The Ideal Gas Law

An ideal gas satisfies these conditions:

1. It consists of **a large** number of identical particles (atoms, molecules).
2. The volume occupied by the particles themselves is negligible compared to the volume of the container they're in (**particles are dots or made of dots**).
3. The particles move in **random** motion.
4. The particles obey Newton's laws of motion; they experience forces only during collisions; any collisions are completely elastic, and instantaneous (take a negligible amount of time).

Avogadro's Number

An amount of a matter having **6.02×10^{23}** particles is named a **MOLE**

A mole is very similar to a dozen, in the sense that it stands for a certain number of things.
A dozen means 12, while a mole means 6.02×10^{23}.

This number is also known as *Avogadro's number*, N_A.

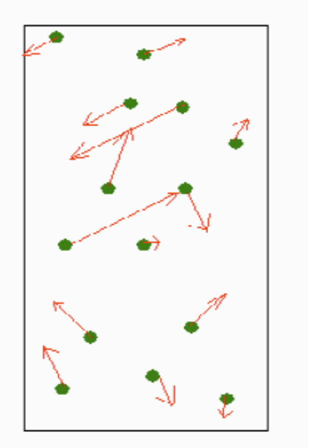
$$\mathbf{N_A = 6.02 \times 10^{23}}$$

1 mole = N_A particles

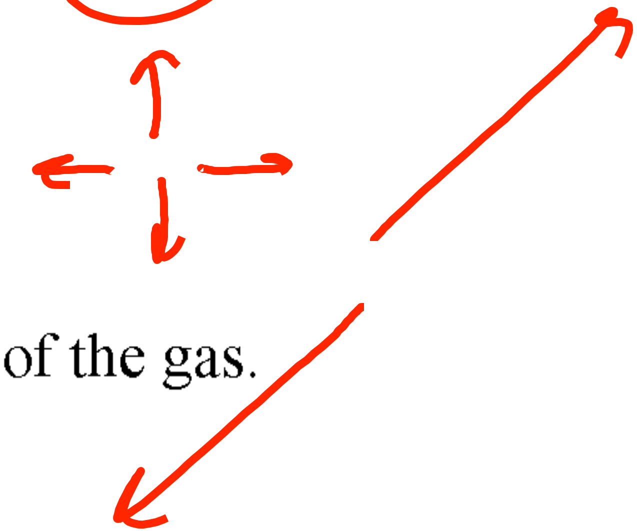
Every material has the same number of particles in 1 mole, but the mass of 1 mole is different to different materials.

A molar mass **$\mu = m_0 N_A$** ***is the mass of 1 mole of the substance.***

Question



What is the average velocity of the ideal gas particles?

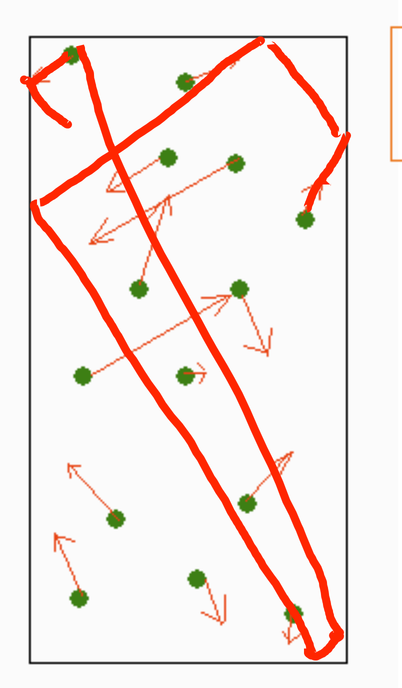


1) It depends on the temperature of the gas.

2) Zero.

3) Impossible to tell.

Average Kinetic Energy and Temperature



The absolute temperature of an object T

$$\boxed{E_{K\text{-}AVE} = \frac{3}{2}kT} \quad \text{or} \quad T = \frac{2}{3k}E_{K\text{-}AVE}$$

$T - 273 = t°$

Ⓕ

Here, the constant $\boxed{k = 1.38 * 10^{-23}\ \text{J/K}}$

k is called the *Boltzmann* constant.

According to this simple approach, at the absolute zero temperature (T = 0) particles are not moving (E_K=0)

The unit of the absolute temperature is Kelvin (K)

Light and Heavy Particles

A box of ideal gas consists of light particles and heavy particles (the heavy ones have 16 times the mass of the light ones). Initially all the particles have the same speed. When equilibrium is reached what will be true? V_1 • ● V_2

1) All the particles will still have the same speed. $V_1 = V_2$

2) The average speed of the heavy particles will be equal to the average speed of the light particles. $V_{1A} = V_{2A}$

3) The average speed of the heavy particles will be larger than the average speed of the light particles. $V_2 > V_1$

4) The average speed of the heavy particles will be smaller than the average speed of the light particles. $V_2 < V_1$

Light and Heavy Particles

D. The average speed of the heavy particles will be smaller than the average speed of the light particles (by a factor of 4**).**

Coming to equilibrium means coming to a particular temperature, which means the average kinetic energy of the particles is a particular value, the same for all particles, no matter how heavy they are.

The average kinetic energy of the light particles equals the average kinetic energy of the heavy particles

$$E_{\text{K-AVE-heavy}} = E_{\text{K-AVE-light}}$$

this can only happen if the average speed of the heavy particles is smaller than that of the light particles.

T = [illegible]

$T_1 = T_2$

$\cancel{\frac{2}{3k}} E_1 = \cancel{\frac{2}{3k}} E_2$

$E_{\text{K-AVE-heavy}} = E_{\text{K-AVE-light}}$

$$\tfrac{1}{2} m_{\text{heavy}} v^2_{\text{rms-heavy}} = \tfrac{1}{2} m_{\text{light}} v^2_{\text{rms-light}}$$

or

$$16 * m_{\text{light}} v^2_{\text{rms-heavy}} = m_{\text{light}} v^2_{\text{rms-light}}$$

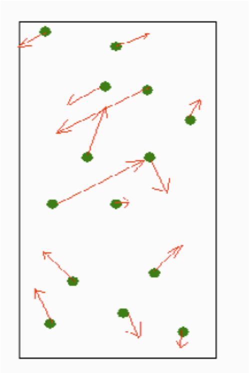
or

$$16 v^2_{\text{rms-heavy}} = v^2_{\text{rms-light}}$$

or

$$4 v_{\text{rms-heavy}} = v_{\text{rms-light}}$$

Average Kinetic Energy and Temperature



The average kinetic energy of a particle is:

$$E_{K\text{-}AVE} = \frac{E_{K1} + E_{K2} + E_{K3} + \ldots + E_{KN}}{N} = \tfrac{1}{2} m_0 * \{v_{rms}\}^2$$

A hot object has more fast moving ("hot") particles then a cool object!

Temperature shows how many "hot" (fast) particles there are in an object. Temperature is directly proportional to the average kinetic energy of a particle!

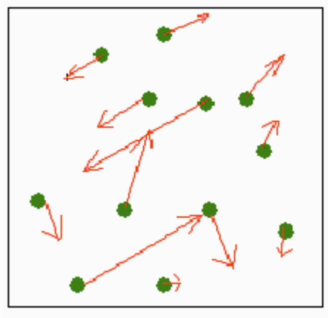
Definition: the absolute temperature of an object T

$$E_{K\text{-}AVE} = \frac{3}{2} kT$$

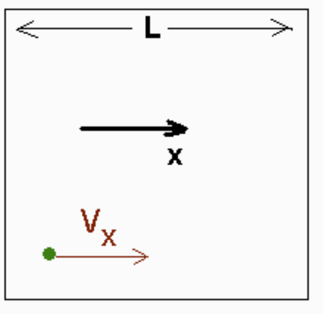
or

$$T = \frac{2}{3k} E_{K\text{-}AVE}$$

Kinetic Theory



Consider a cube, L on each side. The box contains N molecules of ideal gas of mass m_0



Let's consider ONE particle only **(SIM. 2C)**

$$F_0 = \frac{2m_0 v_x}{\Delta t}$$

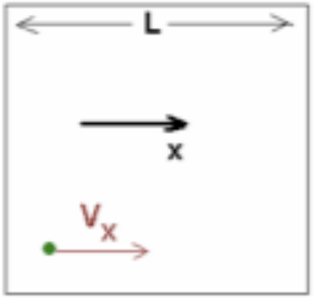
F_0 is the average force exerted by *one molecule* when it collides with a wall of the box that is perpendicular to the x-axis

Our single molecule collides with *this* wall once every Δt seconds:

$$\Delta t = \frac{2L}{v_x}$$

This gives an average force for one particle of:

$$F_0 = \frac{m_0 v_x^2}{L}$$



This gives:

$$F = \frac{m_0 N}{3L} v_{rms}^2$$

Dividing by the area, L^2, of the wall gives the pressure:

$$P = \frac{F}{A} = \frac{F}{L^2} = \frac{m_0 N}{3L^3} v_{rms}^2$$

which is ($L^3 = V$; $E_{K\text{-}AVE} = \tfrac{1}{2} m_0 * \{v_{rms}\}^2$):

$$\underline{P} = \frac{m_0 N}{3V} v_{rms}^2 = \frac{2N}{3V} (\tfrac{1}{2} m_0 v_{rms}^2) = \underline{\frac{2N}{3V} E_{K\text{-}AVE}}$$

or:

$$\boxed{PV = \frac{2N}{3} E_{K\text{-}AVE}}$$

If we use the definition of the absolute temperature:

$$\boxed{E_{K\text{-}AVE} = \frac{3}{2} kT}$$

We finally have:

the ideal gas law

$$\boxed{PV = NkT}$$

The ideal gas law

$$\boxed{PV = NkT}$$

$n \cdot R = 100$

$n = \frac{100}{.31}$

It is convenient to rewrite the law:

$\boxed{N = n * N_A,}$ n is the number of moles: $PV = nN_AkT$

Let's define $\boxed{N_Ak = R,}$ the universal gas constant:

R = 8.31 J/(mol K)

$$\boxed{PV = nRT}$$

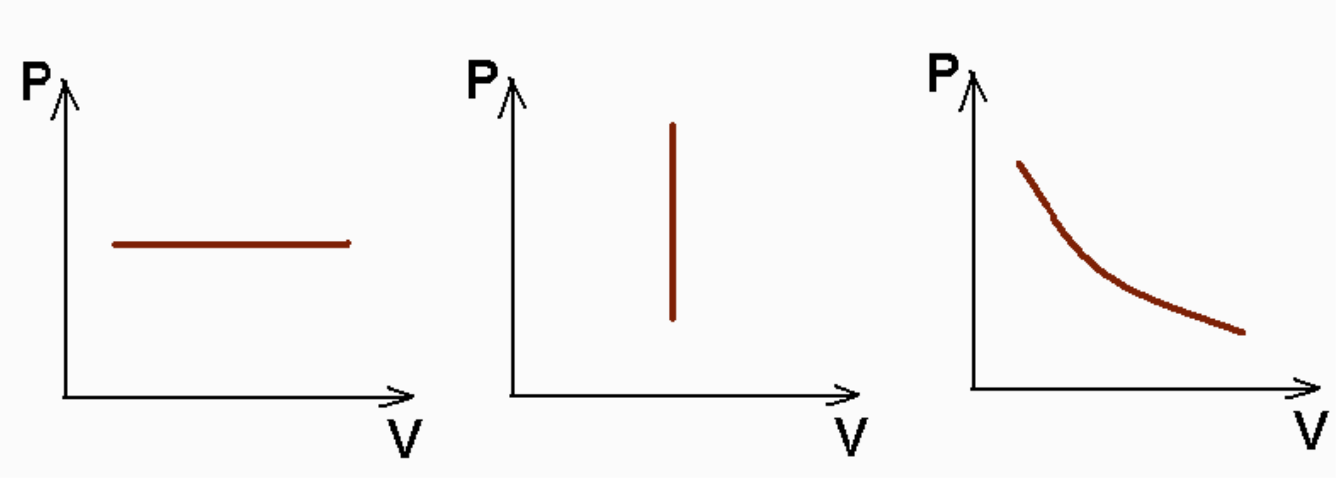
$n = \frac{m}{M}$

Processes

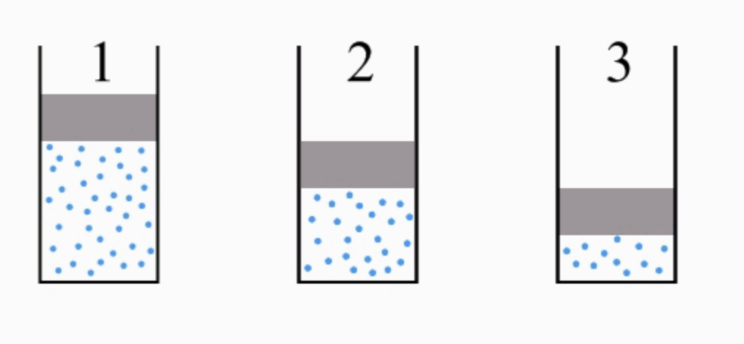
$PV = nRT$

Three standard processes (n = const):

Isobaric	Isochoric	Isothermal
P = const	V = const	T = const
V/T = const	P/T = const	PV = const



(Try to plot graphs for the same processes using PT and VT axes)



Three Cylinders

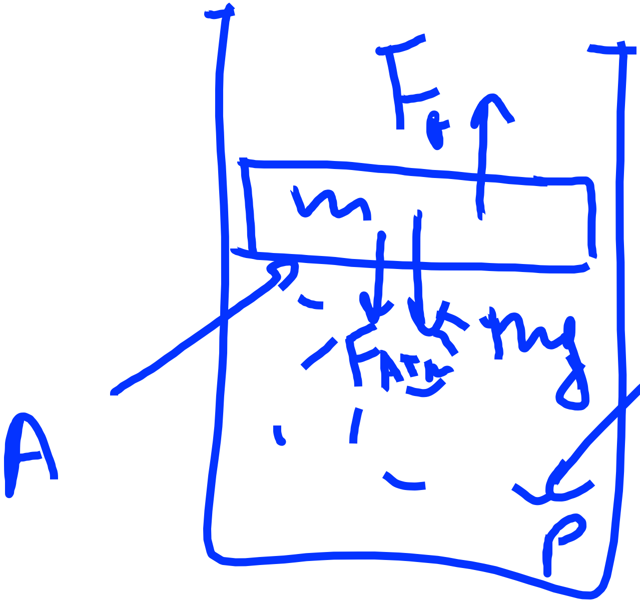
The three cylinders shown are identical. The piston in each cylinder is free to slide up and down without friction. The pistons are identical. Each piston is shown in its equilibrium position, and the top of each piston is exposed to the atmosphere.

Rank the cylinders based on the pressure of the gas confined inside.

1) $1 > 2 > 3$
2) $3 > 2 > 1$
3) $1 = 2 = 3$
4) There is not enough information given to answer the question.

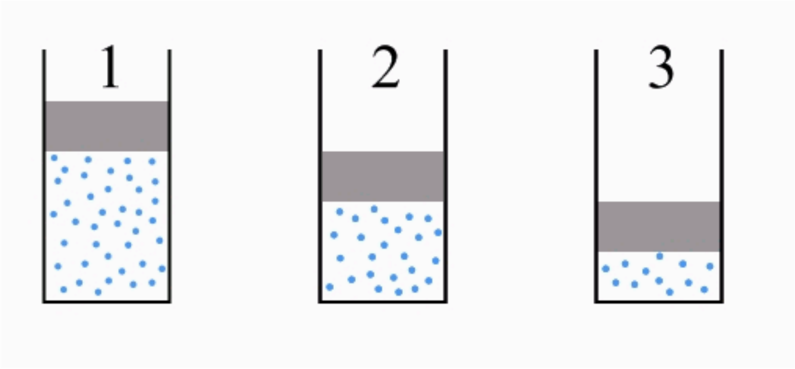
$$\boxed{PV = NkT}$$

$P_{ATM} = 100000\ Pa$



$$F_c = mg + F_{ATM}$$

$$\underline{P} = \frac{F}{A} = \frac{F_c}{A} = \frac{mg + F_{ATM}}{A}$$



Three Cylinders

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~~If the temperature is the same, choose ranking based on the number of particles.~~

1) 1 > 2 > 3
2) 3 > 2 > 1
3) 1 = 2 = 3
4) There is not enough information given to answer the question.

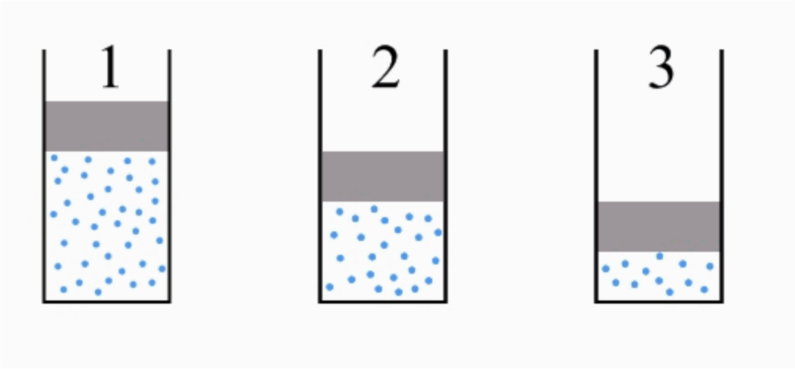
$\frac{T}{V} = \text{cons}$

T cons

$= \frac{P}{kN} = \frac{T}{V}$

PV = NkT

$\frac{T_1}{V_1} = \frac{T_2}{V_2}$



Three Cylinders

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If the temperature is the same, choose ranking based on the number of particles.

1) $1 > 2 > 3$
2) $3 > 2 > 1$
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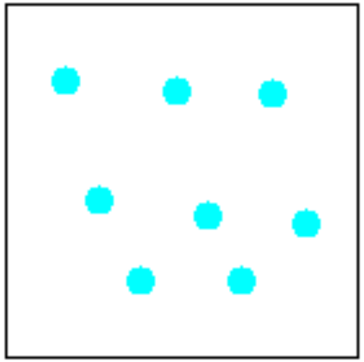
4) There is not enough information given to answer the question.

$$\boxed{PV = NkT}$$

An ideal gas is in a container at the pressure of 600 kPa.



Find a new gas pressure in the container if the volume of the container was increased twice of the initial, the new absolute temperature of the gas is a half of the initial and a third of the gas was lost to the environment.

$$PV = NkT$$

1. 100 kPa   2. 200 kPa   3. 300 kPa etc   9. 900kPa

0. any other number

$PV = nRT$

$\frac{PV}{nT} = R = \text{const}$

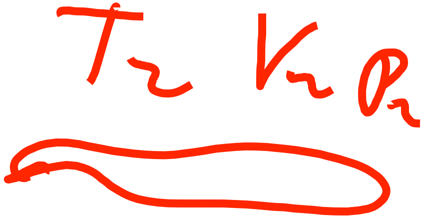
$1)\ V_2 = 2 \cdot V_1$

$T_2 = \frac{1}{2} T_1$

$\boxed{n_2 = \frac{2}{3} n_1}$

$\frac{P_1 V_1}{n_1 T_1} = \frac{P_2 V_2}{n_2 T_2}$

1. Yes   2. No



$P_1 V_1 = n_1 R T_1$

$n_1 = n_2$

$P_2 V_2 = n_2 R T_2$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

Question

The temperature of an ideal gas in a *closed* container is raised from 10°C to 20°C. If the pressure at 10°C is P_i, what is P_f, the pressure at 20°C?

$$PV = nRT$$

But we have to use the absolute temperature scale!

$T_i = 10 + 273 = 283$ K $\qquad$ $T_f = 20 + 273 = 293$ K

When V, n = const

$$P/T = nR/V = \text{const}$$

or

$P_i/T_i = P_f/T_f$ or $P_f = P_i{*}T_f/T_i = P_i{*}1.053$

$$\mathbf{P_i < P_f < 2P_i}$$

Equipartition of Energy and Internal Energy

A single molecule might have *different* energies:

1) kinetic energy of translational motion.

2) kinetic energy of rotational motion.

3) energy of oscillations (vibrations and interactions).

The internal energy E_{int} (or U) of an object is the total sum of all the energies of all the molecules.

Equipartition of Energy: Each contribution to the internal energy contributes an equal amount of energy.

Monatomic ideal gas ●

Each particle is a *dot*. ●

Only translational kinetic energy does exist, hence the internal energy is equal to just total average kinetic energy of all the particles, therefore:

Monatomic ideal gas: $E_{int} = E_{K\text{-}AVE}N = \frac{3}{2}NkT = \boxed{3} * \frac{1}{2}NkT$

Each direction (x, y, and z – 3 directions!) contributes ½NkT to the energy.

We can say that: each (or *any*) type of energy contributes *exactly* ½NkT Jules in the system.

For *a diatomic* (two atoms make one molecule) molecule there are *three translation directions*, and *rotational kinetic energy also contributes*, but *only* for rotations about two of the three perpendicular axes. The *five* contributions to the energy (five degrees of freedom) give:

$$\text{Diatomic ideal gas: } E_{int} = \frac{\boxed{5}}{2}NkT$$

This consideration applies at intermediate temperatures only. At low temperatures only the translational kinetic energy contributes, and at higher temperatures two additional contributions (kinetic and potential energy) come from vibration.

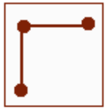
Internal Energy

Ideal gas: $E_{int} = \frac{i}{2}NkT$

i (the number of degrees of freedom) depends on the type of the particles:

$i = 3$  for point-like particles

$i = 5$  for dumbbell-like particles (no oscillations)

$i = 6$  for big particles made of 3 or more atoms (no oscillations)

$PV = nRT$

Ideal gas: $U = E_{int} = \frac{i}{2}\frac{N}{N_A}N_AkT = \frac{i}{2}nRT$

Ideal gas: $U = \frac{i}{2}nRT$

$\underline{U = \frac{i}{2} \cdot PV}$

The First Law of Thermodynamics

Thermodynamics: the study of systems involving energy in the form of heat and work.

The First Law of Thermodynamics: $\boxed{Q = \Delta U + W}$

Q represents heat added to a system (or removed if it is negative)

U is the internal energy of the system (the energy associated with the motion of the atoms and/or molecules), so ΔU is the change in the internal energy. ΔU is directly proportional to the change in temperature ΔT.

W is the work done ***by the system***. ($W = - W_{\text{on-system}}$)

The First Law is often written: $\boxed{\Delta U = Q - W = Q + W_{\text{on-system}}}$

Two ways to change U!

A) by heating it or cooling it up; or B) by doing work

Work (at constant pressure)

We defined work previously as:

$W = F*\Delta x$ (true if the force is constant)

$F = P*A$, so:

$$W = P*A*\Delta x = P*\Delta(Ax) = P*\Delta V$$

At constant pressure the work done by the system is the pressure multiplied by the change in volume.

If there is no change in volume, no work is done.

$\Delta V > 0 \Rightarrow W > 0$ $\Delta V < 0 \Rightarrow W < 0$

Work (in general)

If there is a change in volume and the pressure changes the work done by the system is the area under the P-V graph.

This is why P-V diagrams are so useful in thermodynamics.

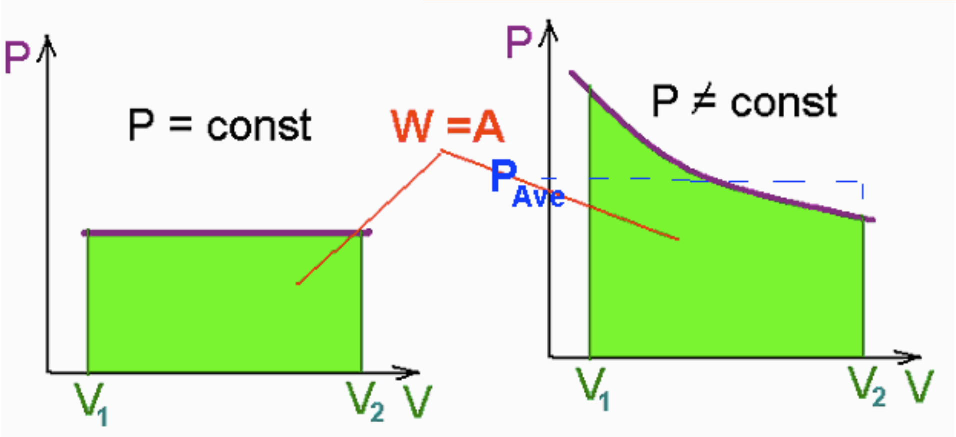
1. $W > 0$
2. $W < 0$
3. $W = 0$

$P = \text{const}$

$W = P^*\Delta V = P^*(V_2 - V_1)$

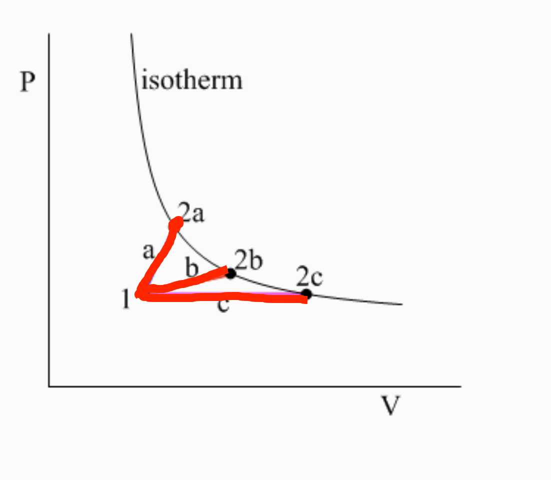
$P \neq \text{const}$

$W = P_{Ave}{}^*\Delta V = \text{Area (P-V graph)}$



$\Delta V > 0 \Rightarrow W > 0$ $\quad \Delta V < 0 \Rightarrow W < 0$

Question



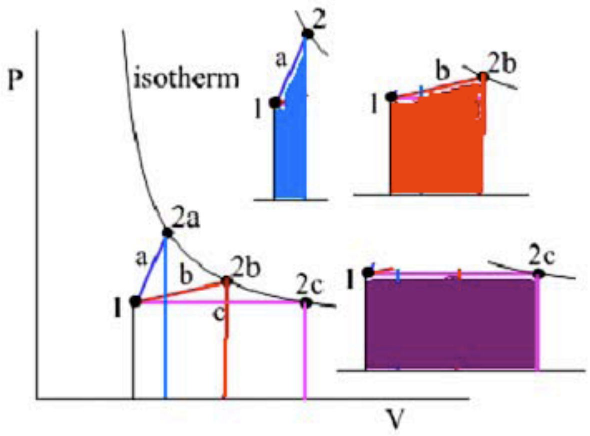
An ideal gas initially in state 1 progresses to a final state by one of three different processes (a, b, or c). Each of the possible final states has the same temperature.

For which process is the work done by the system is larger?

$W = P_{Ave} * \Delta V = \text{Area (P-V graph)}$

1 a
2 b
3 c
4 Equal for all three
5 Not enough information is given to answer this question.

Question



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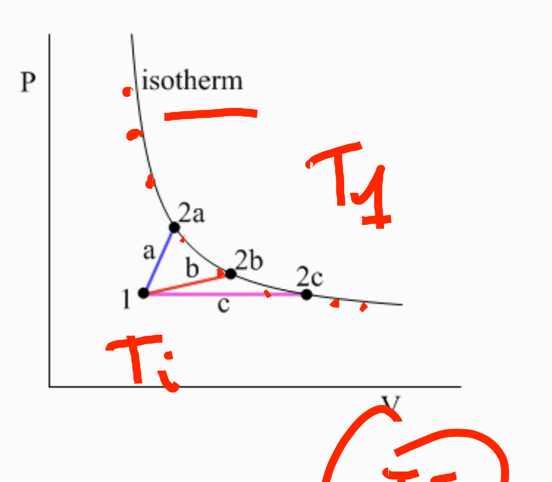
For which process is the work done by the system is larger?

$$W = P_{Ave} * \Delta V = \text{Area (P-V graph)}$$

By looking at the areas under the curves for each process we can see that **more work is done for process c**.

C. c

Question



An ideal gas initially in state 1 progresses to a final state by one of three different processes (a, b, or c). Each of the possible final states has the same temperature.

For which process is the change in internal energy larger?

$\Delta U = U_f - U_i$

1 a

2 b

3 c

[✓ 4] Equal for all three

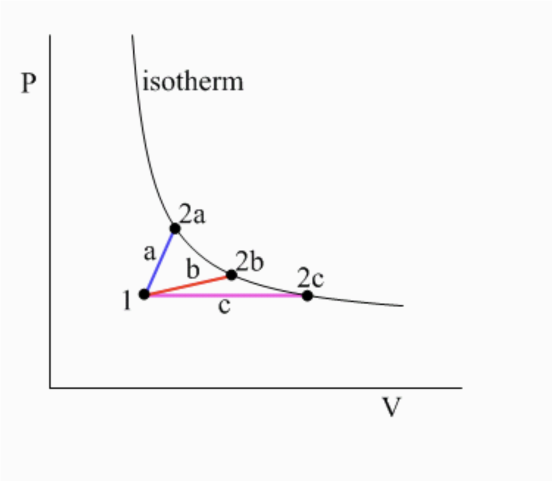
5 Not enough information is given to answer this question.

Ideal gas: $U = \frac{i}{2} nRT$

i

1

Question



Because *the change in temperature* is the same, hence *the change in internal energy* is the same for all three processes.

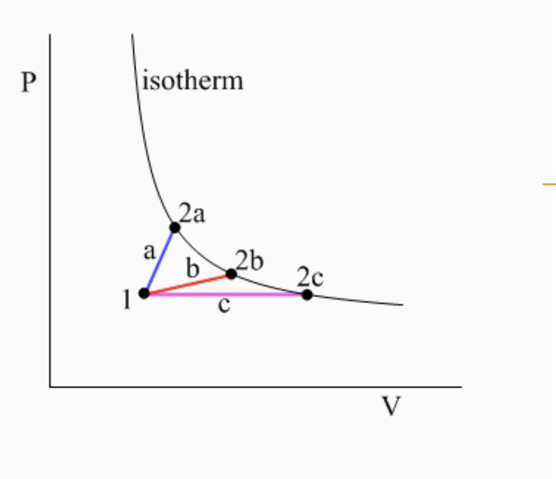
D. Equal for all three

Ideal gas: $U = \frac{i}{2} nRT$

$$\Delta U = \frac{i}{2} nR\Delta T$$

$\Delta T = T_2 - T_1$ is the same => ΔU is the same

Question

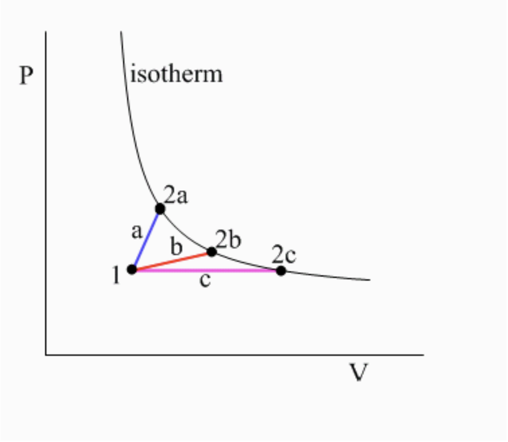


For which process is more heat transferred into the ideal gas?

$$Q = \Delta U + W$$

1 a
2 b
3 c
4 Equal for all three
5 Not enough information is given to answer this question

Question



Q = the change in internal energy + work

$$\underline{Q} = \boxed{\Delta U} + \underline{W}$$

By looking at the areas under the curves for each process we can see that more work is done for process c.

W_c is the greatest

ΔU is the same for all

Q_c is the largest.

C. c

Heat Capacity of a Gas

The heat capacity of a substance tells us how much heat is required to raise a certain amount of the substance by one degree.
In general: The heat capacity of a substance c is defined by

$$\underline{Q = C\Delta T}$$

For a gas we define *a molar* heat capacity c - the heat required to increase the temperature of *1 mole* of the gas by 1 K.

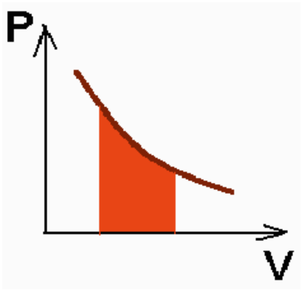
$$\underline{Q = c\,n\,\Delta T} \text{ (} n \text{ is the number of moles)}$$

The value of the heat capacity depends on whether the heat is added at constant volume, constant pressure, etc.

Constant Temperature

A constant temperature process is called *an isothermal process*.

An example is when a gas in a container that is immersed in a constant-temperature bath is allowed to expand slowly, or is compressed slowly.



At constant temperature there is no change in internal energy. $\Delta U = 0$ (c does not exist for this process!)

The First Law: $Q = W$ => $Q = \text{Area}_{(P\text{-}V)}$ $P = \{nRT\}/V$

The P-V diagram for this process follows an isotherm, a line of constant temperature

For an ideal gas at constant temperature, the pressure is inversely proportional to the volume. This makes it straight-forward to do an integration to find that:

$$Q = W = nRT\ln\left(\frac{V_f}{V_i}\right)$$

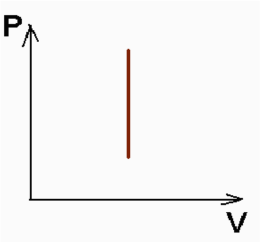
Constant Volume

A constant volume process is called as an *isochoric process*. An example is when heat is added to a gas in a container with fixed walls or/and pistons.

Because the walls can't move, the gas can not do work:

$W = P_{ave}\Delta V = P_{ave}*0 = 0$

In that case the First Law states: $Q = \Delta U$



The P-V diagram for this process is simple - it's a vertical line going up if heat is added, and going down if heat is removed.

In the case of an ideal gas: $U = \frac{i}{2}NkT = \frac{i}{2}nRT = \frac{i}{2}PV \leftarrow$ $PV = nRT$

$$\text{Therefore } Q = \Delta U = \frac{i}{2}nR\Delta T = U_f - U_i =$$

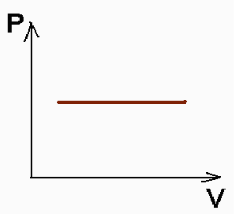
$$= \frac{i}{2}\underline{P_f V_f} - \frac{i}{2}\underline{P_i V_i}$$

$C_V = \frac{i}{2}Rn$

Constant Pressure

A constant pressure process is called *an isobaric process*. An example is a gas in a container sealed with a piston that is free to slide up and down. If heat is added the temperature goes up and the system expands, so work is done.

The P-V diagram for this process is a horizontal line, so the work done is simply:



$$W = P\,\Delta V = nR\,\Delta T$$

$$PV = nRT$$

$$\to \frac{i+2}{2}(P_f V_f - P_i V_i)$$

For an ideal gas:

$$\Delta U = \frac{i}{2}\,nR\,\Delta T$$

Plugging this into the First Law gives: $Q = \Delta U + W$

$$Q = \frac{i}{2}\,nR\,\Delta T + nR\,\Delta T = \boxed{\frac{i+2}{2}\,nR\,\Delta T}$$

$$C_P = \frac{i+2}{2}Rn$$

The ratio C_P / C_V

It turns out that the ratio of the specific heats is an important number. The symbol we use for the ratio is γ.

For an ideal gas we have:

$$\gamma = \frac{C_P}{C_V} = \frac{(i+2)R}{2} * \frac{2}{iR} = \frac{(i+2)}{2}$$

For an ideal gas the variable

$$\gamma = \frac{C_P}{C_V}$$

depends on the value of the *i* only!

Adiabatic Process

An adiabatic process is a process in which no heat is transferred. This can happen if the process happens so quickly that there is no time to transfer heat, or if the system is very well insulated from its surroundings.

By a definition for an adiabatic process: $Q = 0$

In that case the First Law states: $\Delta U = -W$

The P-V diagram for this process (it looks alike with the isothermal process) shows that the system cooling as it expands.

It can be shown that the equation governing this path on the P-V diagram is: $PV^{\gamma} = \text{constant}$

In addition, for an adiabatic process: $TV^{\gamma-1} = \text{constant}$

A Summary of Thermodynamic Processes

Process	**Basic Idea**	**Work (W)**	**ΔU**	**Heat (Q)**
Isochoric (Constant Volume)	$W = 0$ $P/T = \text{const}$	0	$n\,C_V\,\Delta T$	$Q = \Delta U$
Isothermal (Constant Temp.)	$\Delta T = \Delta U = 0;$ $PV = \text{const}$	$nRT\,\ln(\frac{V_f}{V_i})$	0	$Q = W$
Adiabatic (No Heat)	$Q = 0$	$-n\,C_V\,\Delta T$	$n\,C_V\,\Delta T$	0
Free Expansion	$Q = \Delta T = 0;$ $P_iV_i = P_fV_f$	0	0	0
Isobaric (Constant Pressure)	no zeroes! $V/T = \text{const}$	$P\Delta V = nR\,\Delta T$	$n\,C_V\,\Delta T$	$n\,C_P\,\Delta T$

molar!

Sample Problem

A container of monatomic ideal gas contains just the right number of moles so that $nR = 20$ J/K. The gas is in state 1 such that: $P_1 = 20$ kPa $V_1 = 100 \times 10^{-3}$ m^3

(a) What is the temperature T_1 of the gas?

(b) If $Q = 2500$ J of heat is added to the gas, and the gas expands at constant pressure, the gas will reach a new equilibrium state 2. What is the final temperature T_2?

(c) How much work was done by the gas during the expansion?

(d)What is the final volume V_2?

(e) What is the change in the internal energy?

Sample Problem

A container of monatomic ideal gas contains just the right number of moles so that $nR = 20$ J/K. The gas is in state 1 such that: $P_1 = 20$ kPa $V_1 = 100 \times 10^{-3}$ m^3

(a) What is the temperature T_1 of the gas?

Use the ideal gas law: $PV = nRT$, so:

$$T_1 = P_1V_1/nR = 2000/20 = 100 \text{ K}$$

(b) If $Q = 2500$ J of heat is added to the gas, and the gas expands at constant pressure, the gas will reach a new equilibrium state 2. What is the final temperature T_2?

We've already seen that, at constant pressure for a monatomic ideal gas: $Q = \Delta U + W = (3/2)nR\Delta T + nR\Delta T = (5/2)nR\Delta T$

Therefore $\Delta T = (2/5)Q/nR = 1000/20 = 50$ K.

$T_2 = T_1 + \Delta T = 100 + 50 = 150$ K

(c) How much work was done by the gas during the expansion?

$W = nR\Delta T = 20 * 50 = 1000$ J
(This equation is true only for a constant pressure process)

(d)What is the final volume V_2?

One approach is to bring in the ideal gas law again:

$V_2 = nRT_2/P_2 = 20(150)/20\text{x}10^3 = 150 \text{ x } 10^{-3}\ \text{m}^3$