Experiments with an electroscope

We make an electroscope being negatively charged.

Now we bring a negatively charged rod close to its plate (but not touching the plate).

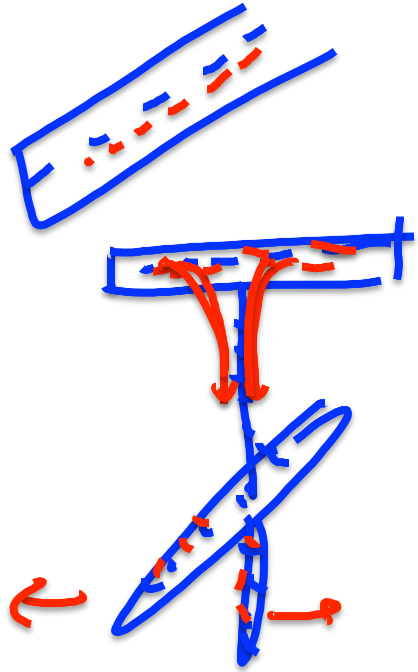
What is happening to the moving arm of the electroscope?

1. Nothing

2. The arm deflects even more

3. The arm deflects less

(Other options: what if the rod is positive; a neutral insulator; a neutral conductor; what if the electroscope is positive?)



2. The arm deflects even more

Two electrons are separated by 1 m distance.

Do they repel or attract each other?

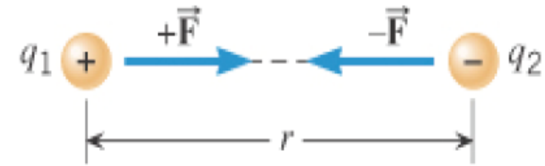
Experiment shows the following relation between the magnitude of the force and the distance:

D, m	**1**	**2**	**3**	**4**	**5**	**6**
F, $*10^{-28}$ **N**	2.3	0.575	0.256	0.144	0.092	0.064

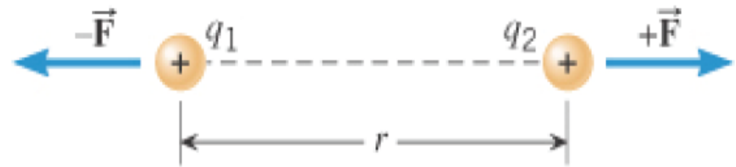
Sketch the graph for the magnitude of the force as a function of the distance.

Is it increasing or decreasing function? Which function fits the best the graph: $\sim x$, $\sim x^2$, $\sim \frac{1}{x}$, $\sim \frac{1}{x^2}$ **?**

Two small spheres are charged; one is missing $7*10^{22}$ electrons; another one has extra $3*10^{22}$ electrons. If the distance between the spheres is 4 m, find the force (magnitude and direction) between the spheres.



(a)



(b)

The **magnitude** of the electrostatic force exerted by one point charge on another point charge is directly proportional to the magnitude of the charges and inversely proportional to the square of the distance between them.

$$F = k\frac{|q_1||q_2|}{r^2}$$

$$\varepsilon_o = 8.85\times10^{-12}\ \mathrm{C^2/(N\cdot m^2)}$$

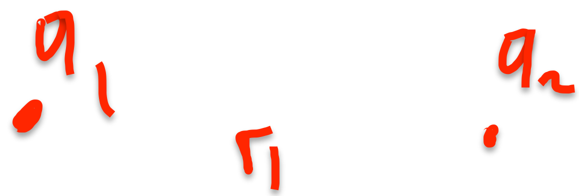
$$k = 1/(4\pi\varepsilon_o) = \quad 9\times10^{9}\ \mathrm{N\cdot m^2/C^2}$$

Coulomb's Law

$$|F_{el}| = k\frac{|q_1 \bullet q_2|}{r^2}$$

(An individual force acting between 2 charges)

Change	Force
Make q_1 n times large	n times stronger
Make q_2 n times smaller	n times weaker
Make r n times longer	n^2 times weaker
Make r n times shorter	n^2 times stronger

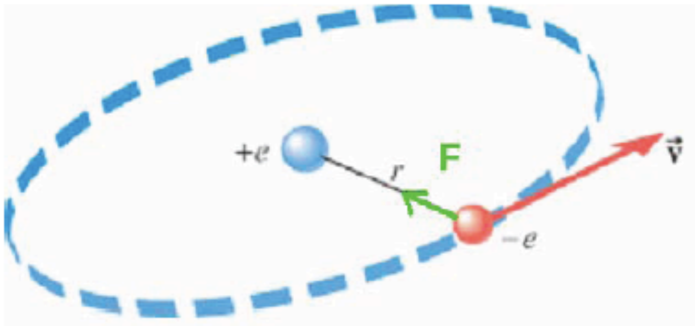


$$F_1 = k\frac{|q_1 \cdot q_2|}{r_1^2} = 16N$$

$$r_2 = 4 \cdot r_1$$

$$F_2 = k\frac{|q_2 \cdot q_2|}{(4 \cdot r_2)^2};$$

$$F_2 = \frac{16}{4^2} = \frac{4N}{4} = 2N$$



According the III N L
The force acting on the proton is equal in magnitude to the force acting on the electron:

$$F_p = F = 8.22\times10^{-8}\,\text{N}$$

The electric force acting on the electron is inversely proportional to the radius squared:

$$F \sim \frac{1}{r^2} \qquad F_1 \sim \frac{1}{r_1^2} \qquad F_2 \sim \frac{1}{r_2^2} = \frac{1}{(3r_1)^2} = \frac{1}{9r_1^2} = \frac{F_1}{9}$$

When the radius is three times greater, the force is nine times smaller!

$$\text{F} = \frac{8.22\times10^{-8}}{9} = 0.91\times10^{-8} = 9.1\times10^{-9}\,N$$

Two identical tiny metal balls carry charges of +3 μC and -12 μC. They are 3 m apart. We brought the charges together and then placed again at the same distance. (a) Compare the force between the charges before and after the touching. (b) What should be the new distance so the force would not change?

The new distance should be …

1.the same

2. larger then before

3. smaller than before

$$F_2 = K \frac{|q_1' q_2'|}{3^2}$$

$$F_1 = K \frac{|q_2 \cdot q_1|}{3^2}$$

$$F_1 = \frac{K}{3^2} |3 \cdot 12|$$

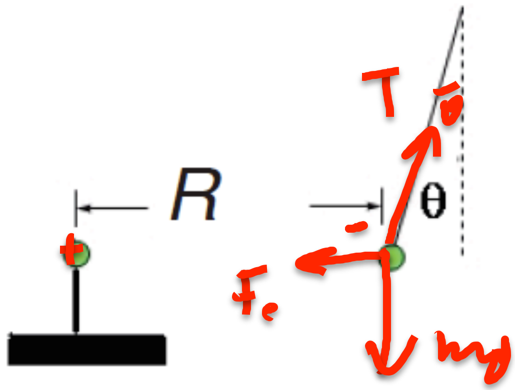
$$36$$

$$F_2 = \frac{K}{3^2} \cdot |4{,}5 \cdot 4{,}5|$$

$$20{,}25$$

$$F_2 < F_1$$

Two balls



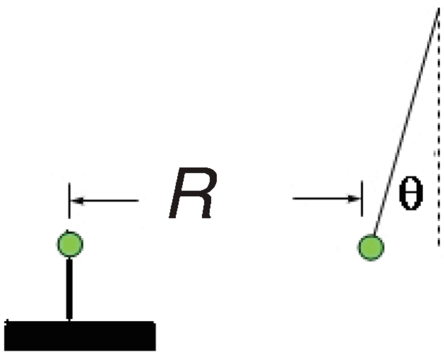
Two identical balls (see the picture) each have a charge of the same magnitude and the same mass

The balls are separated by a distance R

1. charges are both positive
2. charges are both negative
3. the left one is positive, the right one is negative
4. the left one is negative, the right one is positive
5. we do not know the exact polarity but there are definitely of opposite polarity
6. we do not know the exact polarity but there are definitely of the same polarity

Draw FBD

Two balls



Two identical balls (see the picture) each have a charge of the same magnitude and the same mass

The balls are separated by a distance R

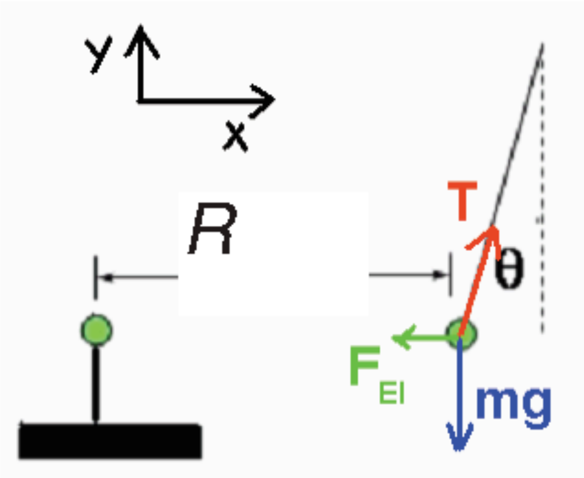
Find the angle θ.

Find the tension in the string.

If we cut the string off the ball what is the initial acceleration of the second ball?

Two balls

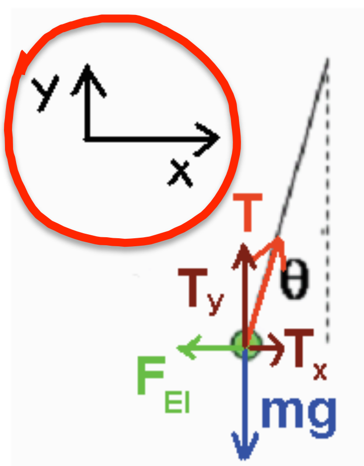
$|q_1| = |q_2| = q$



Three forces are acting on the ball on a string: electric force F_{El}, force of gravity mg, and force of tension T.

At the equilibrium: $\vec{F}_{El} + m\vec{g} + \vec{T} = 0$

In components this gives us:



X: $|T_x| = |F_{El}| = k\dfrac{q^2}{R^2}$

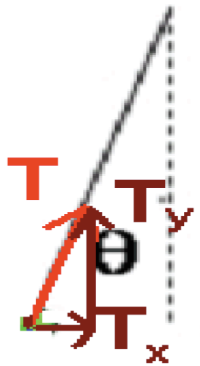
Y: $|T_y| = |mg|$

$$\tan\theta = \left(\frac{mg}{k\frac{q^2}{R^2}}\right)^{-1}$$

From the picture:

$$\tan\theta = \frac{|T_x|}{|T_y|}$$

Using all the given number (converted into SI system), we can find θ.



We know T_x and T_y.

To calculate the tension in the string T we can use the Pythagorean Theorem:

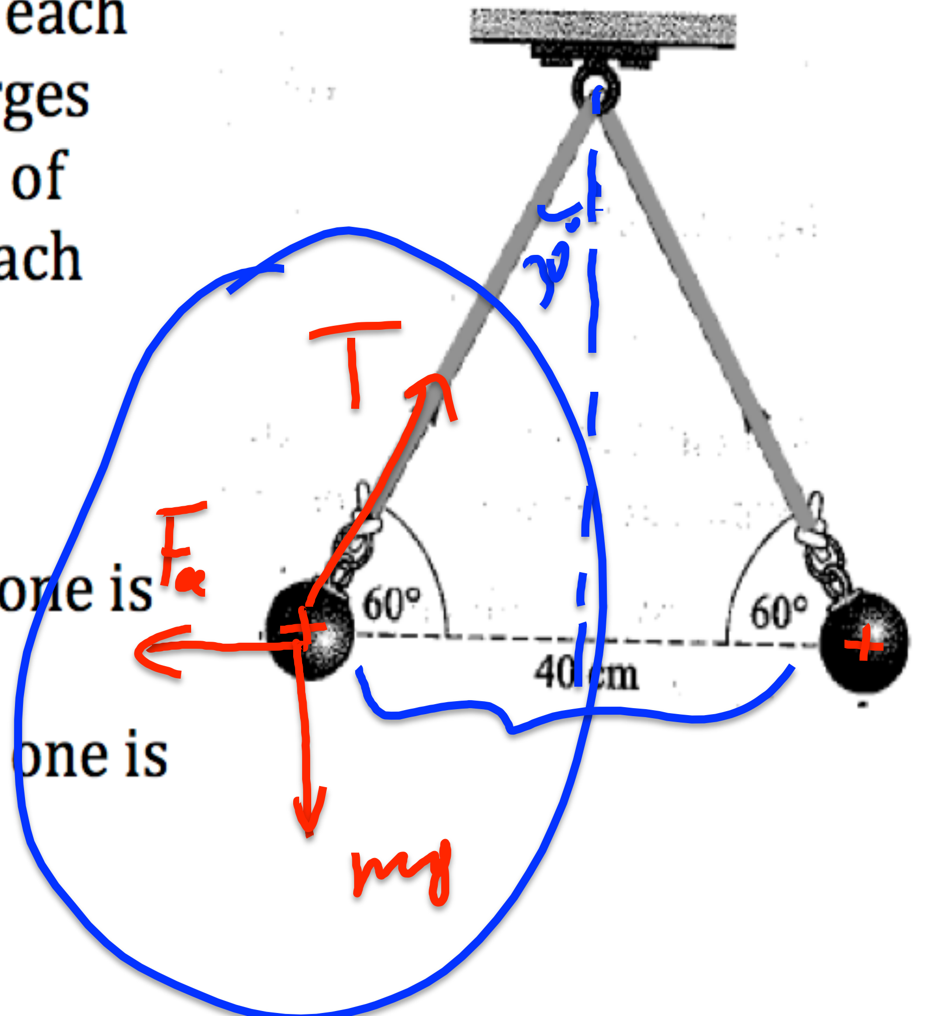
$$T = \sqrt{T_x^2 + T_y^2}$$

If we cut of the string, the ball has only two forces acting on it: electric force F_{El} and mg.

According the Newton's II Law: $m\vec{a} = \vec{F}_{El} + m\vec{g}$

From $\vec{F}_{El} + m\vec{g} + \vec{T} = 0$ we get (at the initial moment when the ball is still at the initial location): $|\vec{F}_{El} + m\vec{g}| = T$: hence $a = \dfrac{T}{m}$

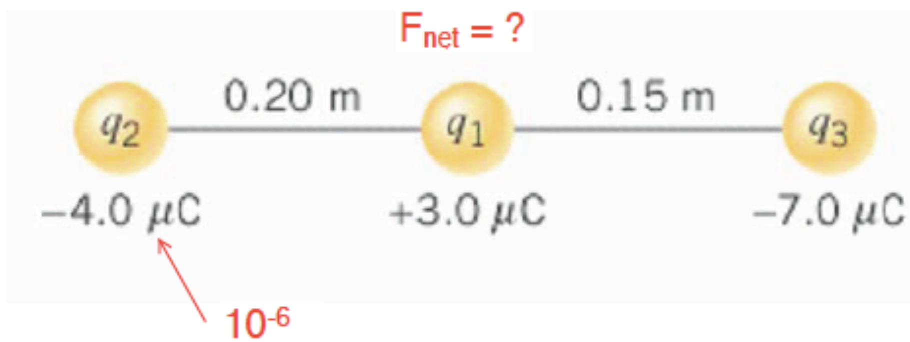
As shown in the picture, two balls, each of mass 0.20 g, carry identical charges and are suspended by two threads of equal length. Find the charge on each ball.



1. both charges are positive
2. the left one is positive, the right one is negative
3. the left one is negative, the right one is positive
4. both charges are negative
5. there might be several options

Coulomb's Force for several charges

Three Charges on a Line

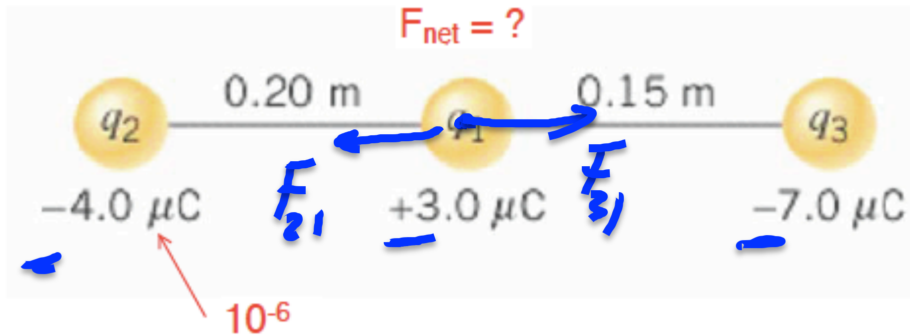


1) Determine the magnitude and direction of the net force on q_1.

2) If you double all the distances and all the charges, find it again

Coulomb's Force for several charges

Three Charges on a Line



1) Determine the magnitude and direction of the net force on q_1.

2) If you double all the distances and all the charges, find it again

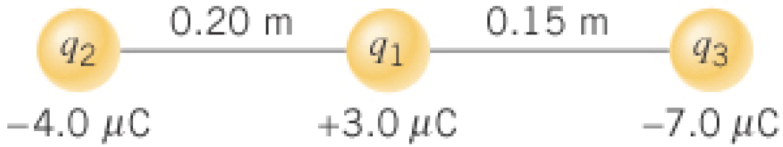
$F_{net_1} = F_{31} + F_{21}$

$F_{31} = \dfrac{9 \cdot 10^9 \cdot 3 \cdot 10^{-6} \cdot 7 \cdot 10^{-6}}{(.15)^2} = 8.4\ N$

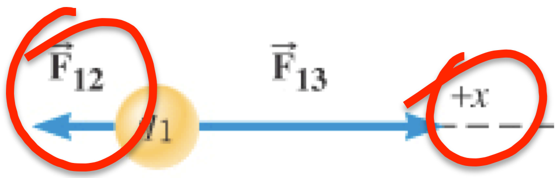
$F_{21} = \dfrac{9 \cdot 10^9 \cdot 3 \cdot 10^{-6} \cdot 4 \cdot 10^{-6}}{0.2^2} = 2.7\ N$

$F_{net_1} = 8.4 + 2.7 = 11.1\ N$

1. Yes 2. No



(*a*)



(*b*) Free-body diagram for q_1

If we double *all* the charges and distances, the net force ...

1. weakens
2. does not change
3. getting stronger

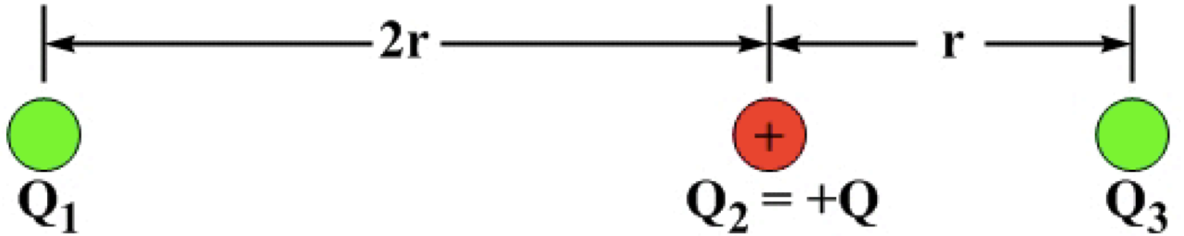
$$F_{12} = k\frac{|q_1||q_2|}{r^2} = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(3.0\times10^{-6}\ \mathrm{C}\right)\left(4.0\times10^{-6}\ \mathrm{C}\right)}{(0.20\mathrm{m})^2} = 2.7\mathrm{N}$$

$$F_{13} = k\frac{|q_1||q_3|}{r^2} = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(3.0\times10^{-6}\ \mathrm{C}\right)\left(7.0\times10^{-6}\ \mathrm{C}\right)}{(0.15\mathrm{m})^2} = 8.4\mathrm{N}$$

Finally, we can find the net force (relative to the x axis):

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_{12} + \vec{\mathbf{F}}_{13} = -2.7\mathrm{N} + 8.4\mathrm{N} = +5.7\mathrm{N}$$

Three charges in a line



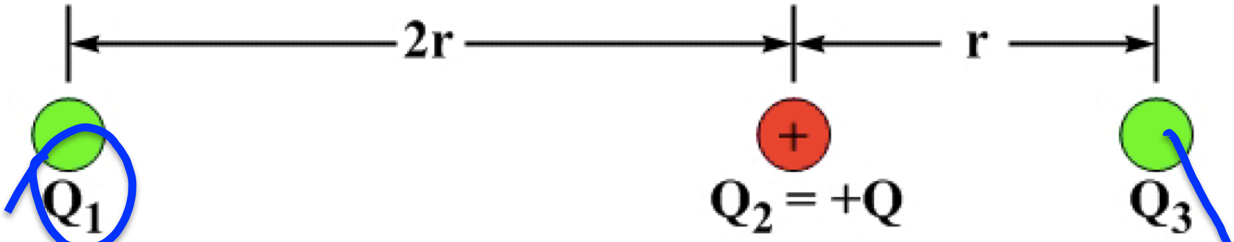
Ball 1 has an unknown charge and sign.
Ball 2 is positive, with a charge of $+Q$.
Ball 3 has an unknown non-zero charge and sign.

Ball 3 is in equilibrium - what does it mean?

What is the sign of the charge on ball 1?

1. Positive
2. Negative
3. We can't tell unless we know the sign of the charge on ball 3.

Three charges in a line



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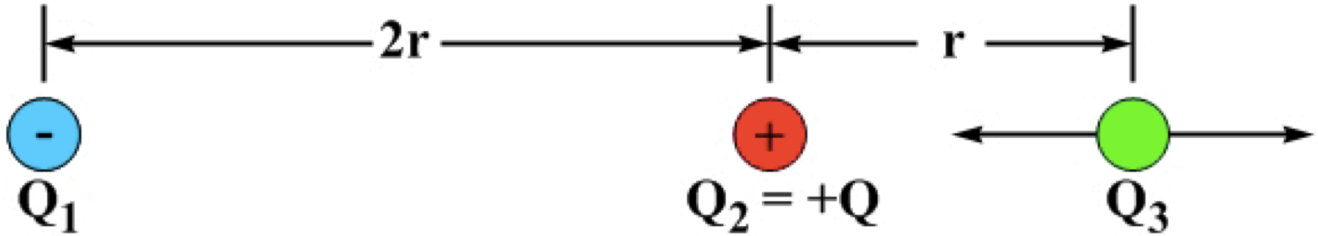
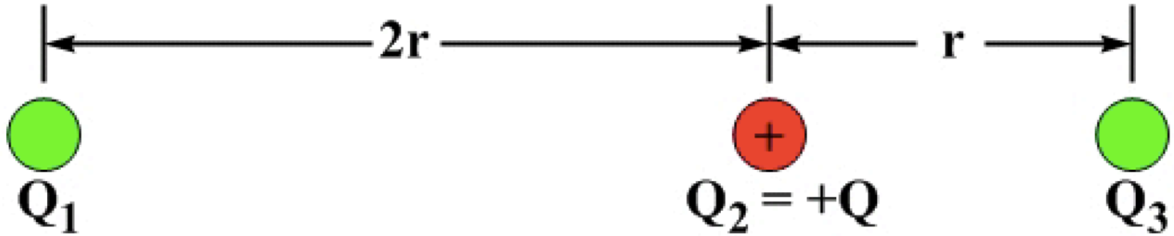


Ball 3 is in equilibrium because it experiences equal-and-opposite forces from the other two balls, so ball 1 must have a negative charge. Flipping the sign of the charge on ball 3 reverses both these forces, so they still cancel.

2. Negative

Three charges in a line

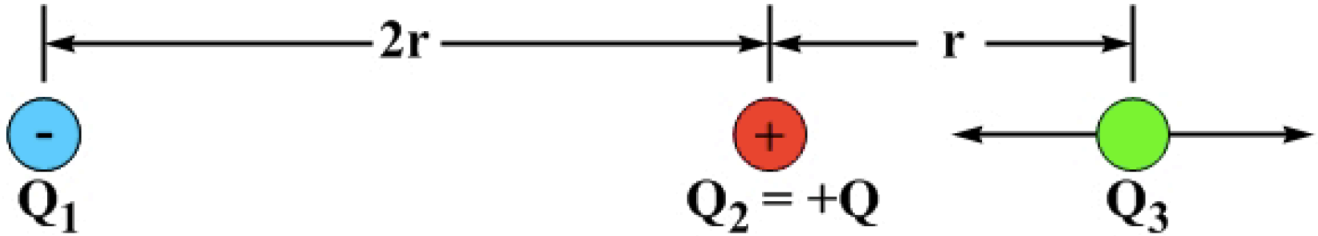


Ball 1 has an unknown charge and sign.
Ball 2 is positive, with a charge of $+Q$.
Ball 3 has an unknown non-zero charge and sign.

Ball 3 is in equilibrium - what does it mean?

Find Q_1

Three charges in a line



Let's do the math. Define to the right as positive.

$$|F_{13}| = |F_{23}|$$

$$\frac{k|Q_1|Q_3}{(3r)^2} = \frac{kQQ_3}{r^2}$$

$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23} = 0$$

$$-\frac{k|Q_1|Q_3}{(3r)^2} + \frac{kQQ_3}{r^2} = 0 \leftarrow$$

what do we use in this equation, magnitudes or components?

$$\frac{|Q_1|}{9} = Q$$

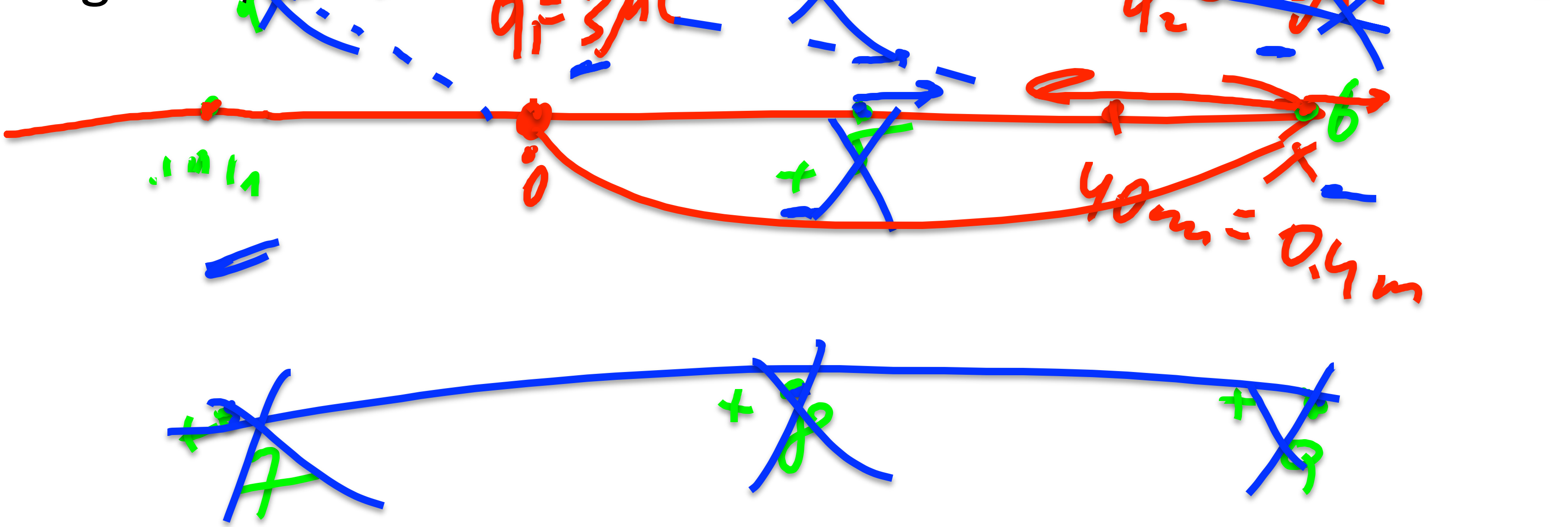
$$|Q_1| = 9Q$$

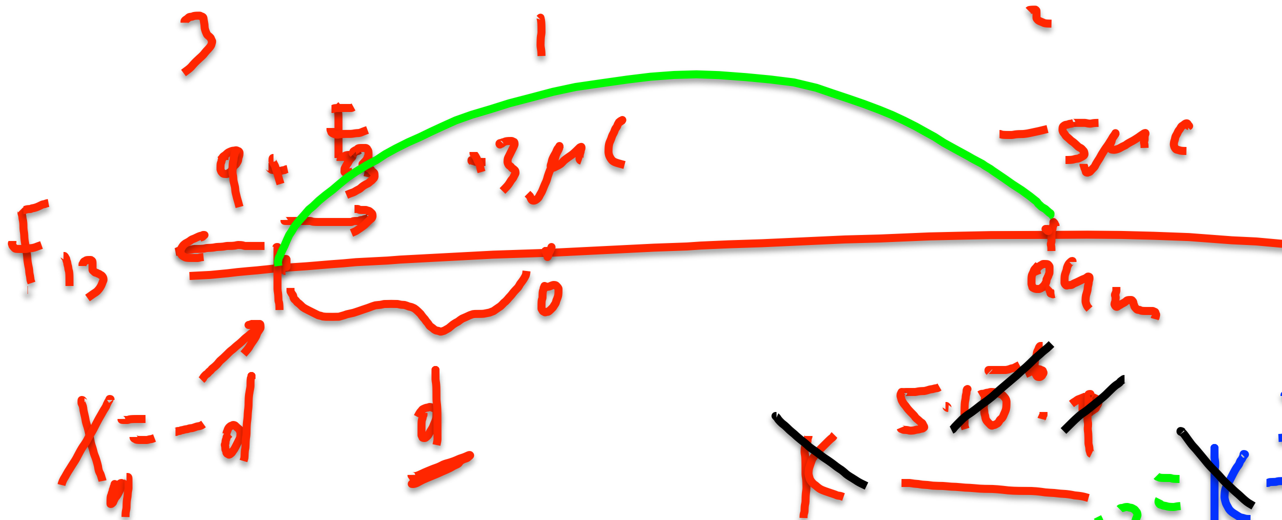
the ratio $|Q_1/Q_2| = 9$

$Q_1/Q_2 = -9$

Two small charged spheres are placed on the x-axis: +3.0 μC at x = 0 and -5.0 μC at x = 40 cm. Where must a third charge q be placed if the force it experiences is to be zero? Also solve the problem when both charges are positive (or negative).

Two small charged spheres are placed on the x-axis: +3.0 μC at x = 0 and -5.0 μC at x = 40 cm. Where must a third charge q be placed if the force it experiences is to be zero? Also solve the problem when both charges are positive (or negative).





$$k\frac{5\cdot10^{-6}\cdot q}{(0.4+d)^2} = k\frac{3\cdot10^{-6}\cdot q}{d^2}$$

$$|F_{23}| = |F_{13}|$$

$$\sqrt{\frac{5}{(0.4+d)^2}} = \sqrt{\frac{3}{d^2}}$$

$$\frac{\sqrt{5}}{0.4+d} = \frac{\sqrt{3}}{d}$$

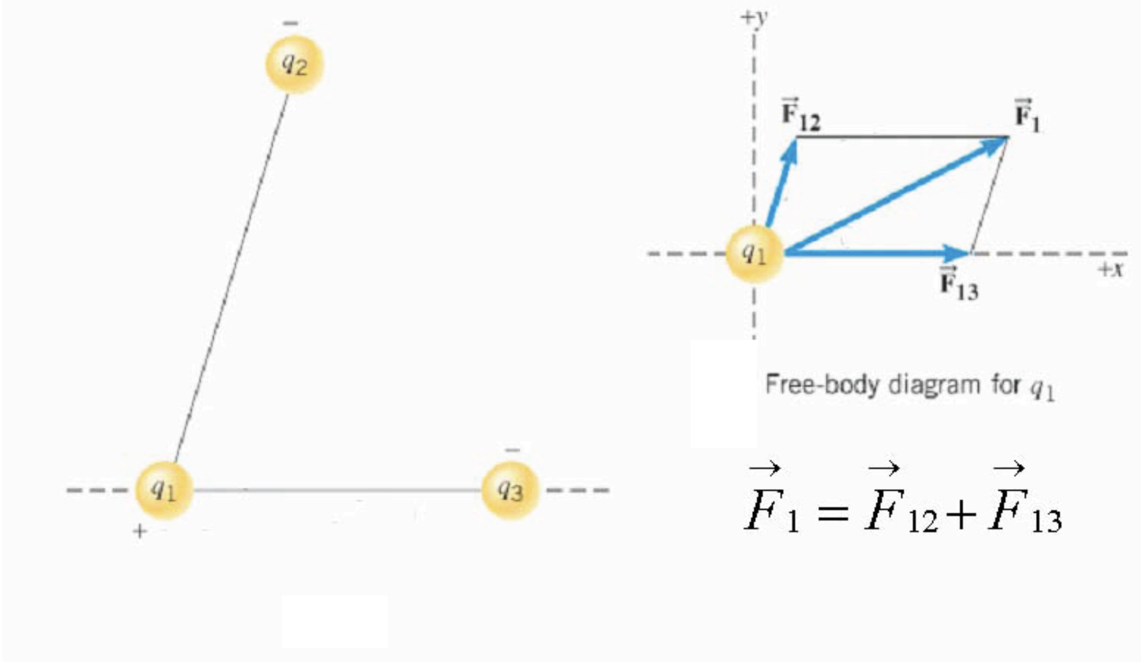
1. Yes

~~2. no~~

...... d = ...

Superposition principle

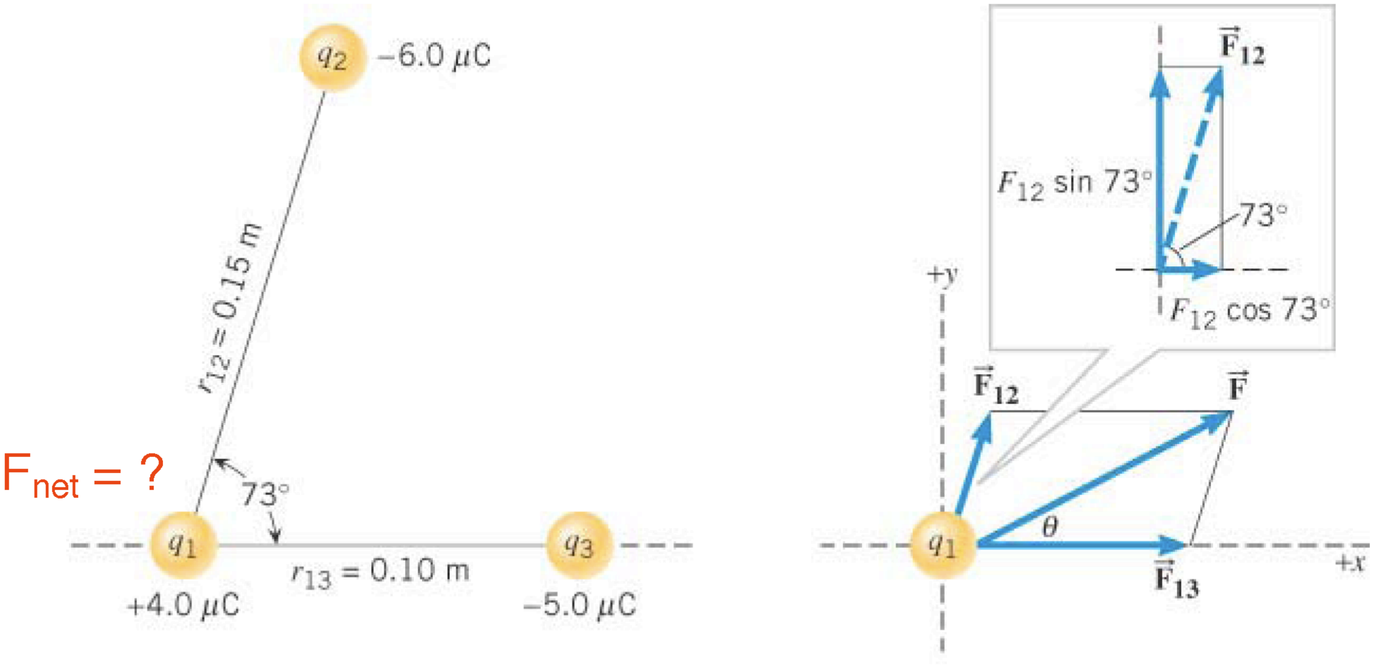
2D case



$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13}$$

The net electric force acting on the charge q_1 is equal to the vectorial sum of the forces produced by the individual elementary charges acting on q_1.

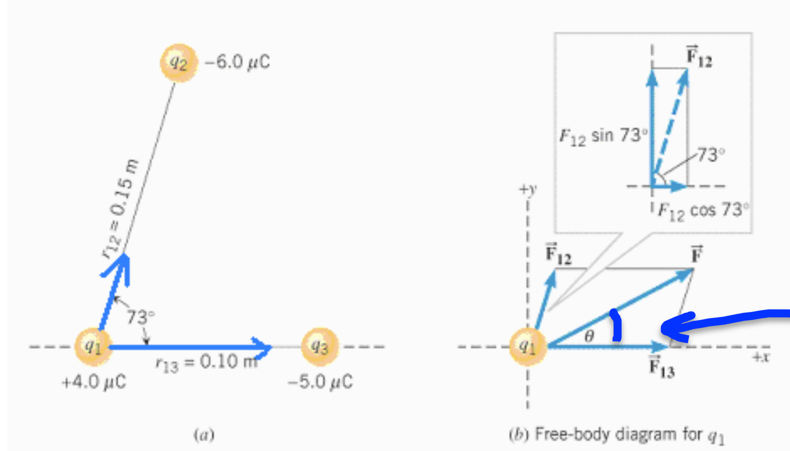
FBD



(a)

(b) Free-body diagram for q_1

Each force can be resolved into x and y components.



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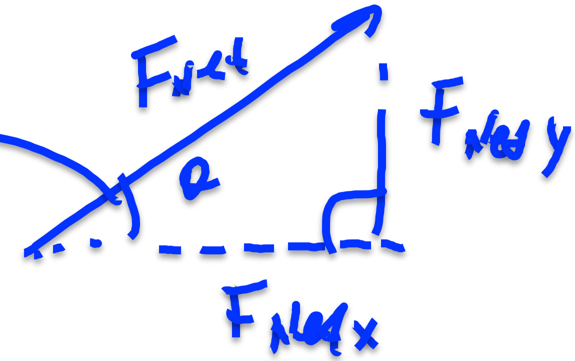


Calculation

Force	x component	y component
$\vec{F}_{12}$	$+(9.6\text{ N})\cos 73° = +2.8\text{ N}$	$+(9.6\text{ N})\sin 73° = +9.2\text{ N}$
$\vec{F}_{13}$	$+18\text{ N}$	0 N
$\vec{F}$	$\vec{F}_x = +21\text{ N}$	$\vec{F}_y = +9.2\text{ N}$

The magnitude F and the angle θ of the net force are

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(21\text{ N})^2 + (9.2\text{ N})^2} = \boxed{23\text{ N}}$$

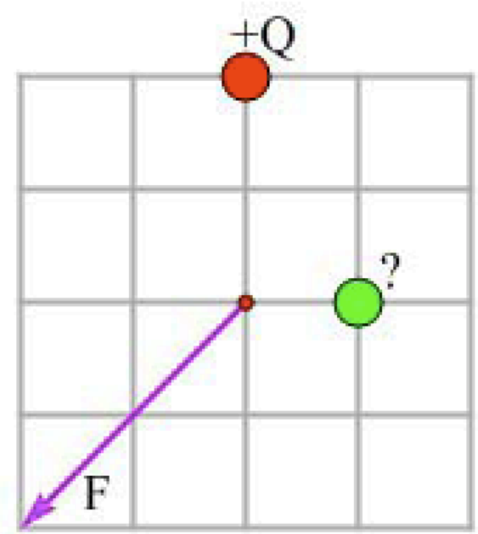
$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{9.2\text{ N}}{21\text{ N}}\right) = \boxed{24°}$$

The net force on a test charge

The diagram shows the net force experienced by a positive test charge located at the center of the diagram. The force comes from two nearby charged balls, one with a charge of +Q and one with an unknown charge. What is the sign and magnitude of the charge on the second ball?

1. +Q/4
2. +Q/2
3. +Q
4. +2Q
5. +4Q
6. none of these

Draw individual forces on the test charge (which is stronger?)



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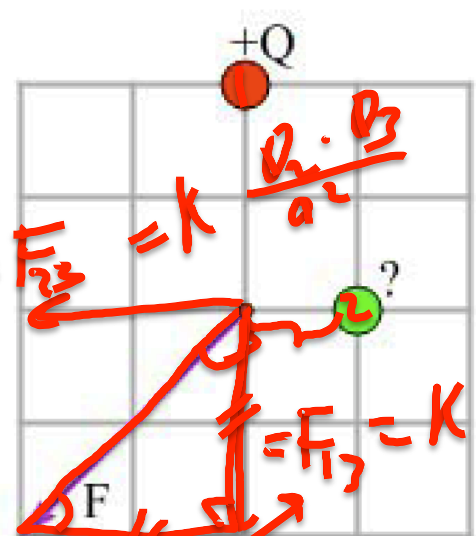
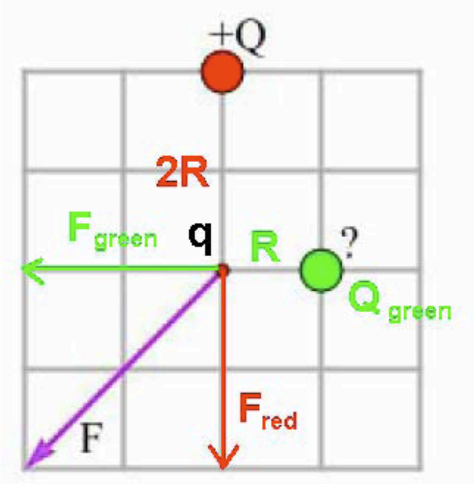




What is the sign and the magnitude of the charge on the second ball?

1. Q/4

The net force (the purple arrow) can be resolved as the sum of individual forces (the green and the red arrows).

To make the force from the red charge directed downward the red charge must be positive. In this case to make the force from the green charge directed to the left *the green charge must be positive* as well.

Magnitudes of the individual forces can be calculated by the Coulomb's Law. We see from the picture that the magnitudes of the individual forces are the same, but the green charge is twice closer then the red one; hence to make the same magnitude it has to be *four times smaller* than the red one.

The net force on a test charge, II

The diagram shows the net force experienced by a test charge located at the center of the diagram. The force comes from two nearby charged balls, one with a charge +Q and one with an unknown charge. What is the sign and the magnitude of the charge on the second ball?

Draw individual forces on the test charge (what do they tell about the sign of the charge?

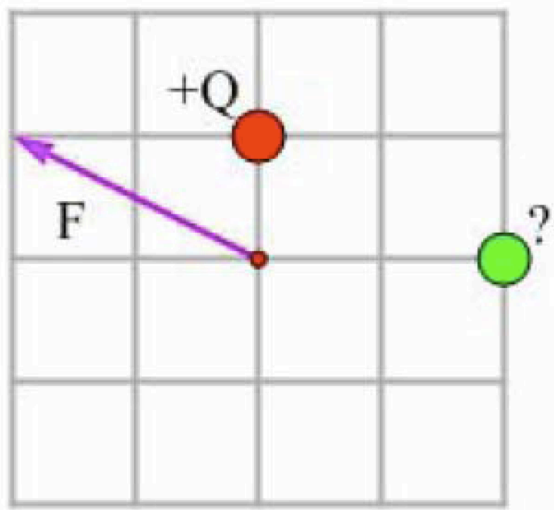
A. +Q

B. $+Q \times \sqrt{2}$

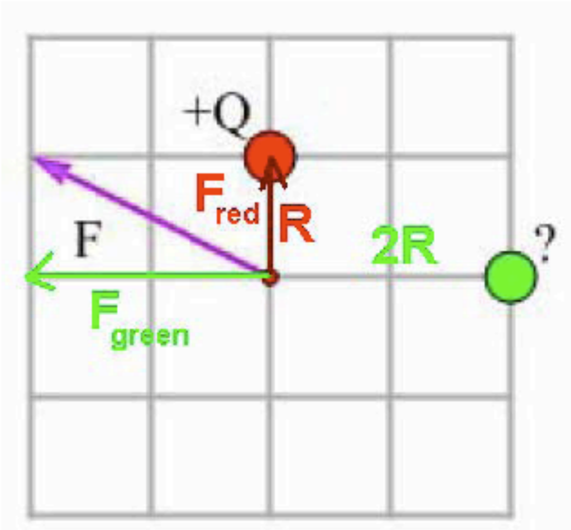
C. +2Q

D. $+2Q \times \sqrt{2}$

E. none of these



What is the sign and the magnitude of the charge on the second ball?



E. none of these

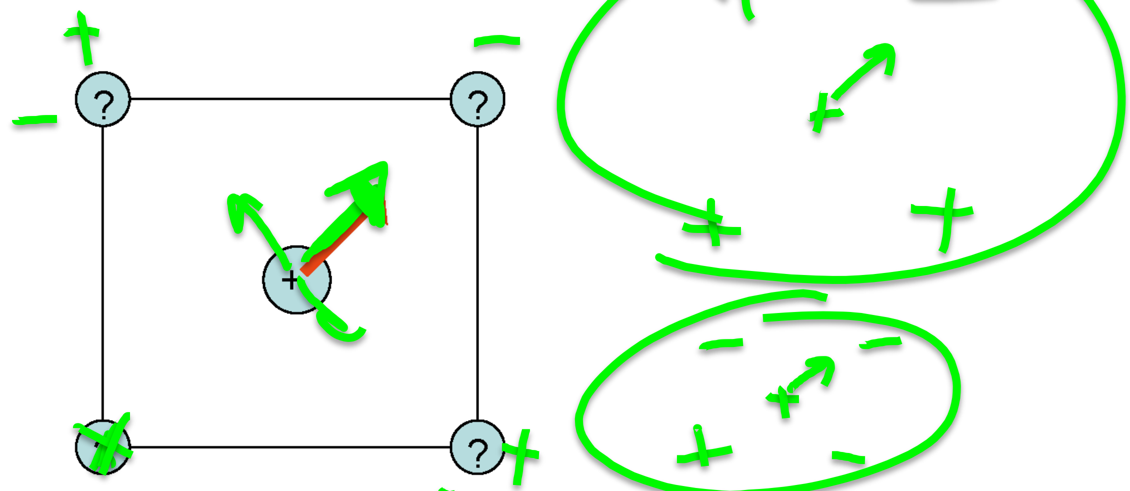
The net force (the purple arrow) can be resolved as the sum of individual forces (the green and the red arrows).
To make the force from the red charge directed upward the red charge must be negative.

In this case to make the force from the green charge directed to the left *the green charge must be negative* as well.

But all the answers provide positive charges.

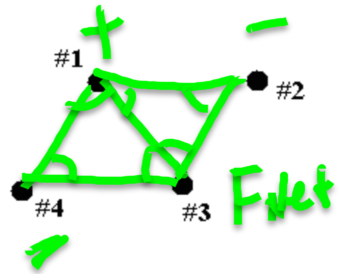
A two-dimensional situation

Case 1: There is an object with a charge of $+Q$ at the center of a square. Can you place a charged object at each corner of the square so the net force acting on the charge in the center is directed toward the top right corner of the square? Each charge has a magnitude of Q, but you get to choose whether it is + or – .



PRS

How many configurations did you find?

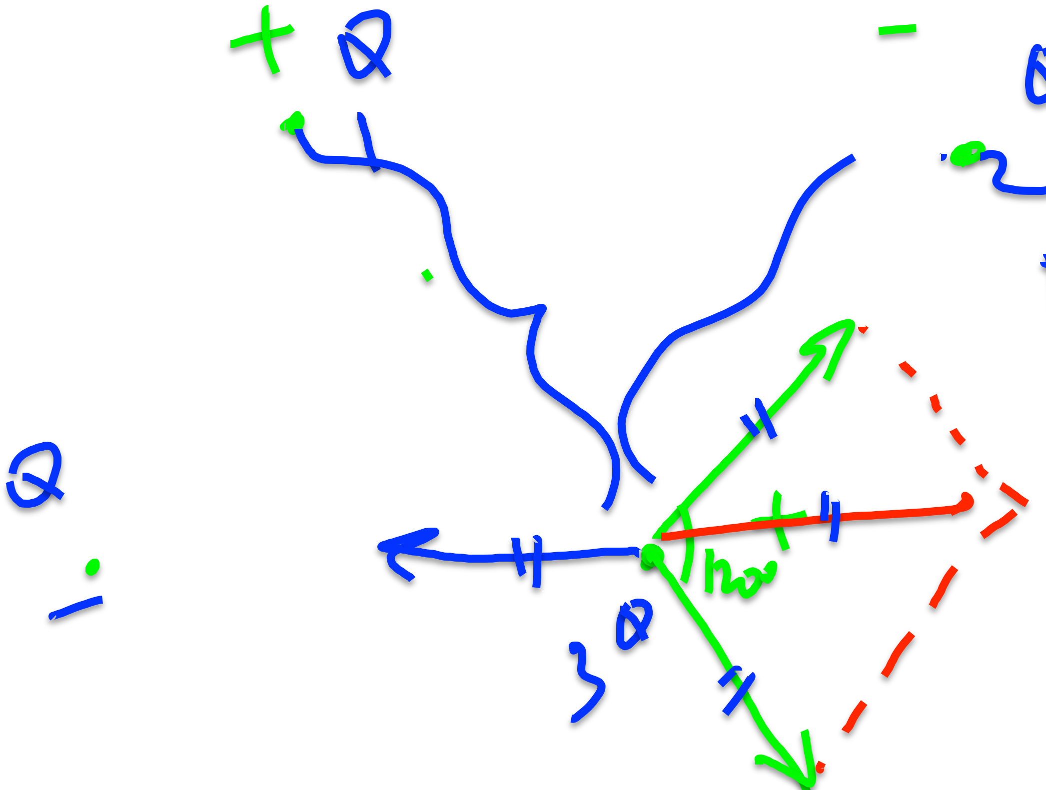


You can see in the picture four charges.

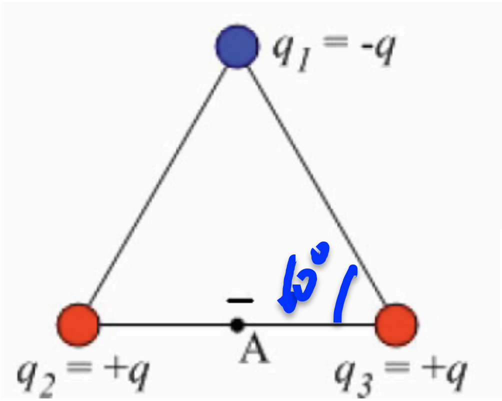
The distance from the charge #1 to the charge #2 is equal to d, as well as the distance between the charges #1 and #4, same between the charges #2 and #3, and between the charges #3 and #4 (if you are interested, the four charges together form a rhombus, so the distance between the charges #1 and #3 is also d).

If the charge #1 is +Q, and the charge #2 is –Q, what should be the charge #4, so the charge #3 would have zero net force acting on it?

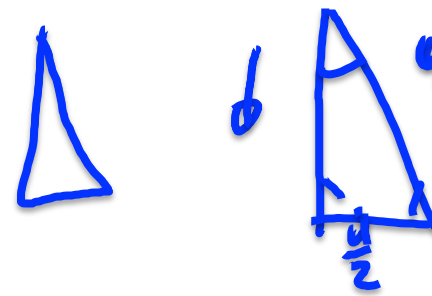
Let's discuss the strategy for solving this problem.



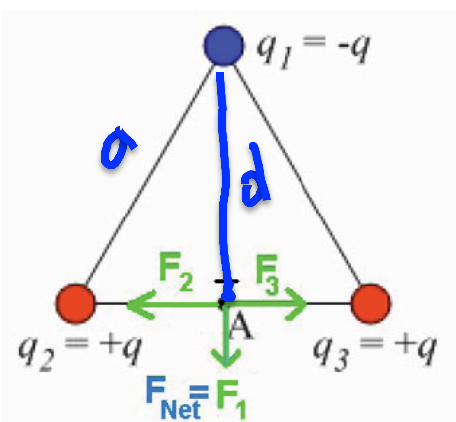
$$F_{13} = k\frac{Q \cdot Q}{a^2}$$



In what direction is the net force acting on a negative charge located at the Point A, halfway between the positive charges?



The net force is the sum of three forces acting on the charge, two of which just cancel each other out.

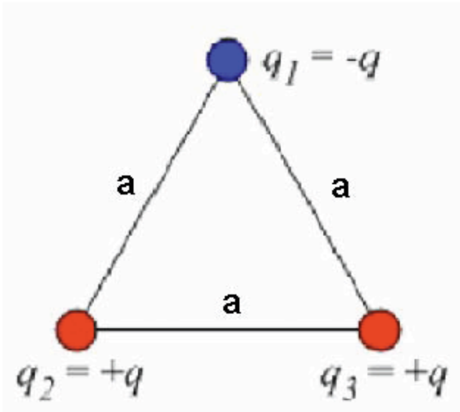


$$\vec{F}_2 = -\vec{F}_3 \qquad \vec{F}_2 + \vec{F}_3 = 0$$

$$\vec{F}_{Net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{F}_1$$

$$F_1 = K \frac{q_1 \cdot q_2}{d^2}$$

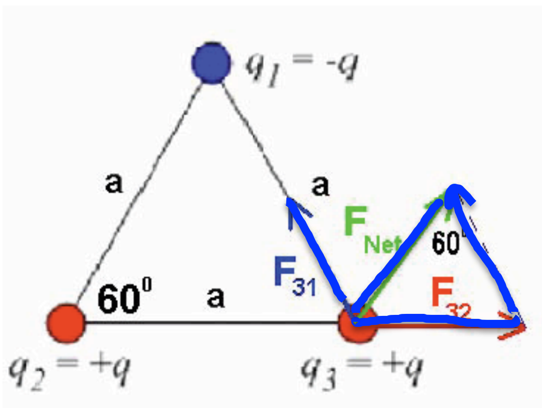
$$\frac{d}{a/2} = \tan 60°$$



Find the force on the red charge.

The system is symmetrical; both red charges experience the force of the same magnitude.

The net force acting on the right red charge (for example) is the sum of two forces.



$$\vec{F}_{Net} = \vec{F}_{31} + \vec{F}_{32}$$

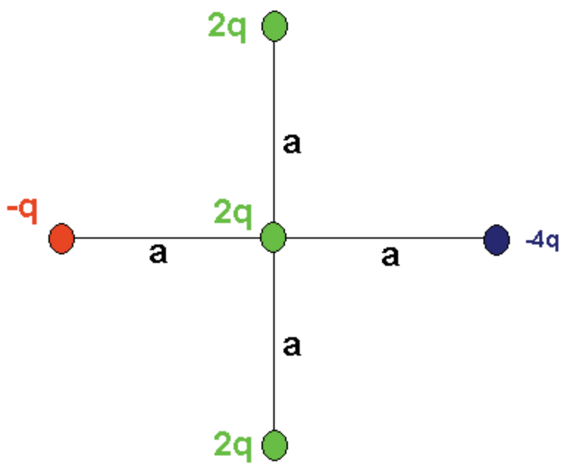
According the Coulomb's Law:

$$|F_{31}| = |F_{32}| = k\frac{q^2}{a^2}$$

Because we deal with equilateral triangles (for charges as well as for forces), the absolute values of the forces are the same;

$$|F_{Net}| = |F_{31}| = |F_{32}| = k\frac{q^2}{a^2}$$

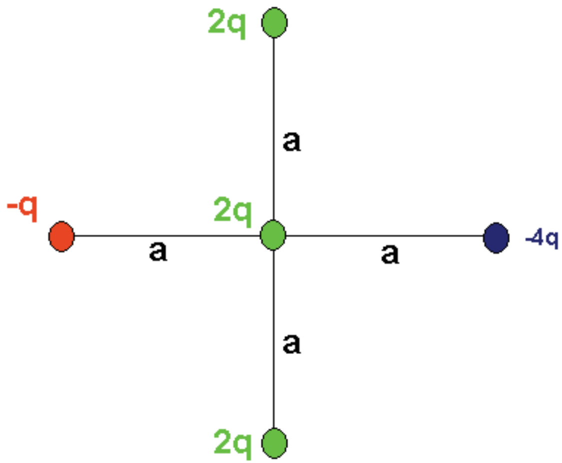
Four charges



If $F = k\dfrac{q^2}{a^2} = 10$

a) find the net force acting on the charge at the origin.

b) find the net force acting on the blue charge.

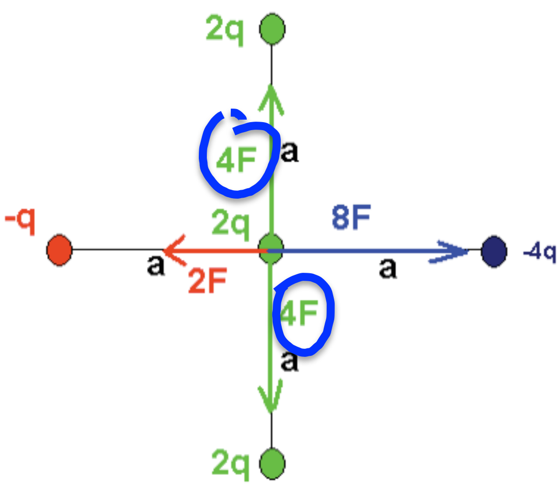


Four charges

If $F = k\dfrac{q^2}{a^2} = 10N$

a) find the net force acting on the charge at the origin.

b) find the net force acting on the blue charge.



Solution, part a:

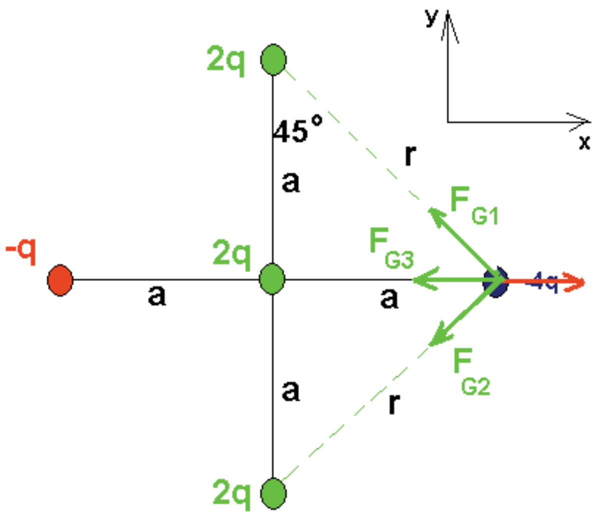
Always start from FBD.

Green forces cancel out each other, so

$$\vec{F}_{Net} = \vec{F}_B + \vec{F}_R + \vec{F}_{G1} + \vec{F}_{G2} = \vec{F}_B + \vec{F}_R = F_B - F_R$$

Hence, $F_{Net} = 8F - 2F = 6F$

Solution, part b:



Always start from FBD.

$$\vec{F}_{Net} = \vec{F}_{G1} + \vec{F}_{G2} + \vec{F}_{G3} + \vec{F}_R$$

We can see that F_{Net} does not have the y-component.

$$F_{Net} = F_{Net\,x} = -F_{G1x} - F_{G2x} - F_{G3} + F_R$$

Let's find all the forces (magnitudes!):

$$F_R = k\frac{q \cdot 4q}{(2a)^2} = k\frac{4q^2}{4a^2} = F$$

$$F_{G3} = k\frac{2q \cdot 4q}{a^2} = k\frac{8q^2}{a^2} = 8F$$

We need to know r to find F_{G1} or F_{G2}: $r^2 = a^2 + a^2 = 2a^2$

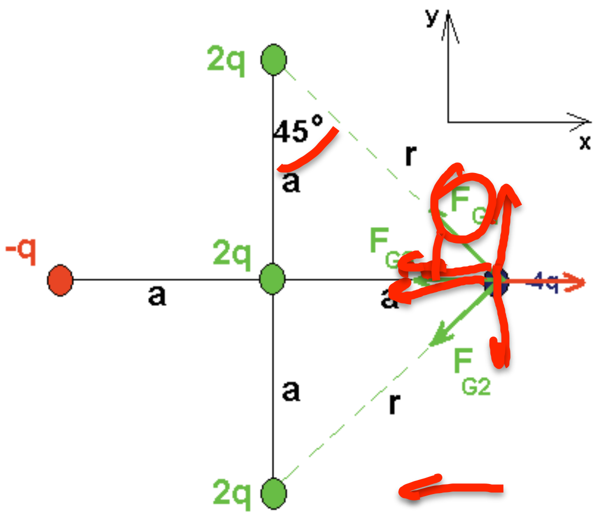
$$F_{G1} = F_{G2} = k\frac{2q \cdot 4q}{r^2} = k\frac{8q^2}{2a^2} = 4F$$

Now we can find: $F_{G1\,x} = F_{G1} * \sin 45^0 = 4F\sin 45^0$ (and same for $F_{G2\,x}$).

That gives us: $F_{Net} = -4F\sin 45^0 - 4F\sin 45^0 - 8F + F = (-8\sin 45 - 7)F$

If we are looking for the magnitude, we have to take the $|(-8\sin 45 - 7)F|$.

Solution, part b:



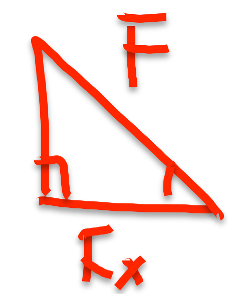
Always start from FBD.

$$\vec{F}_{Net} = \vec{F}_{G1} + \vec{F}_{G2} + \vec{F}_{G3} + \vec{F}_{R}$$

We can see that F_{Net} does not have the y-component.

$$F_{Net} = F_{Net\,x} = -F_{G1x} - F_{G2x} - F_{G3} + F_{R}$$

Let's find all the forces (magnitudes!):



$$F_R = k\frac{q\cdot 4q}{(2a)^2} = k\frac{4q^2}{4a^2} = F$$

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We need to know r to find F_{G1} or F_{G2}: $r^2 = a^2 + a^2 = 2a^2$

$$F_{G1} = F_{G2} = k\frac{2q\cdot 4q}{r^2} = k\frac{8q^2}{2a^2} = 4F$$

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