!

$$|\vec{E}_1| = |\vec{E}_2|$$

$$\frac{3kQ}{x^2} = \frac{kQ}{(x - 4 \text{ cm})^2}$$

First, let's flip the equation over:

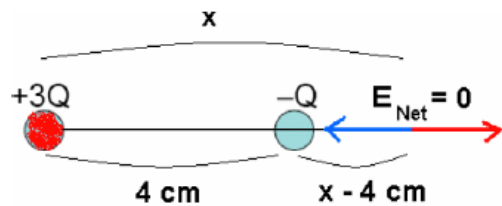
$$\frac{x^2}{3} = \frac{(x - 4)^2}{1}$$

Now, let's take a square root of it:

$$\sqrt{\frac{x^2}{3}} = \sqrt{\frac{(x - 4)^2}{1}}$$

From the definition:  $\sqrt{x^2} = |x|$  hence,

$$\frac{|x|}{\sqrt{3}} = \frac{|x - 4|}{1}$$



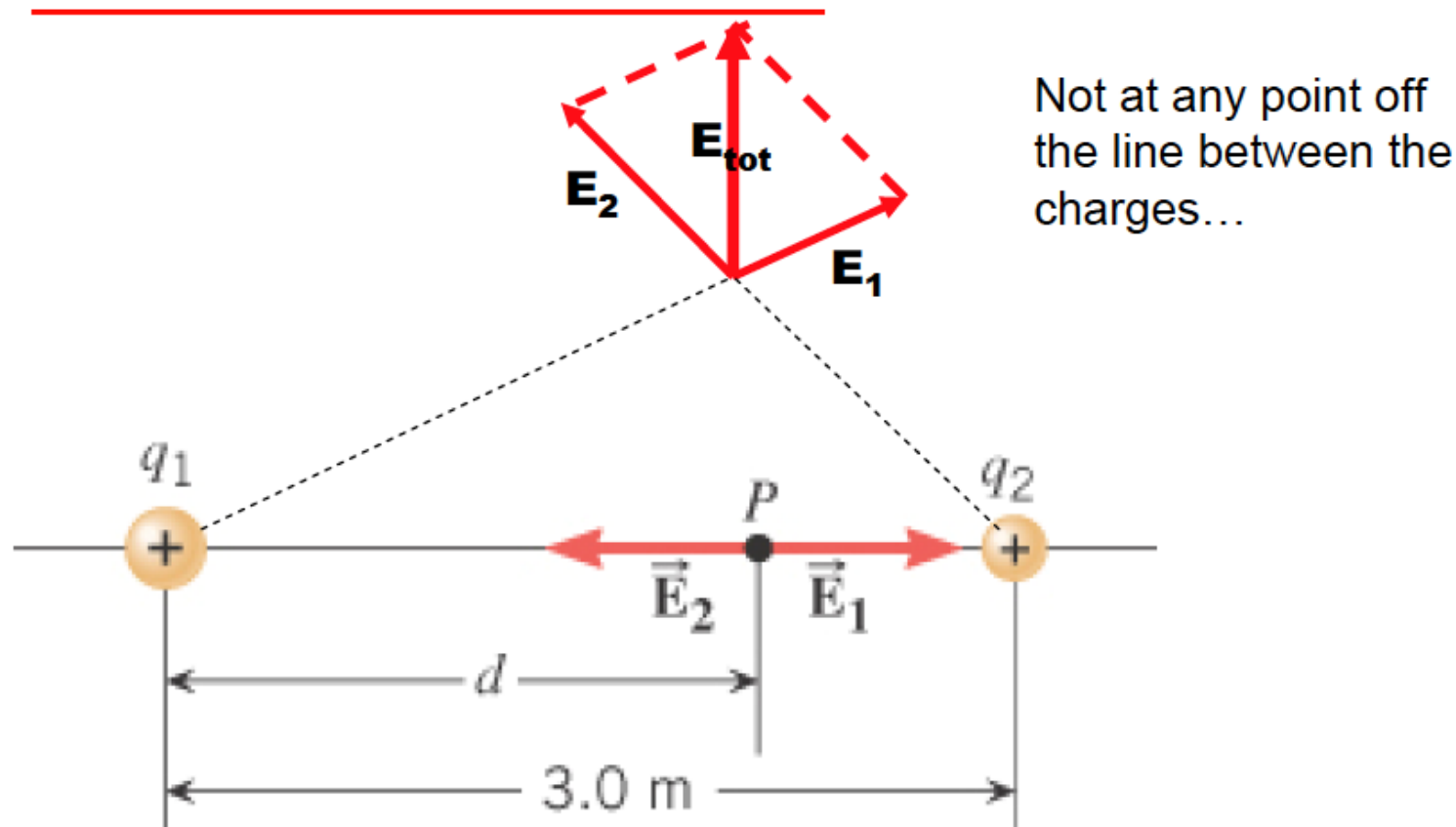
From the definition:  $|x| = \begin{cases} x, & \text{if } x > 0 \\ -x, & \text{if } x < 0 \end{cases}$

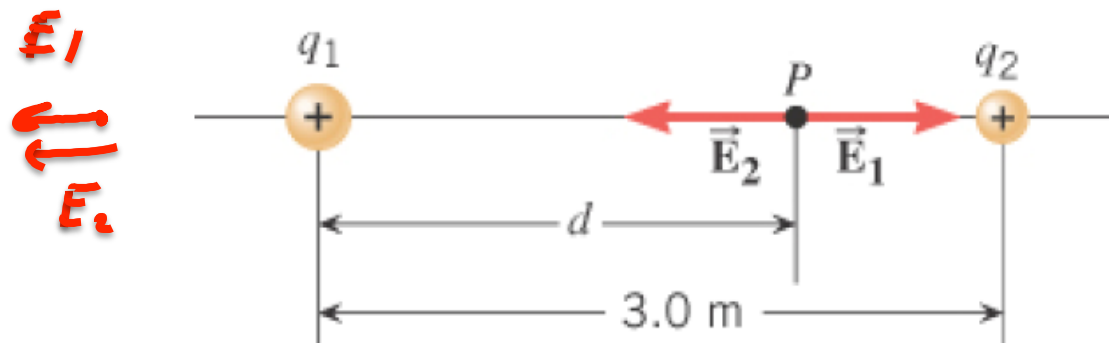
In our case  $x$  is the positive distance  $x > 0$  and  $x > 4$  ( $x - 4 > 0$ ),  
 so  $|x| = x$ , and  $|x - 4| = x - 4$ , hence we have

$$\frac{x}{\sqrt{3}} = \frac{x - 4}{1} \quad \text{or} \quad 0.577x = x - 4 \quad \text{and} \quad \underline{x = 9.46 \text{ cm}}$$

## The Electric Fields from Separate Charges May Cancel

$q_1 = +16\mu\text{C}$  and  $q_2 = +4.0\mu\text{C}$  are separated by 3.0m.  
Where is the net electric field zero?





$$E = k \frac{|q|}{r^2}$$

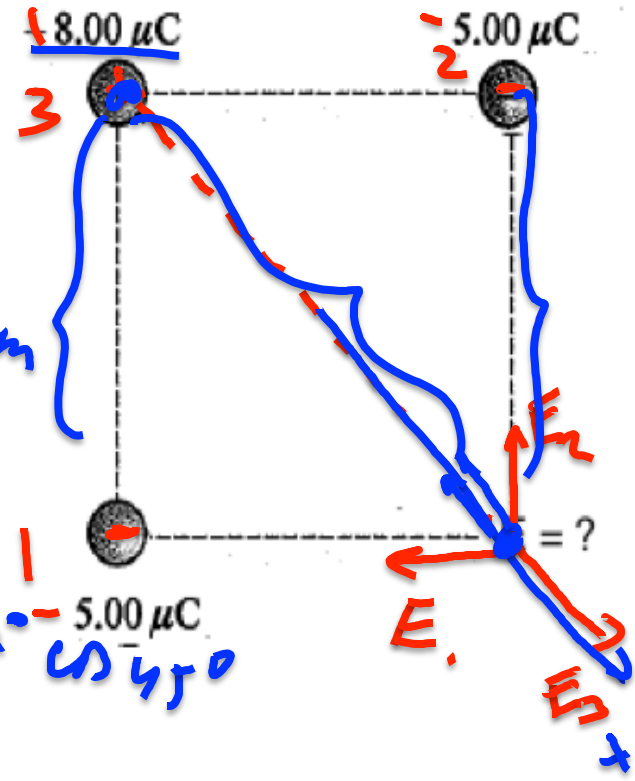
$$E_1 = E_2$$

$$k \frac{(16 \times 10^{-6} \text{ C})}{d^2} = k \frac{(4.0 \times 10^{-6} \text{ C})}{(3.0 \text{ m} - d)^2}$$

$$4.0(3.0 \text{ m} - d)^2 = d^2$$

$$d = +2.0 \text{ m}$$

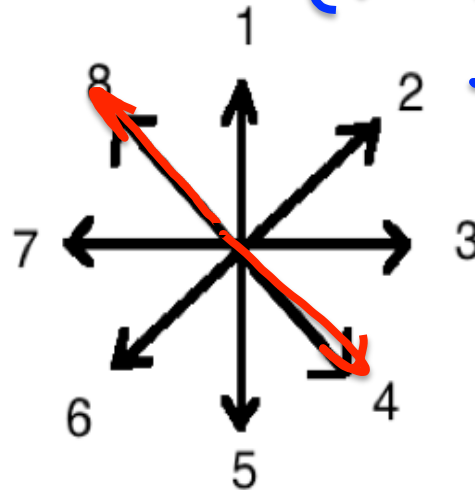
Three charges are placed on three corners of a square, as shown in the picture. Each side of the square is 30.0 cm. Compute  $E$  at the fourth corner. What would be the force on a  $-6.00 \mu\text{C}$  charge placed at the vacant corner?



$$F_{\text{net}} = \frac{k \cdot 8 \mu\text{C}}{(\sqrt{2} \cdot 0.3)^2} -$$

$$- 2 \cdot k \cdot \frac{5 \mu\text{C}}{(0.3)^2} \quad \cos 45^\circ$$

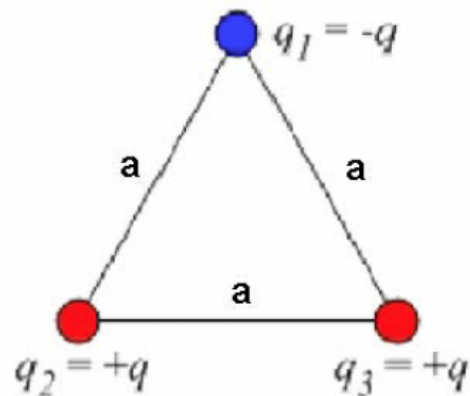
The direction of  $E$ -field is



The direction of  $F$  is

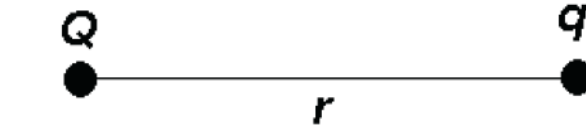
9. it is 0

Find net E field at the location of the red charge.



$$E = k \frac{|q|}{r^2}$$

# Potential energy of a charge in a field of another charge



$$W = V_i - V_f$$

The old knowledge; the magnitude of the force:  
(always positive!)

$W$   $F \cdot s \cdot \cos \theta$

$$F = k \frac{|Qq|}{r^2}$$

A graph with force  $F$  on the vertical axis and distance  $r$  on the horizontal axis. A blue curve represents the inverse-square relationship. A green shaded area is shown under the curve between two points  $r_1$  and  $r_2$  on the horizontal axis. A blue arrow points from the shaded area up towards the handwritten  $W$ .

The new knowledge; the potential energy for two charges:

$$EPE = U = k \frac{Qq}{r}$$

We can think of it as: A) the potential energy of the charge  $q$  in the field of the charge  $Q$ ; B) the potential energy of the charge  $Q$  in the field of the charge  $q$ ; C) the potential energy of interaction between the charges  $Q$  and  $q$ .

Two charges: *Absolute* or *Actual*?

$$U = \frac{kqQ}{r}$$

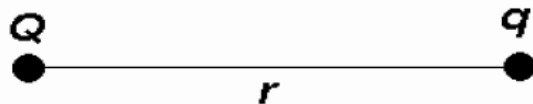
In the expression

1.  $U$  is the *absolute* value of the potential energy,  $q$  and  $Q$  are *actual* values of the charges
2.  $U$  is the *absolute* value of the potential energy,  $q$  and  $Q$  are *absolute* values of the charges
3.  $U$  is the *actual* value of the potential energy,  $q$  and  $Q$  are *actual* values of the charges
4.  $U$  is the *actual* value of the potential energy,  $q$  and  $Q$  are *absolute* values of the charges





## Problem



Find the potential energy of the system consisting of a  $2\ \mu\text{C}$  charge and a  $-4\ \mu\text{C}$  charge, separated by the distance of  $10\ \text{cm}$ .

By the definition, for the system of two charges:  $U = k \frac{Qq}{r}$

Using the SI system of units:  $Q = 2 \cdot 10^{-6}\ \text{C}$ ,  $q = -4 \cdot 10^{-6}\ \text{C}$   
 $r = 0.1\ \text{m}$

Calculation gives

$$U = 8.99 \cdot 10^9 \frac{2 \cdot 10^{-6} \cdot (-4 \cdot 10^{-6})}{0.1} = -71.92 \cdot 10^{-2} = -0.72\ \text{J}$$

A charge of  $0.20 \mu\text{C}$  is  $30 \text{ cm}$  from another point charge. It is known that  $0.027 \text{ J}$  of work is required to bring the second charge  $18 \text{ cm}$  closer to the  $0.20 \mu\text{C}$  charge. Find the value of the second charge.

1.  $1 \mu\text{C}$

2.  $2 \mu\text{C}$

3.  $3 \mu\text{C}$

4.  $4 \mu\text{C}$

5.  $5 \mu\text{C}$

6.  $6 \mu\text{C}$

$$U = \frac{kqQ}{r}$$

$$V_2 = k \frac{q_1 q_2}{r_2}$$

$$q_1 = 2 \mu\text{C} \quad q_2 = ?$$

$$r_1 = 0.3 \text{ m}$$

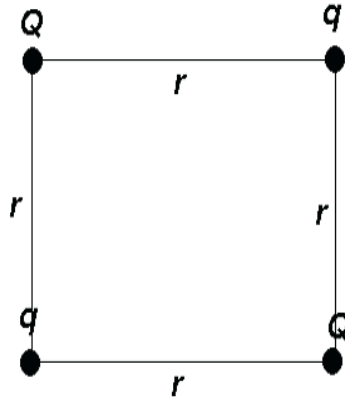
$$q_1 \quad q_2$$

$$r_2 = 0.12 \text{ m}$$

$$W_{\text{conservative}(\text{field})} = PE_{\text{initial}} - PE_{\text{final}}$$

$$0.027 \text{ J} = V_1 - V_2$$

### Problem



Four charges are placed at the corners of a square.

$$Q = 2 \cdot 10^{-6} \text{ C}, \quad q = -4 \cdot 10^{-6} \text{ C} \quad r = 0.1 \text{ m}$$

Find the electrostatic potential energy of the system.

$$U = \frac{kqQ}{r}$$

Each pair of charges gives an input.

Let's numerate the charges.

Now, let's show all the different pairs of charges.

How many different pairs of charges do you have?

A) 3

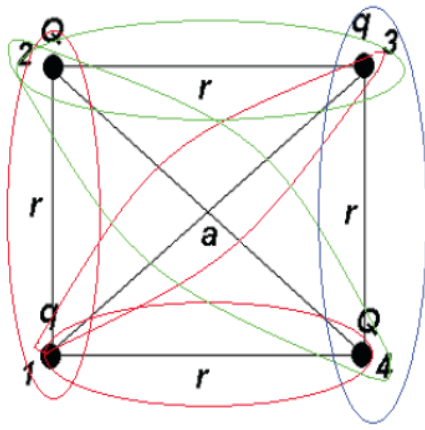
B) 4

C) 5

D) 6

E) 7

## Problem



Four charges are placed at the corners of a square.

$$Q = 2 \cdot 10^{-6} \text{ C}, \quad q = -4 \cdot 10^{-6} \text{ C} \quad r = 0.1 \text{ m}$$

The electrostatic potential energy of the system:

$$U_{\text{total}} = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$$

$$U_{12} = k \frac{Qq}{r} = -0.72 \text{ J} = U_{14} = U_{23} = U_{34}$$

To find  $U_{13}$  we need to know the distance  $a$ ;  $a = \sqrt{r^2 + r^2} = r\sqrt{2} = 0.14 \text{ m}$

$$U_{13} = k \frac{qq}{a} = 8.99 \cdot 10^9 \frac{(-4 \cdot 10^{-6})^2}{0.14} = 1.028 \text{ J}$$

$$U_{24} = k \frac{QQ}{a} = 8.99 \cdot 10^9 \frac{(2 \cdot 10^{-6})^2}{0.14} = 0.257 \text{ J}$$

That gives:  $U_{\text{total}} = -1.595 \text{ J}$

If you are to assemble this system by bring the charges from infinity one by one, what work would you do?

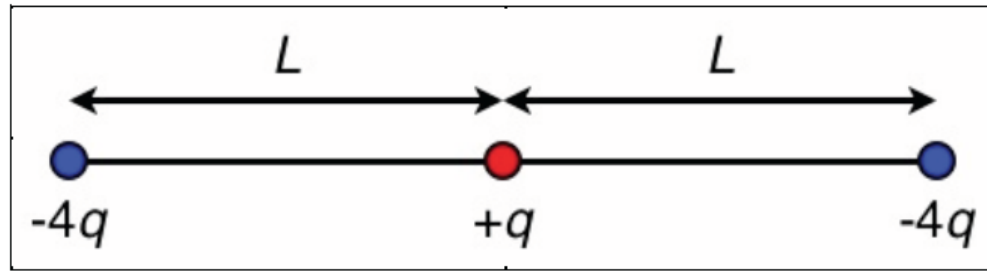
- 1) 1.595 J
- 2) 0 J
- 3) -1.595 J

$$V_i = 0$$

$$V_4 = -1.595$$

$$W_4 = V_i - V_4$$

$$W_i = -W_4 = V_4 - V_i$$

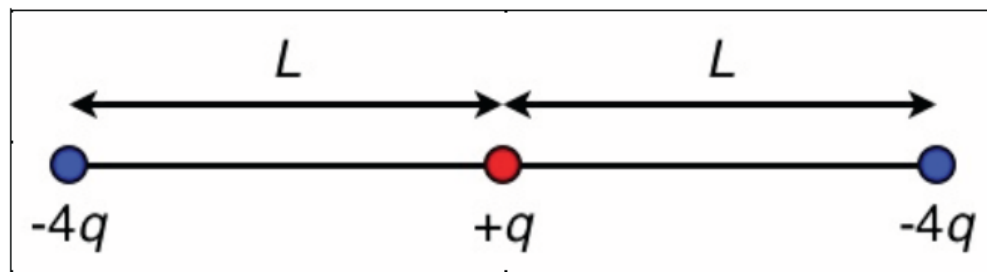


In order to find the electric potential energy of the configuration we must consider the interacting pairs in the system. In this case, there are three ways to pair up the objects. We use the equation for the potential energy for the interaction between two charges ( $U = (kqQ)/r$ ) to find the potential energy for each pair, and then we add these together to find the total potential energy.

1.  $U < 0$

2.  $U = 0$

3.  $U > 0$

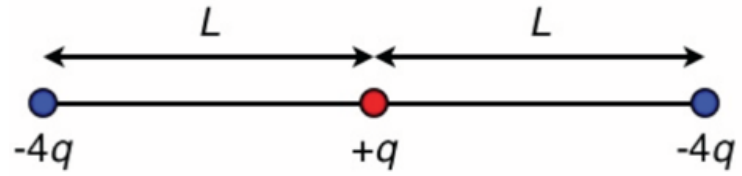
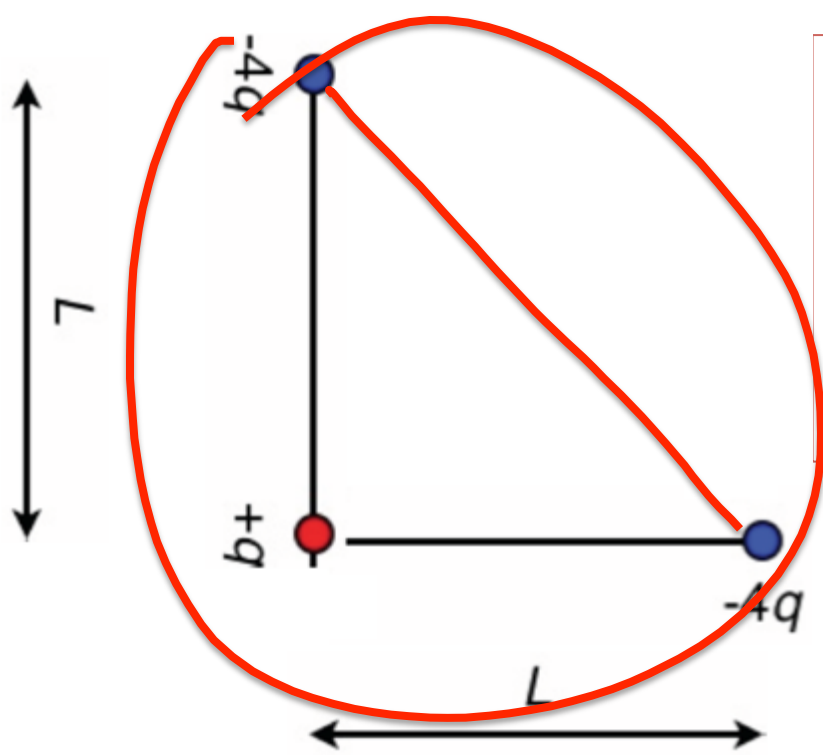


$$-4kq^2/L - 4kq^2/L + 16Kq^2/2L = 0$$

$$k \frac{-4q * q}{L} + k \frac{-4q * q}{L} + k \frac{-4q * -4q}{2L} = 0$$

In order to find the electric potential energy of the configuration we must consider the interacting pairs in the system. In this case, there are three ways to pair up the objects. We use the equation for the potential energy for the interaction between two charges ( $U = (kqQ)/r$ ) to find the potential energy for each pair, and then we add these together to find the total potential energy.

$$k \frac{-4q * q}{L} + k \frac{-4q * q}{L} + k \frac{-4q * -4q}{2L} = 0$$



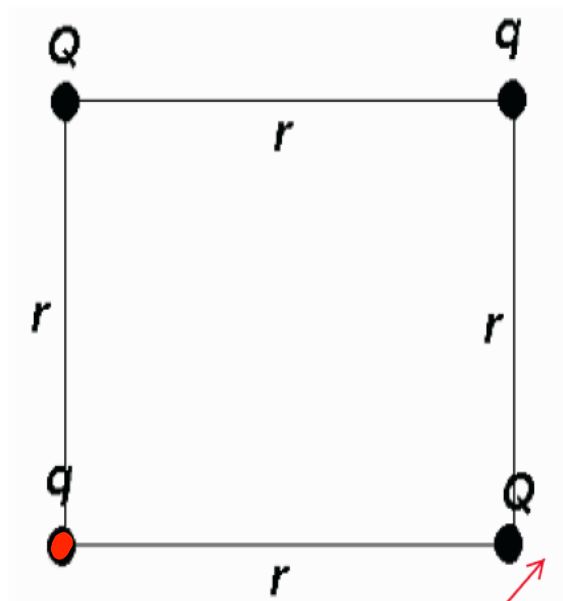
$$k \frac{-4q * q}{L} + k \frac{-4q * q}{L} + k \frac{-4q * -4q}{2L} = 0$$

**Before!**

The energy for the shown configuration of charges is ...

1. zero
2. some positive number
3. some negative number

### Four charges again



Potential energy of charge  $q$  in the field created by the three other charges

IS EQUAL TO

Potential energy of the *system* comprised of four charges.

PRS

Is this statement correct?

1. Yes!

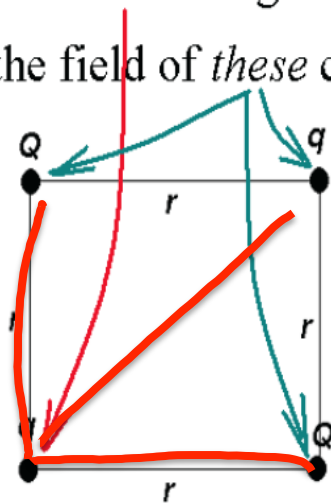
2. No!

3. Maybe, depending on the charges



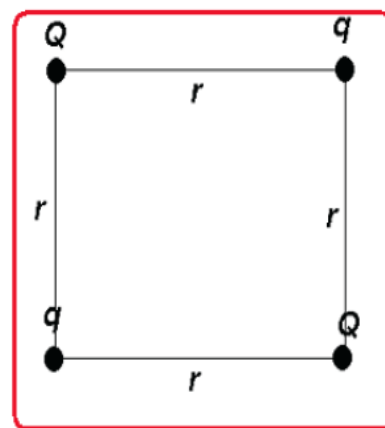
## Important!

The potential energy  
of *this* charge  
in the field of *these* charges



Three charges act on one!  
Three charges contribute into  
the potential energy of the  
fourth charge.

The potential energy of  
all *these* charges

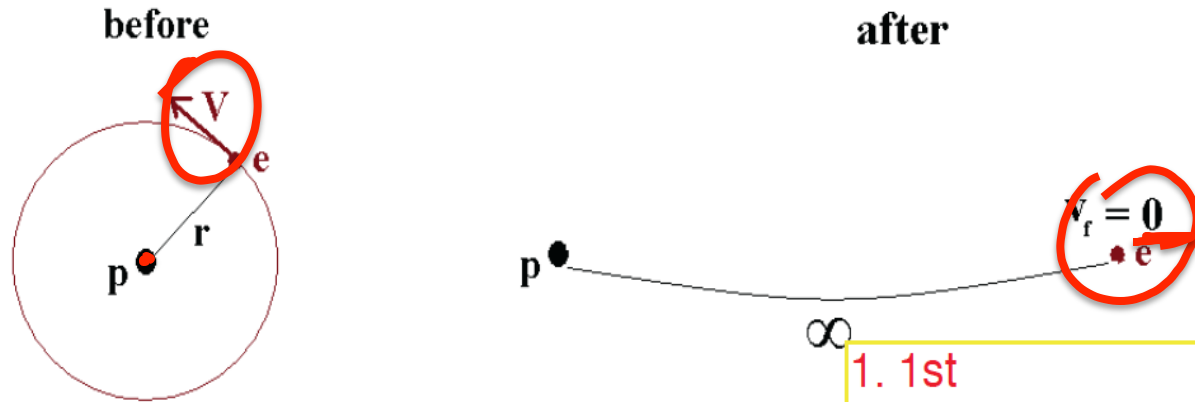


$\neq$

Four charges interact!  
Six pairs of charges contribute  
into the potential energy  
of the system.

# Escape speed

Find the minimum speed an electron, which starts some distance  $r$  from a proton, must have to escape from the proton.  
Assume the proton remains at rest the whole time.



Let's start with the conservation of energy equation

$$U_i + K_i + W_{nc} = U_f + K_f$$

Which terms can we cross out immediately?

1. 1st
2. 2nd
3. 3d
4. 4th
5. 5th
6. there are two terms
7. there are three terms

## Escape speed

$$U_i + K_i + W_{nc} = U_f + K_f$$

Which terms can we cross out immediately?

Assume no resistive forces, so  $W_{nc} = 0$

Assume the electron barely makes it to infinity, so both  $U_f$  and  $K_f$  are zero.

This leaves:  $U_i + K_i = 0$

or  $-\frac{ke^2}{r} + \frac{1}{2}mv_{\text{escape}}^2 = 0$  and  $v_{\text{escape}} = \sqrt{\frac{2ke^2}{mr}}$

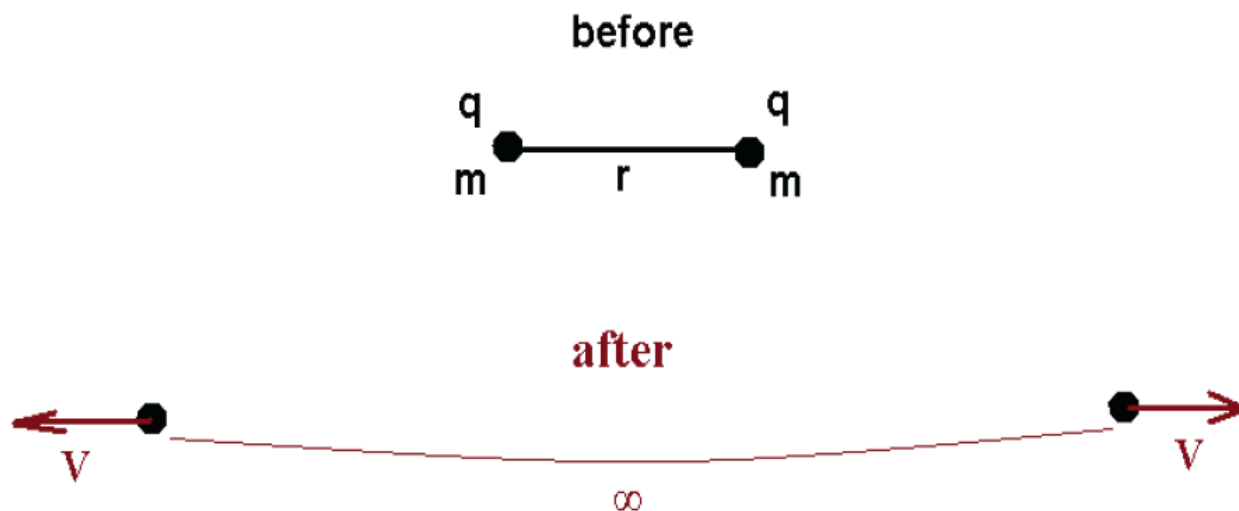
For:  $m = 9.1 \times 10^{-31}$  kg,  $e = 1.6 \times 10^{-19}$  C,  $r = 0.05 \times 10^{-9}$  m

We have:

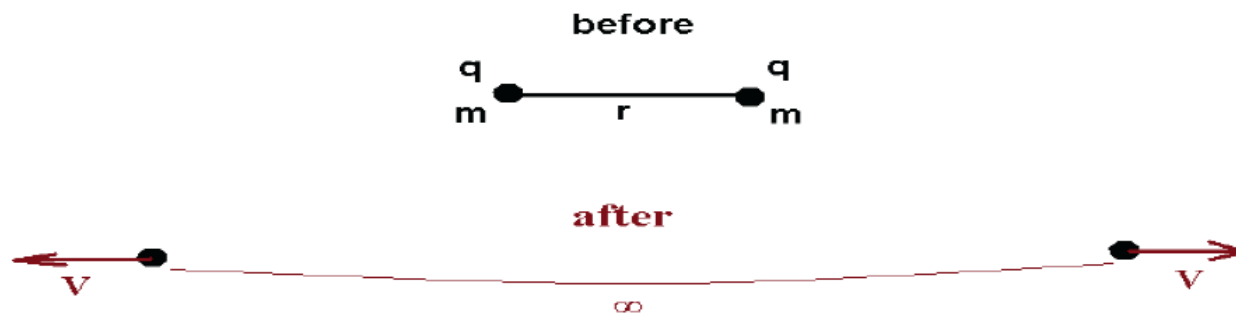
$v_{\text{escape}} \sim 3 \times 10^6$  m/s

Two small identical balls with the mass of 10 gram and the charge of  $10\text{ }\mu\text{C}$  are tied up by a string with the length of 10 cm.

If we cut the string and let the charges moving away what will be the speed of each charge when the distance between them is infinitely large?



**v is about 1. 3 m/s    2. 30 m/s    3. 300 m/s    4. 3000 m/s**



$$E_{\text{total}} = E_k + E_p \quad E_{\text{total}} = \text{const}$$

$$E_{\text{total-initial}} = E_{\text{total-final}} \Rightarrow$$

$$E_{k-i} + E_{p-i} = E_{k-f} + E_{p-f} \Rightarrow$$

$$\begin{array}{ccccccc}
 \underbrace{E_{k-i} = 0} & \underbrace{E_{p-i} = k \frac{q^2}{r}} & E_{k-f} = \underbrace{\frac{1}{2}mv^2} + \underbrace{\frac{1}{2}mv^2} & \underbrace{E_{p-f} = 0} \\
 \Rightarrow & & \Rightarrow & 
 \end{array}$$

$$\begin{array}{ccc}
 \Rightarrow & k \frac{q^2}{r} = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 & \Rightarrow k \frac{q^2}{r} = mv^2
 \end{array}$$

$$\Rightarrow v = \sqrt{\frac{kq^2}{mr}} = \sqrt{\frac{8.99 \cdot 10^9 (10 \cdot 10^{-6})^2}{0.01 \cdot 0.1}} = 29.98 \text{ m/s}$$

Two small identical balls with the mass of 10 gram and the charge of  $10\ \mu\text{C}$  are shot towards each other with the speed 30 m/s (each). If initially they are very far away from each other, find the minimum distance between the balls.



نب



A charge of  $+2.0\text{ }\mu\text{C}$  is placed at the origin. Suppose that a proton is released at  $r = 10\text{ cm}$  from the  $+2.0\text{ }\mu\text{C}$  charge. How fast will it be moving as it passes a point at  $r = 50\text{ cm}$ ?

1.  $5.0\text{ Mm/s}$

2.  $5.1\text{ Mm/s}$

3.  $5.2\text{ Mm/s}$

4.  $5.3\text{ Mm/s}$

5.  $5.4\text{ Mm/s}$

6.  $5.5\text{ Mm/s}$

An electron gun shoots electrons at a metal plate that is 4.0 mm away in vacuum. The electric field in the region between the plate and the gun is 1.25 kV/m. What is the minimum speed electrons should have in order to reach the plate?

1.  $0.13 \times 10^6 \text{ m/s}$       2.  $1.3 \times 10^6 \text{ m/s}$       2.  $13 \times 10^6 \text{ m/s}$



## Electric potential of a charge

$$V = \frac{V}{q}$$

It can be proved that

The electric potential  $V$  of a charge  $Q$   
at the distance  $r$  from the charge

$$V = k \frac{Q}{r}$$

(this works for one point-like charge ONLY!)

PRS

In order to use this expression the zero level is to be chosen:

1. at the location of the charge
2. at infinity

## The Potential of a Point Charge; an Example

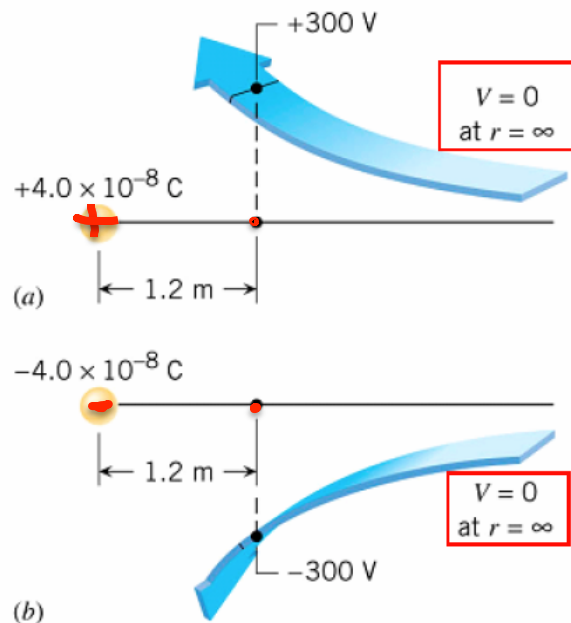
Using a zero reference potential at infinity, determine the amount by which a point charge of  $4.0 \times 10^{-8} \text{ C}$  alters the electric potential at a spot 1.2 m away when the charge is (a) positive and (b) negative.

(a)

$$V = \frac{kq}{r} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(+4.0 \times 10^{-8} \text{ C})}{1.2 \text{ m}} = \underline{+300 \text{ V}}$$

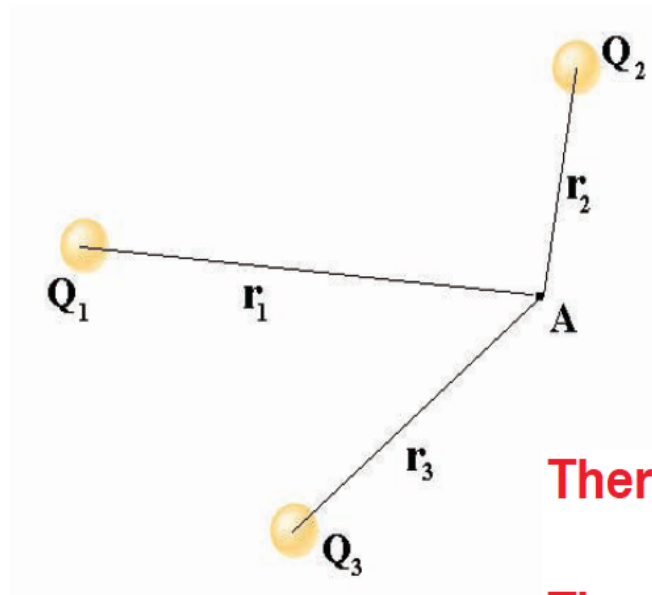
(b)

$$V = \underline{-300 \text{ V}}$$



# The Total Electric Potential

Electric potential at a given point  
created by a system of charges  
is equal to the algebraic sum of individual potentials  
created by each individual charge at the point.



$$V_A = V_1 + V_2 + V_3$$

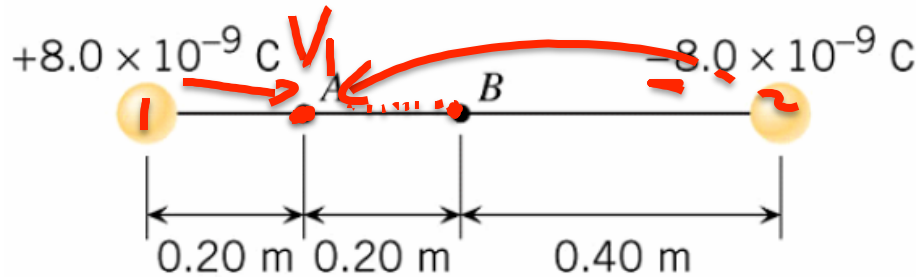
$$V_A = k \frac{Q_1}{r_1} + k \frac{Q_2}{r_2} + k \frac{Q_3}{r_3}$$

**There are NO vectors!**

**There are NO magnitudes!**

## Example

At locations A and B, find the total electric potential.



$$V_A = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(+8.0 \times 10^{-8} \text{ C})}{0.20 \text{ m}} + \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(-8.0 \times 10^{-8} \text{ C})}{0.60 \text{ m}} = +240 \text{ V}$$

$$V_B = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(+8.0 \times 10^{-8} \text{ C})}{0.40 \text{ m}} + \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(-8.0 \times 10^{-8} \text{ C})}{0.40 \text{ m}} = 0 \text{ V}$$

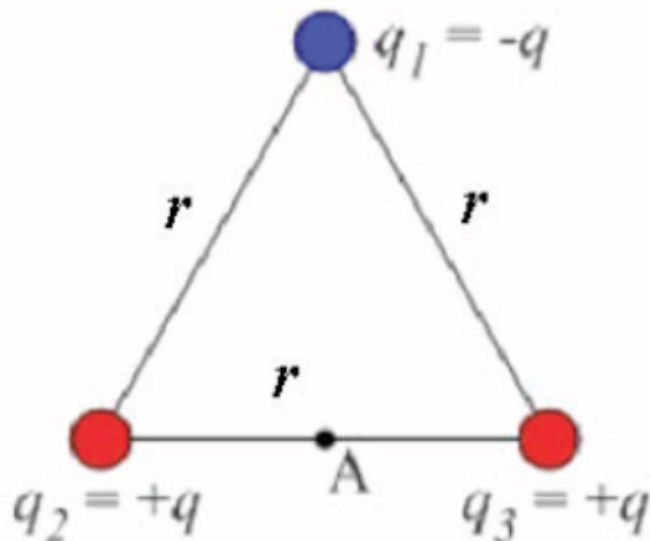
A point charge of  $-2.0\ \mu\text{C}$  is placed on the y-axis at  $y = -100\ \text{cm}$ . A second, of  $3.0\ \mu\text{C}$ , is placed on the y-axis at  $y = 200\ \text{cm}$ . If a test charged is moving from a point A to a point B (see below) what is the potential difference the charges is falling through? Points A and B are on the y-axis: point-A at  $y = 0.1\ \text{m}$  and point-B at  $y = 0.9\ \text{m}$ .

1.  $-17\ \text{kV}$     2.  $-8.5\ \text{kV}$     3.  $0\ \text{V}$     4.  $8.5\ \text{kV}$     5.  $17\ \text{kV}$

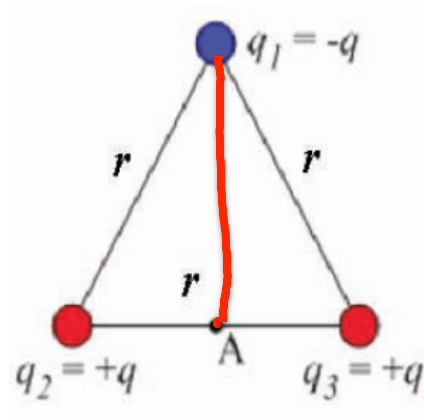
## Practice Exercise

### Three charges

What is the potential at the point A?



## Three charges

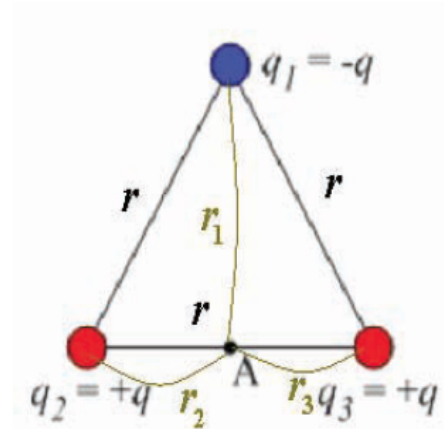


What is the potential at the point A?

$$V = V_1 + V_2 + V_3$$

$$r_2 = \frac{1}{2}r \quad r_3 = \frac{1}{2}r$$

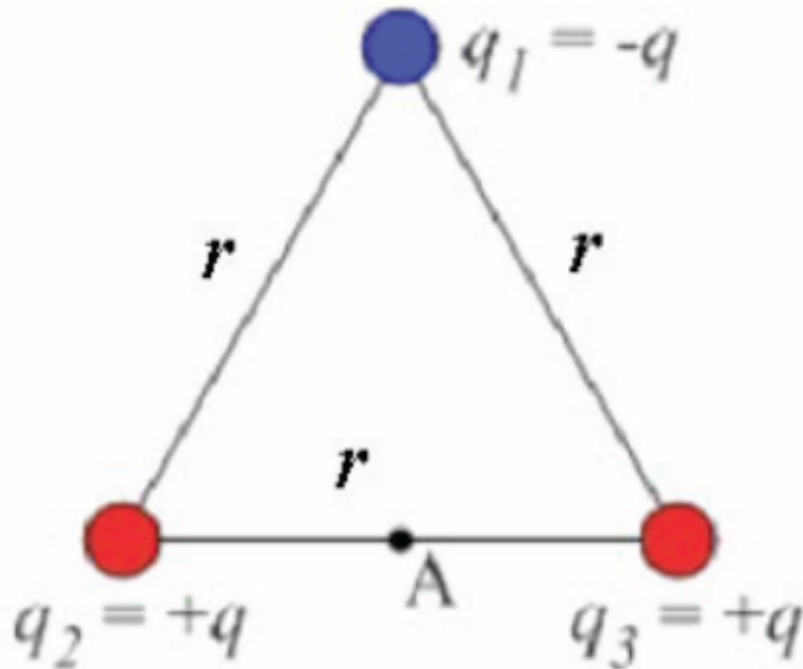
$$r_1 = \sqrt{r^2 - r_2^2} = \frac{\sqrt{3}}{2}r$$



$$V = k \frac{-q}{r_1} + k \frac{q}{r_2} + k \frac{q}{r_3} = k \frac{-q}{r_1} + k \frac{q}{r_2} + k \frac{q}{r_3} = k \frac{q}{r} \left( -\frac{2}{\sqrt{3}} + 2 + 2 \right) = \left( 4 - \frac{2}{\sqrt{3}} \right) k \frac{q}{r}$$

# Three charges

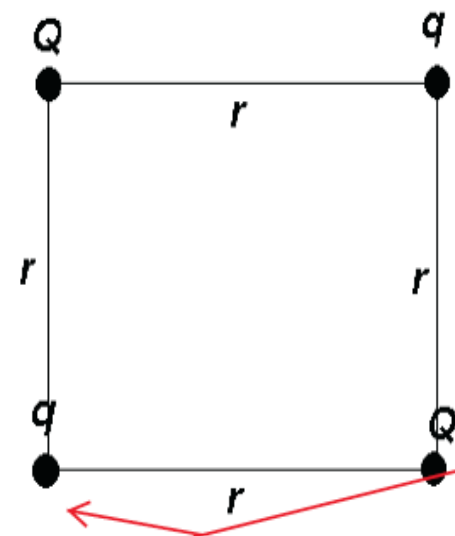
What is the potential at the location of a red charge?



1. Some positive number
2. 0
3. Some negative number



### Four charges again



If  $Q = 2 \mu\text{C}$  and  $q = -4 \mu\text{C}$  and  $r = 10 \text{ cm}$ .

What is the value of the electrostatic field potential three other charges make at the location of the charge  $q$ ?

Potential  $V$  is about

1. 10 V

2. 100 V

3. 1000 V

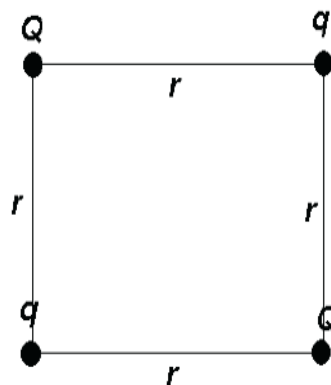
4. 10000 V

5. 100000 V

6. 1000000 V

7. 10000000 V

## Four charges again

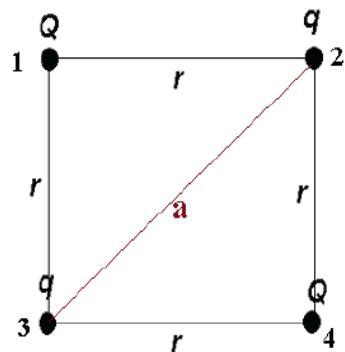


If  $Q = 2 \mu\text{C}$  and  $q = -4 \mu\text{C}$  and  $r = 10 \text{ cm}$ .

What is the value of the electrostatic field potential three other charges make at the location of the charge  $q$ ?

We need to numerate the charges and introduce the distances.

Now we can write the expression for the total potential at the location of the charge # 3.



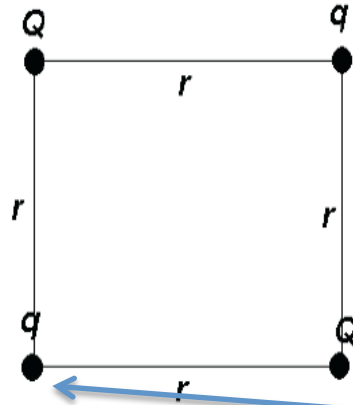
$$V_3 = V_{13} + V_{23} + V_{43} \qquad V_{13} = V_{43} = k \frac{Q}{r}$$

$$V_{23} = k \frac{q}{a} \quad \text{and} \quad a = \sqrt{r^2 + r^2} = r\sqrt{2}$$

Together that gives us:

$$V_3 = k \frac{Q}{r} + k \frac{Q}{r} + k \frac{q}{a} = 8.99 \cdot 10^9 \frac{2 \cdot 10^{-6}}{0.1} \cdot 2 + 8.99 \cdot 10^9 \frac{(-4 \cdot 10^{-6})}{0.1 \cdot \sqrt{2}} = 10.53 \cdot 10^4 \text{ V}$$

## Four charges again



If  $Q = 2 \mu\text{C}$  and  $q = -4 \mu\text{C}$  and  $r = 10 \text{ cm}$ .

What is the value of the electrostatic field *potential* three other charges make at the location of the charge  $q$ ?

We just found it; it is  $V = 10.53 \cdot 10^4 \text{ V}$

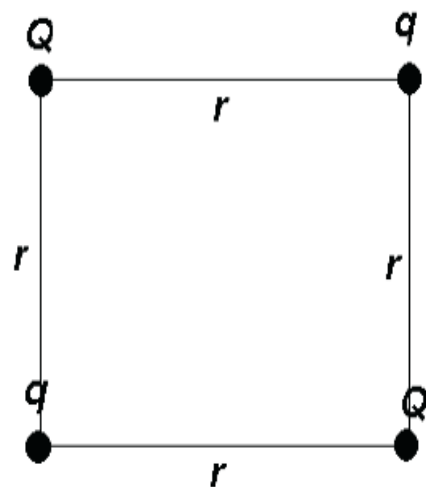
What is the *potential energy* of the charge  $q$  in the field created by the three other charges?

This potential energy is ...

1. positive

2. negative

### Four charges again



If  $Q = 2 \mu\text{C}$  and  $q = -4 \mu\text{C}$  and  $r = 10 \text{ cm}$ .

What is the value of the electrostatic field *potential* three other charges make at the location of the charge  $q$ ?

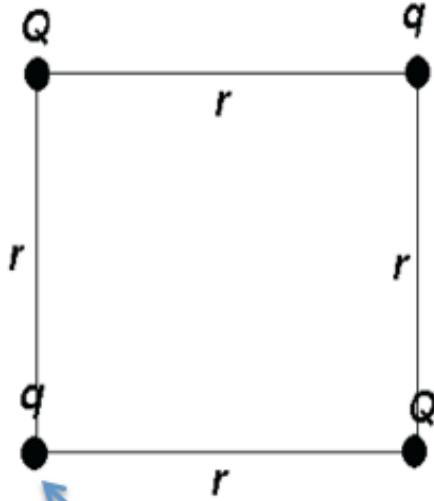
We just found it; it is  $V = 10.53 \cdot 10^4 \text{ V}$

What is the *potential energy* of the charge  $q$  in the field created by the three other charges?

$$EPE = q \cdot V \Rightarrow$$

$$EPE = -4 \cdot 10^{-6} \cdot 10.53 \cdot 10^4 = -0.4212 \text{ J}$$

## Four charges again



If  $Q = 2 \mu\text{C}$  and  $q = -4 \mu\text{C}$  and  $r = 10 \text{ cm}$ .

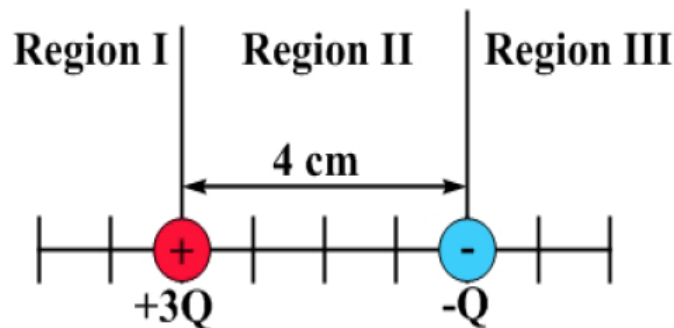
What is the value of the electrostatic field *potential* three other charges make at the location of the charge  $q$ ?

$$\text{EPE} = -4 * 10^{-6} * 10.53 * 10^4 = -0.4212 \text{ J}$$

How much work do we have to do to bring this charge to infinity?

1.  $-0.4212 \text{ J}$       2.  $0 \text{ J}$       3.  $0.4212 \text{ J}$

## Worksheet: where is the potential zero?

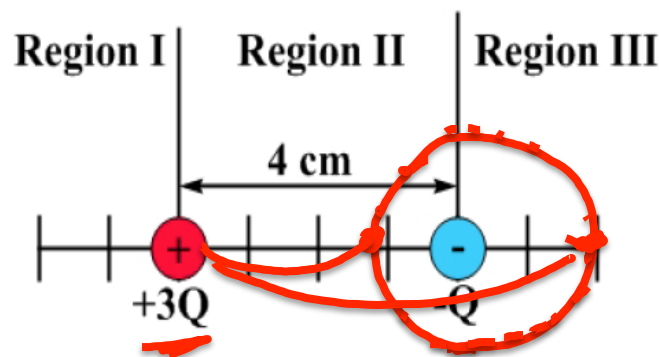


Two charges,  $+3Q$  and  $-Q$ , are separated by  $4\text{ cm}$ .

Where is a point along the line passing through them (and a finite distance from the charges) where *the net electric potential is zero*?

1. Region I only
2. Region II only
3. Region III only
4. Region I and II
5. Region II and III

## Worksheet: where is the potential zero?



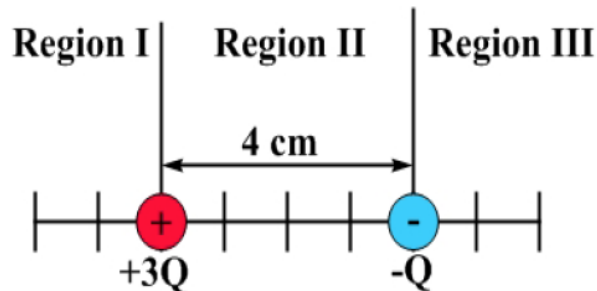
Two charges,  $+3Q$  and  $-Q$ , are separated by 4 cm. At some point  $V = 0$ .

$$V = 0 = V_{+3Q} + V_{-Q} = k \frac{3Q}{r_1} + k \frac{-Q}{r_2} = k \frac{3Q}{r_1} - k \frac{Q}{r_2} \Rightarrow$$

$$k \frac{3Q}{r_1} - k \frac{Q}{r_2} = 0 \quad \text{or} \quad k \frac{3Q}{r_1} = k \frac{Q}{r_2}$$

$$\Rightarrow \frac{3}{r_1} = \frac{1}{r_2} \Rightarrow 3r_2 = r_1$$

## Worksheet: where is the potential zero?



Two charges,  $+3Q$  and  $-Q$ , are separated by 4 cm.

Where is a point along the line passing through them (and a finite distance from the charges) where the net electric potential is zero?

Remember:  $V = V_{+3Q} + V_{-Q} = k \frac{3Q}{r_1} + k \frac{-Q}{r_2} = k \frac{3Q}{r_1} - k \frac{Q}{r_2}$

$$3r_2 = r_1$$

To make  $V = 0$  the right point should be three times closer to the charge  $-Q$  than to the charge  $3Q$ !

How many points like that are there on the line?

1. 1

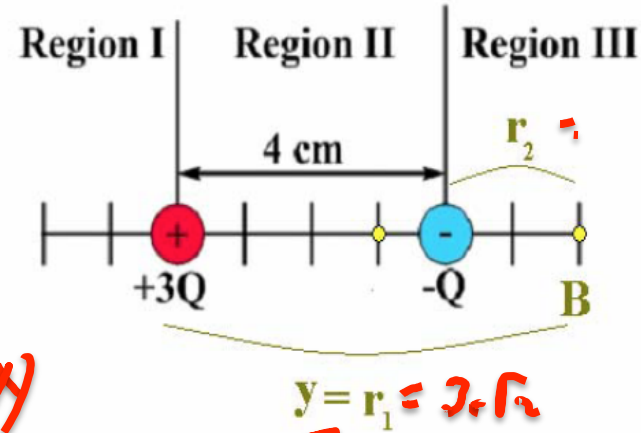
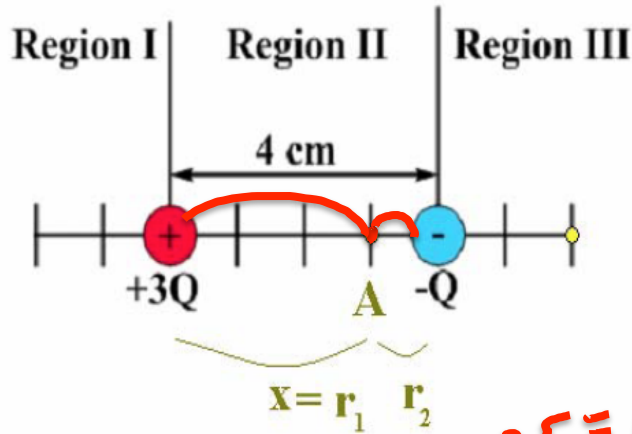
2. 2

3. 3

4. 4



# Worksheet: where is the potential zero?



We have TWO points where  $3r_2 = r_1$  (see the pictures).

Hence we have to write TWO equations from one equation  $3r_2 = r_1$

For the point A:

$$3(4 - x) = x$$

$$12 - 3x = x$$

$$x = 3 \text{ cm}$$

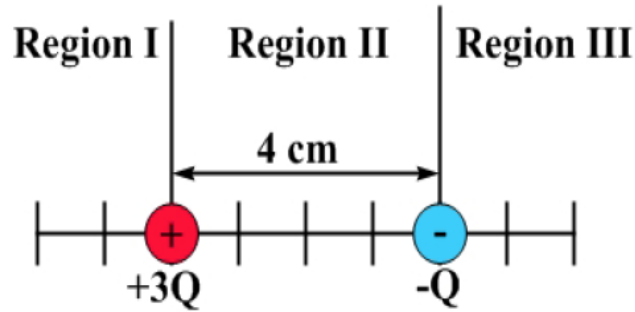
For the point B:

$$3(y - 4) = y$$

$$3y - 12 = y$$

$$y = 6 \text{ cm}$$

## Worksheet: where is the potential zero?

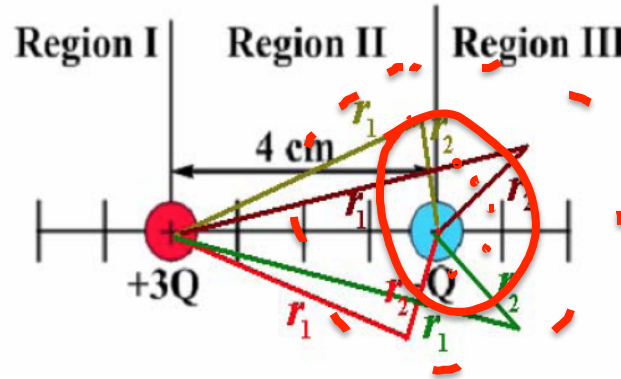


Are there other locations which are NOT on the line connecting the charges, but where the net potential is zero?

1. Yes
2. No
3. This questions is too hard

Hint: think of the condition  $3r_2 = r_1$

## Worksheet: where is the potential zero?



Are there are places that are *not* on the straight line joining the charges, a finite distance away, where the potential is *zero*?

**A. Yes**

As long as the condition  $3r_2 = r_1$  is fulfilled, the net potential at that point is 0!

There is an *infinite* number of points where  $V = 0$ !

**There is a whole equipotential *line* with  $V = 0$  (as well as  $V > 0$  and  $V < 0$ )**

Which statement is true?

1. field lines are *parallel* to equipotentials and directed from a *higher* potential to a *lower* potential
2. field lines are *parallel* to equipotentials and directed from a *lower* potential to a *higher* potential
3. field lines are *perpendicular* to equipotentials and directed from a higher potential to a *lower* potential
4. field lines are *perpendicular* to equipotentials and directed from a *lower* potential to a *higher* potential

# Electric potential of a charge

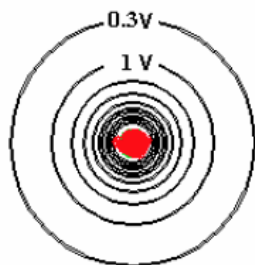
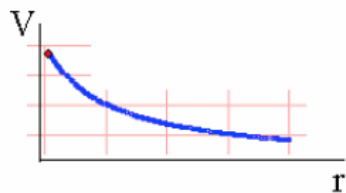
$$V = k \frac{Q}{r}$$

$$Q > 0$$

$$Q = 10^{-9}/8.99$$

$$V = 1/r$$

r (m)	V (V)
1	1
2	0.5
3	0.33



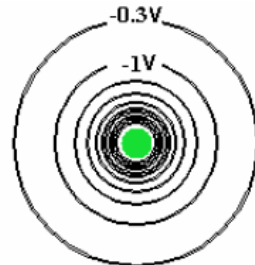
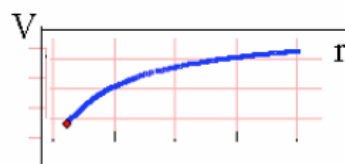
The *equipotentials*  
are circles!

$$Q < 0$$

$$Q = -10^{-9}/8.99$$

$$V = -1/r$$

r (m)	V (V)
1	-1
2	-0.5
3	-0.33



The *equipotentials*  
are circles!

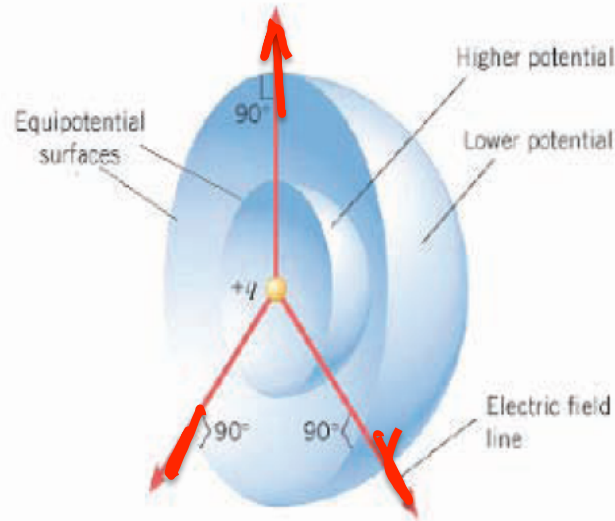
# Electric potential of a charge

## Three-dimensional representation (Equipotential surfaces)

Electric potential from a point charge :

$$V = \frac{kq}{r}$$

$q > 0$



Notice: Electric field (red arrows) is ALWAYS:

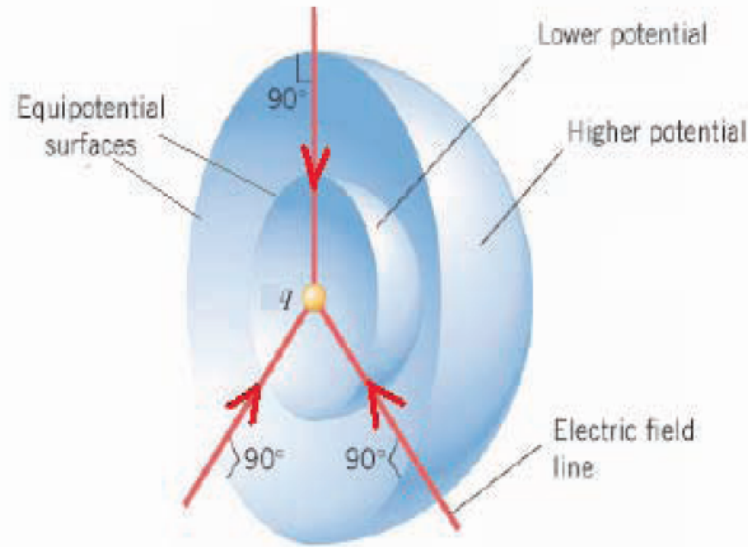
- 1) *perpendicular* to equipotentials!
- 2) directed from a higher potential to a *lower* potential!

## Three-dimensional representation (Equipotential surfaces)

Electric potential from a point charge :

$$V = \frac{kq}{r}$$

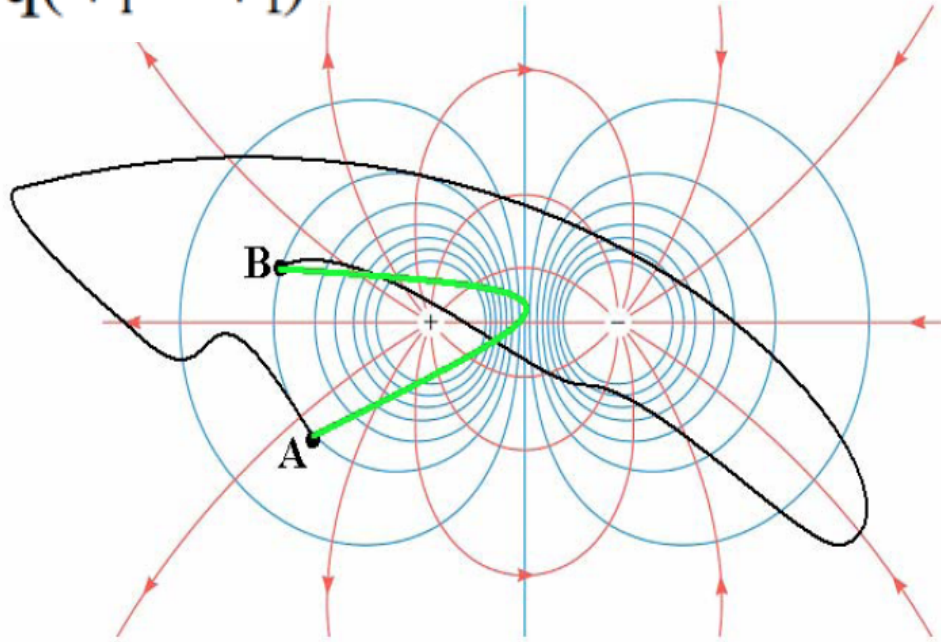
$$q < 0$$



Notice: Electric field (red arrows) is ALWAYS:

- 1) *perpendicular* to equipotentials!
- 2) directed from a higher potential to a *lower* potential!

$$W_E = q(V_i - V_f)$$



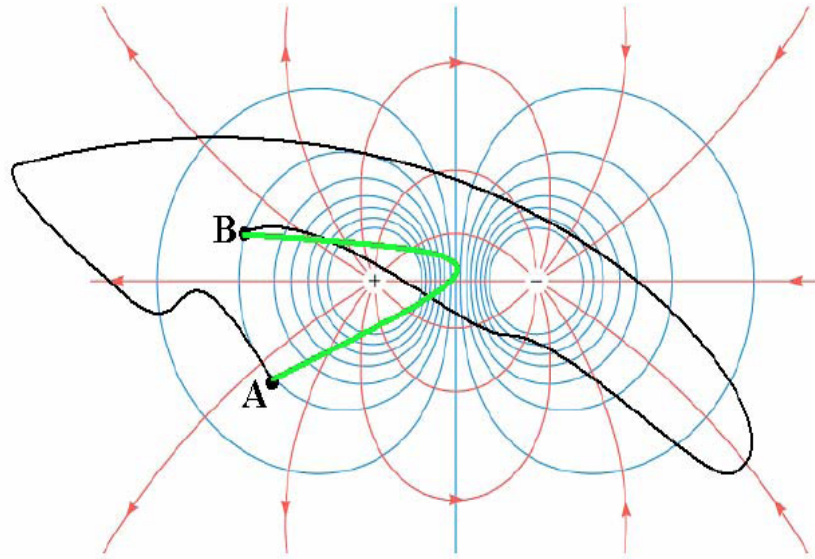
A charge  $Q$  was moved from the point A to the point B twice:

1. at first time it was moving along the path shown by a black curved line.
2. then it was moving along the green line.

The work done by the field on the charge in both cases was:

1. some positive number
2. some negative number
3. zero
4. not enough information





A charge  $Q$  was moved from the point A to the point B twice:

1. at first time it was moving along the path shown by a black curved line.
2. then it was moving along the green line.

The work done by the field on the charge in both cases was:

**C. zero**

$$W_{El} = q(V_A - V_B)$$

Points A and B belong to the same equipotential line,  
hence  $V_A = V_B$  and  $V_A - V_B = 0!$