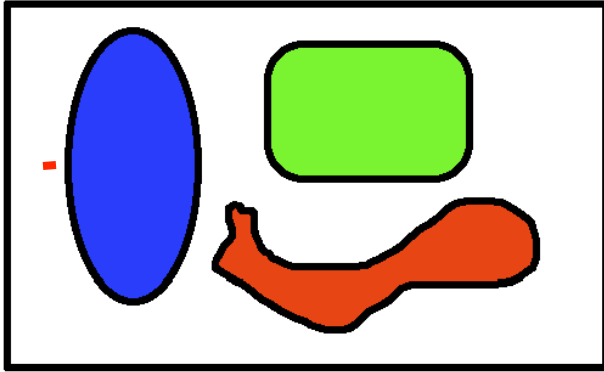


Isolated system



When a system is isolated, there is no energy exchange with an environment =>

Energy does not change =>

Energy can transfer only within the system.

Hot objects get cooler, cold objects get warmer, and/or some objects can change the state. If we account for ALL possible changes and assign an amount of heat to each, the *total* amount of the heat transferred within the system must be equal to **ZERO** (since there is NO energy exchange).

HBE

$$\sum Q = Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + \dots = 0$$

“Heat” is **energy transferred** between a system and its surroundings because of a temperature difference between them.

The heat capacity of an object tells us how much heat is required *to raise a temperature of a certain amount of the substance by one degree.*

In general: The heat capacity of an object C is defined by

$$Q = C\Delta T \text{ (or } C = Q/\Delta T \text{ – the definition)}$$

For gases (and all other substances, too) a *molar* heat capacity C can be introduced - the heat required to increase the temperature of *1 mole* of the gas by 1 K.

$$Q = Cn\Delta T \text{ (n is the number of moles)}$$

In addition, a mass heat capacity c (a little c), or specific heat, can be useful.

$$c = C / m \quad \Rightarrow \quad \underline{Q = cm(T_f - T_i)} \text{ (m is the mass of the object)}$$

Specific heat

The specific heat of a material is the amount of heat required to raise the temperature of 1 kg of the material by 1°C.

The symbol for specific heat is c .

Heat lost or gained by an object is given by:

$$Q = mc\Delta T$$

Material	c (J/(kg °C))	Material	c (J/(kg °C))
Aluminum	900	Water (gas)	1850
Copper	385	Water (liquid)	4186
Gold	128	Water (ice)	2060

A change of state (a phase transition)

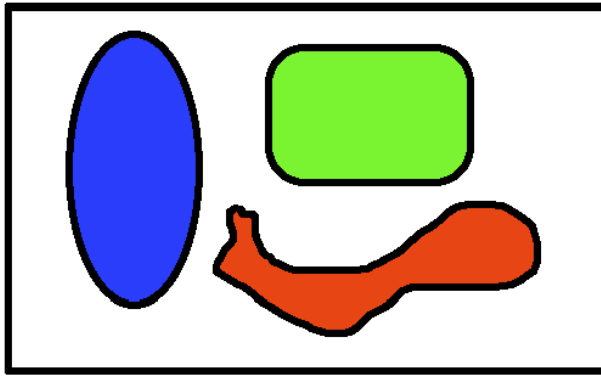
Changes of state occur at particular temperatures, so the heat associated with the process is given by:

Freezing or melting: $|Q| = mL_f$ Melting $Q = mL_f$
where L_f is the latent heat of fusion Freezing $Q = -mL_f$

Boiling or condensing: $|Q| = mL_v$ ← + or - ?
where L_v is the latent heat of vaporization

For water the values are:

$$\begin{aligned} L_f &= 333 \text{ kJ/kg} && = 80 \text{ cal/g} && (T = 0^\circ \text{C}) \\ L_v &= 2256 \text{ kJ/kg} && = 540 \text{ cal/g} && (T = 100^\circ \text{C}) \\ C_{\text{liquid}} &= 4.186 \text{ kJ/(kg }^\circ\text{C)} && = 1 \text{ cal/g}^\circ\text{C} \\ &&& (C_{\text{ice}} \approx C_{\text{steam}} \approx 0.5 \text{ cal/g }^\circ\text{C}) \end{aligned}$$



When an object absorbs some heat (internal energy increases) $\Rightarrow Q > 0$

When an object emits (loses) some heat (internal energy decreases) $\Rightarrow Q < 0$

Heating, cooling $Q = cm(T_f - T_i)$ (you get the right sign automatically)

1. List ALL possible processes

Melting $Q = mL_f$

Freezing $Q = -mL_f$

2. Write ALL

Boiling $Q = mL_v$

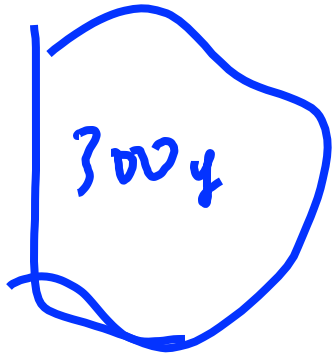
Condensing $Q = -mL_v$

Qs

3. Set HBE

$$\sum Q = Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + \dots = 0$$

4. Solve HBE

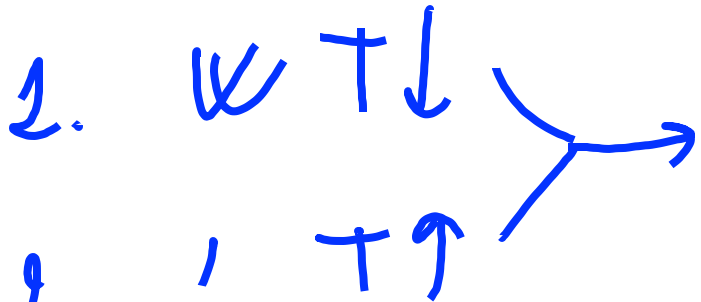


$$t_{i_e} = 22^\circ\text{C}$$

300g \rightarrow 4000 $\frac{\text{J}}{\text{kgK}}$

$$Q_w = C_w \cdot 0.3 \cdot \underline{77}^\circ\text{C}$$

$$t_{i_w} = 100^\circ\text{C}$$



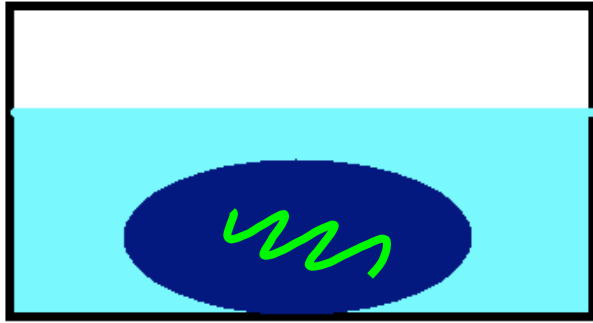
$$Q_L = C_i \cdot 0.3 \cdot \underline{77 - 22}$$

2.

$$C_i = \frac{2 \cdot 4000}{55} = 2000$$

$$Q_w + Q_L = 0 ; -4000 \cdot 0.3 \cdot 23 + C_i \cdot 0.3 \cdot 55 = 0$$

Work Together



Two objects (a liquid and a solid) having initially different temperature are contained in a thermos/thermostat (a closed box which does not take any heat from the system but insulates the system from the environment

preventing the system from a heat exchange with it.

The liquid has its initial temperature of 50°C ; the solid is initially twice warmer than the liquid.

$$T_{iL} = 50^{\circ}\text{C} \quad T_{iS} = 100^{\circ}\text{C}$$

The solid has the mass of a third of the mass of the liquid, but its specific heat is twice of the specific heat of the liquid.

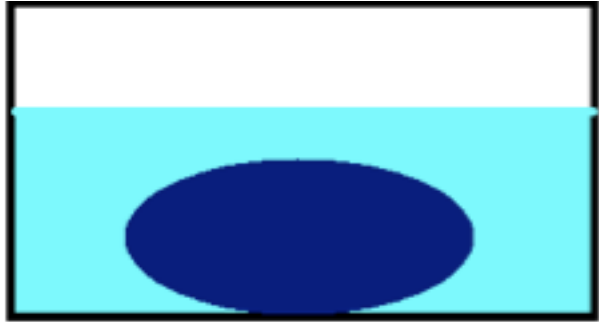
$$m_s = \frac{1}{3} m_L$$

Try to find the final temperature of the system when it is in a thermal equilibrium.

$$c_s = 2 \cdot c_L$$

The liquid has its initial temperature of 50°C ; the solid is initially twice warmer than the liquid.

The solid has the mass of a third of the mass of the liquid, but its specific heat is twice of the specific heat of the liquid.



~~$Q = mL$~~ $Q = cm(T_f - T_i)$

$$\sum Q = 0 \quad Q_2 = c_2 m_2 (T_f - T_{2i})$$

$$Q_1 + Q_2 = 0 \quad m_2 = \frac{1}{3} m_1$$

$$T_{1i} = 50^{\circ}\text{C} \quad T_{2i} = 100^{\circ}\text{C}$$

$$c_1 m_1 (T_f - 50) + 2c_1 \cdot \frac{1}{3} m_1 (T_f - 100) = 0$$

$c_2 = 2 \cdot c_1$

Sample Problem (SIM)

A 300 gram lead ball with a temperature of 80°C is placed in 300 grams of water at a temperature of 20°C .

When the system reaches equilibrium what is the equilibrium temperature?

Assume no energy is exchanged with the surroundings.

1. a little less than 80°C
2. around 50°C
3. a little more than 20°C
4. not sure

$$c_w = 4186 \text{ J}/(\text{kg } ^{\circ}\text{C})$$

$$c_{\text{Pb}} = 130 \text{ J}/(\text{kg } ^{\circ}\text{C})$$

Let's set:

T_w = initial temperature of water

T_{Pb} = initial temperature of lead

T_f = final temperature at equilibrium

Since no heat is exchanged with the surroundings: $\Sigma Q = 0$

$$mc_w(T_f - T_w) + mc_{Pb}(T_f - T_{Pb}) = 0$$

The masses cancel because they're equal in this case. We can determine the specific heat of lead if we know that the specific heat of water is:

$$c_w = 4186 \text{ J/(kg } ^\circ\text{C)}$$

Our equation above becomes:

$$c_{Pb}(T_f - T_{Pb}) = -c_w(T_f - T_w)$$

$$c_{Pb} = c_w * \left(\frac{T_f - T_w}{T_{Pb} - T_f} \right)$$

Or, if we know the specific heat of lead we can find T_f

$$130(T_f - 80) = -4186(T_f - 20) \quad \Rightarrow \quad \underline{T_f = 22 \text{ } ^\circ\text{C}}$$

The accepted value for c_{Pb} is 130 J/(kg $^\circ\text{C}$)

Problem 1. There is 100 g of water in styrofoam cup at 50°C .
What is final temperature going to be after adding 100 g of copper
with initial temperature -200°C ? Assume that no heat is exchanged
with the cup or with the surroundings. Use these approximate values
to determine your answer: Specific heat of water is about $4000\text{ J}/(\text{kg }^{\circ}\text{C})$
Specific heat of copper is about $400\text{ J}/(\text{kg }^{\circ}\text{C})$

Specific heat of ice is about $2000\text{ J}/(\text{kg }^{\circ}\text{C})$

Latent heat of fusion of water is about $3 \times 10^5\text{ J}/\text{kg}$

Let's check if the transition is happening: $Q_w = 0.1 \times 4000 \times (0 - 50) = -20000\text{ J}$

$Q_{cu} = 0.1 \times 400 \times (0 - (-200)) = 8000\text{ J}$

$Q_w + Q_{cu} \neq 0$! So 0°C is NOT final temperature.

$|Q_w| > Q_{cu} \Rightarrow$ when Cu absorbs 8000 J water still will have $t > 0 \Rightarrow t_f > 0$

$$0.1 \times 4000 \times (t_f - 50) + 0.1 \times 400 \times (t_f - (-200)) = 0 \Rightarrow t_f = 27.27^{\circ}\text{C}$$

Ice or Water?

Problem 2. There is 100 g of water in styrofoam cup at 50°C . What is final temperature going to be after adding 800 g of copper with initial temperature -200°C ? Assume that no heat is exchanged with the cup or with the surroundings. Use these approximate values to determine your answer: Specific heat of liquid water is about $4000\text{ J}/(\text{kg }^{\circ}\text{C})$ Specific heat of copper is about $400\text{ J}/(\text{kg }^{\circ}\text{C})$
Specific heat of ice is about $2000\text{ J}/(\text{kg }^{\circ}\text{C})$
Latent heat of fusion of water is about $3 \times 10^5\text{ J}/\text{kg}$

Let's check if the transition is happening: $Q_w = 0.1 \times 4000 \times (0 - 50) = -20000\text{ J}$

$Q_{cu} = 0.8 \times 400 \times (0 - (-200)) = 64000\text{ J}$

$Q_w + Q_{cu} \neq 0$! So 0°C is NOT final temperature.

$|Q_w| < Q_{cu} \Rightarrow$ when water releases 20000 J Cu still have $t < 0 \Rightarrow t_f < 0$ (or at least $t_f = 0$)

\Rightarrow Water freezes let's check if all water freezes: $-20000 + -mL = -20000 + -0.1 \times 300000 = -50000\text{ J}$ still $|Q_w + -mL| < Q_{cu} \Rightarrow$ all water gets frozen and ice is getting colder \Rightarrow

$$0.1 \times 4000 \times (0 - 50) + -0.1 \times 300000 + 0.1 \times 200 \times (t_f - 0) + 0.8 \times 400 \times (t_f - (-200)) = 0 \Rightarrow t_f = -41^{\circ}\text{C}$$

C_n

\approx

? 1)

$C_n \uparrow$

$\approx \downarrow \Rightarrow T_4 > 0$

? 2)

$C_n \uparrow$

$\approx \downarrow \Rightarrow T_4 = 0$

? 3)

$C_n \uparrow$

$\approx \downarrow \Rightarrow T_4 < 0$

T_4

$|Q_{C_n}| = 1$
 $\text{if } T_4 = 0 \dots$

$\langle \rangle$
 $|Q_{C_n}| = 1$
 $\text{if } T_4 = 0$

$$C_{in} > 0$$

$$Q_{in} < 0$$

$$|Q_{in}| = |Q_{in}|$$

$$|Q_{in}| > |Q_{in}|$$

Can 0°

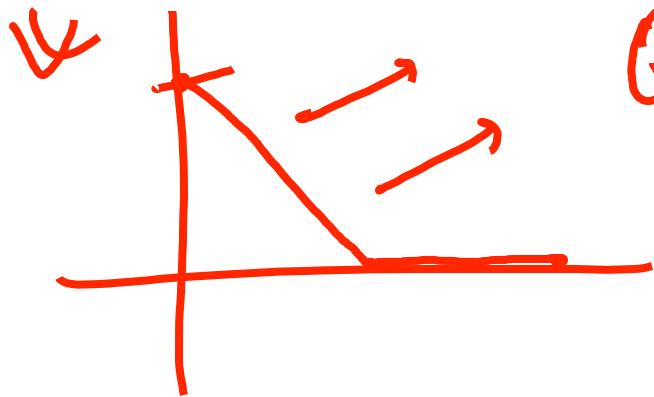
1. Yes

2. No

$$Q_{in} + Q_{in} = 0$$

$$\underline{T_{in} = 0}$$

$$T_{in} > 0$$



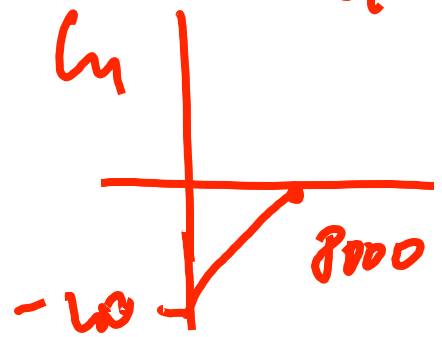
$$Q_1 + Q_2 = c_k \cdot m / (0 - T_i) -$$

$$- \underline{m \cdot L} = \underline{\underline{-10000}}$$

1) ~~8000~~ ↑
 2) 10000 ↓

$$Q_{c_k} = (c_k \cdot m_m (0 - T_{im})) =$$

3) 12000



1) $Q_{ice} + Q_{in} = 0$

2) $Q_{ice} + Q_{in} + -m \cdot c = 0$

3) $Q_{ice} + Q_{in} + -m \cdot L + Q_{ice} = 0$ ⊕

Ice water

1. Fill in the table

	m	c	L	T_i	T_f
Ice					
Water					

2. What *CAN* happen?

3. What final states *CAN* be achieved?

4. Write the expression for the heat for each possible process.

5. Start from the easiest case with *NO* phase changes and try to balance out the heat (use $\sum Q = 0$, remember, the final temperature must be the same for all elements of the system!). If you did it, you are done. If you cannot balance the heat, that means some phase changes are happening (at least one).

6. What phase change can be happening? Set up $\sum Q = 0$ including the term(s) for that change(s). Solve for the unknown.

Ice water (A)

100 grams of ice, with a temperature of -10°C , is added to a styrofoam cup of water. The water is initially at $+10^{\circ}\text{C}$, and has an unknown mass m . If the final temperature of the system is 5°C , what is the unknown mass m ? Assume that no heat is exchanged with the cup or with the surroundings.

Use these approximate values to determine your answer:

Specific heat of liquid water is about $4000 \text{ J}/(\text{kg } ^{\circ}\text{C})$

Specific heat of ice is about $2000 \text{ J}/(\text{kg } ^{\circ}\text{C})$

Latent heat of fusion of water is about $3 \times 10^5 \text{ J}/\text{kg}$

$$Q = cm(T_f - T_i) \quad |Q| = Lm \quad \underline{\Sigma Q = 0}$$

Ice water (B)

100 grams of ice, with a temperature of -10°C , is added to a styrofoam cup of water. The water is initially at $+10^{\circ}\text{C}$, and has an unknown mass m . If the final temperature of the system is -5°C , what is the unknown mass m ? Assume that no heat is exchanged with the cup or with the surroundings.

Use these approximate values to determine your answer:

Specific heat of liquid water is about $4000 \text{ J}/(\text{kg } ^{\circ}\text{C})$

Specific heat of ice is about $2000 \text{ J}/(\text{kg } ^{\circ}\text{C})$

Latent heat of fusion of water is about $3 \times 10^5 \text{ J}/\text{kg}$

$$Q = cm(T_f - T_i) \quad |Q| = Lm \quad \underline{\Sigma Q = 0}$$

100g
ice
-10°C

m-?
10°C

1) W t m
1. 0

2) W i
2. 5

$t_f = -5^\circ\text{C} < T = 0^\circ\text{C}$

3) ice m t
-5

100g
4) ice t
-10

$$m \cdot \cancel{4000} \cdot (0 - 10) +$$

$$+ \underbrace{- m \cdot \cancel{13000} + m \cdot \cancel{2000}(-5 - 0)} +$$

$$+ 100 \cdot \cancel{2000} \underbrace{(-5 - 10)} = 0$$

$$m(-40 - 13 - 10) + 100 \cdot 5 = 0$$

$m = \frac{500}{6}$