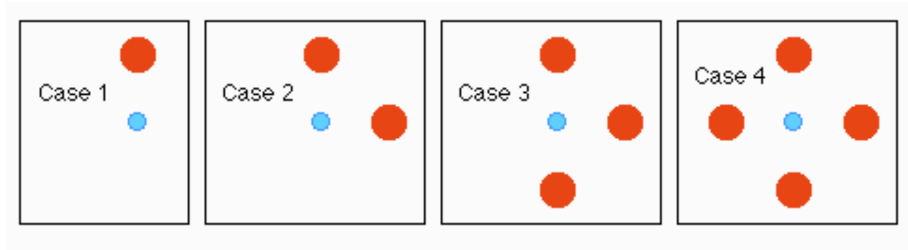
$$F_g = \frac{GmM}{r^2}$$



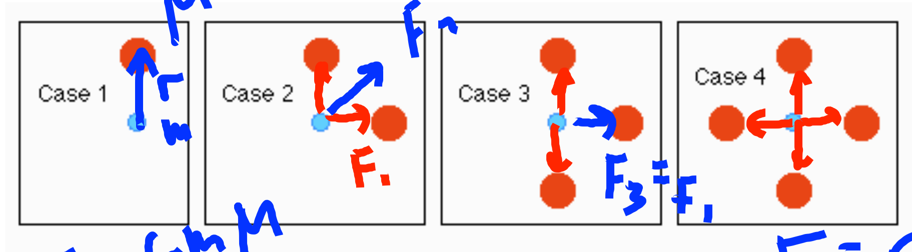
Question

A small blue ball has 1 or more large red balls placed near it. The red balls are all the same mass and the same distance from the blue one. Rank the different cases based on the net gravitational force experienced by the blue ball.

The correct ranking, from largest net force to smallest, is:

1. 1=3>2>4
2. 3>2>1>4
3. 4>3>2>1
4. 2>1=3>4
5. 4>1=2=3

$$F_g = \frac{GmM}{r^2}$$



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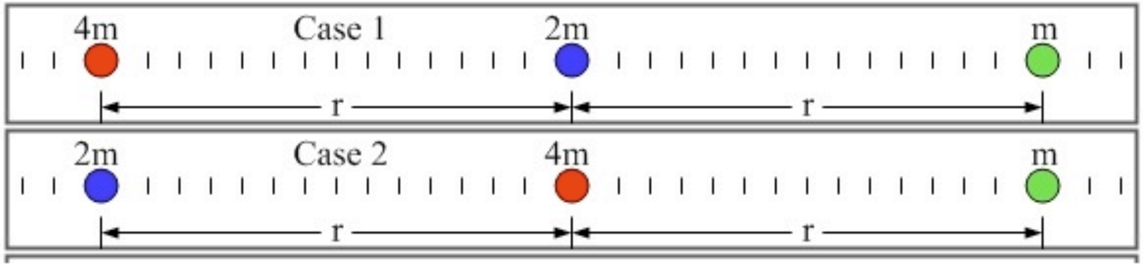
4. 2>1=3>4
5. 4>1=2=3

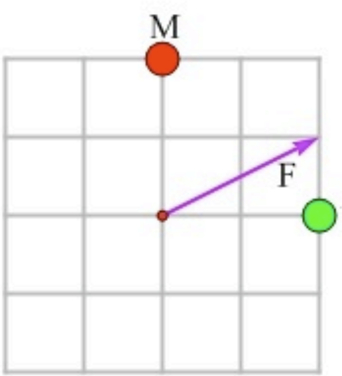
Problem 1

$$F_g = \frac{GmM}{r^2}$$



Find the ratio of the net force acting on the red ball in the case 1 to the one acting on it in the case 2.

Problem 2

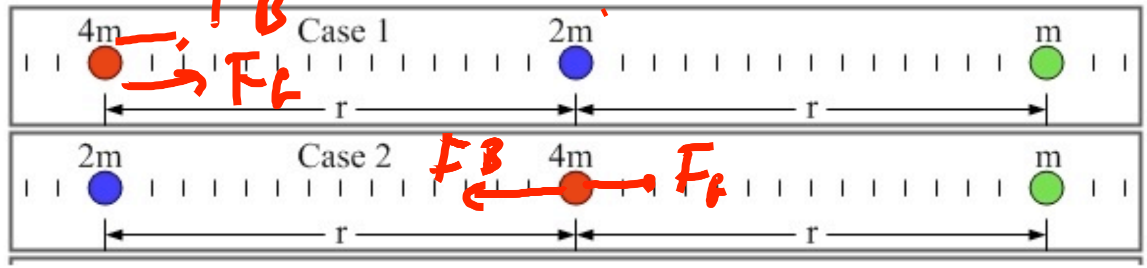


What is the ratio of masses of the balls?

(the arrow shows the net force)

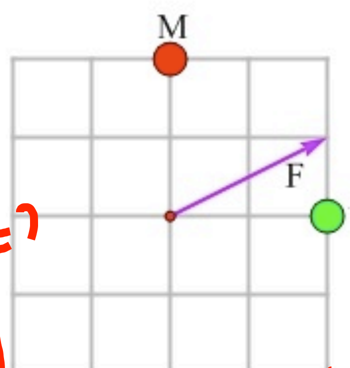
Problem 1

$$F_g = \frac{GmM}{r^2}$$



Find the ratio of the net force acting on the red ball in the case 1 to the one acting on it in the case 2.

Problem 2



What is the ratio of masses of the balls?

(the arrow shows the net force)

Two balls are placed as shown.

$$F_g = \frac{GmM}{r^2}$$

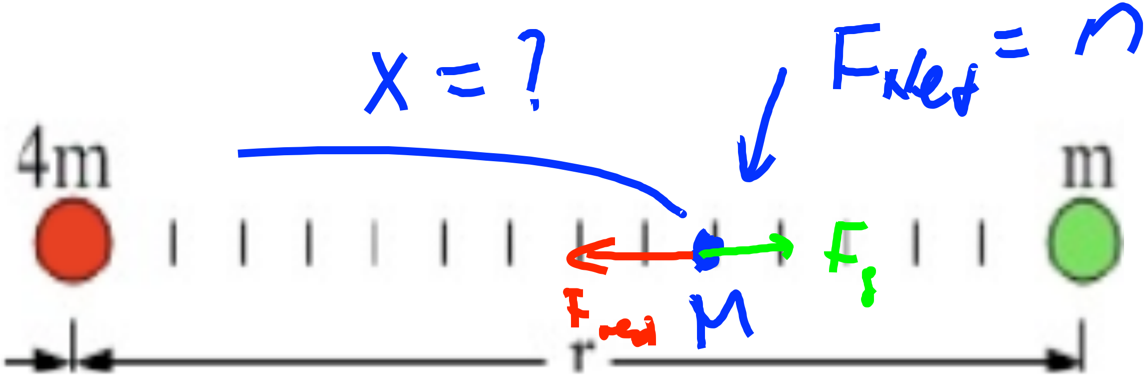
Where should we place a third ball so it would NOT experience any net force of gravity acting on it?

$$F_g = \frac{GmM}{r^2}$$

Two balls are placed as shown.

m, r, G

x = ?



Where should we place a third ball so it would NOT experience any net force of gravity acting on it?

$$\frac{G\,4m\,M}{x^2} = \frac{G\,M\cdot m}{(r-x)^2}$$

1. To the left to the red ball
2. Between the balls
3. To the right to the green ball

$r > x > 0$

$$\sqrt{\frac{4}{x^2}} = \sqrt{\frac{1}{(r-x)^2}} \rightarrow$$

$$4(r-x)^2 = x^2$$

$$4(r^2 - 2rx + x^2) = x^2$$

$$3x^2 - 8rx + 4r^2 = 0$$

$$x = \frac{8r - \sqrt{D}}{2 \cdot 3}$$

$$\frac{2}{x} = \frac{1}{r-x}$$

$$2(r-x) = x; \quad 2r = 3x$$

$$x = \frac{2}{3}r$$

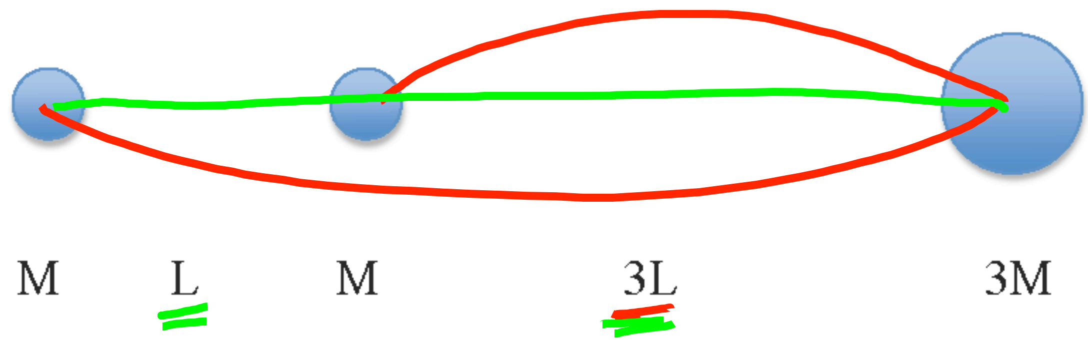
$$F_g = \frac{GmM}{r^2} \qquad U_g = -\frac{GmM}{r}$$

Find the force acting on the big ball.

Find the potential energy of the *big ball*.

Find the potential energy of the *system*.

$$F_g = \frac{GmM}{r^2} \qquad U_g = -\frac{GmM}{r}$$



Find the force acting on the big ball.

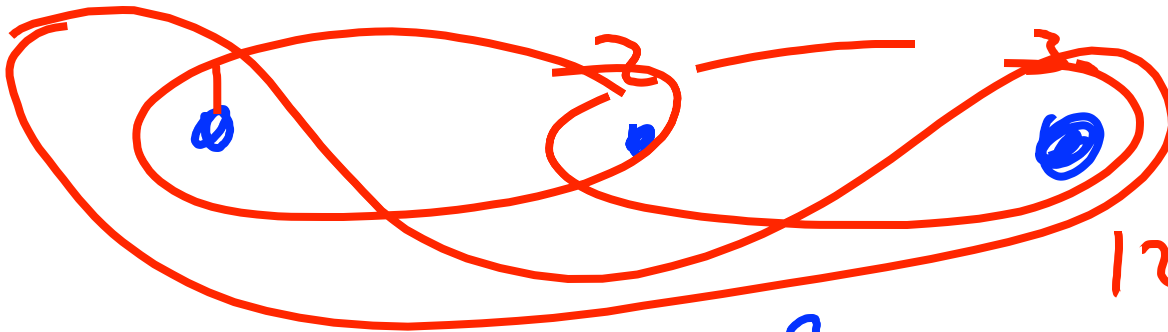
Find the potential energy of the *big ball*.

Find the potential energy of the *system*.

$$U = -\frac{G3M\cdot M}{3L} + -\frac{G3M\cdot M}{4L}$$

$$= -G\frac{M^2}{L}\left(\frac{4}{4} + \frac{3}{4}\right) = -\frac{7GM^2}{4L}$$

$$U = -G\frac{m_1 m_2}{r}$$



12

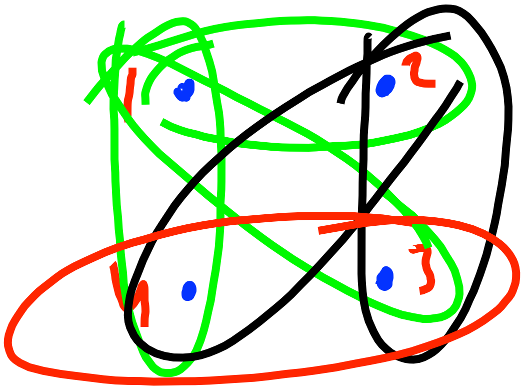
13

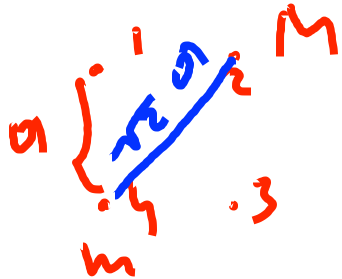
23

How many U?

1
2
3
4
5
6
7
8
9

$U_{sys} =$ +

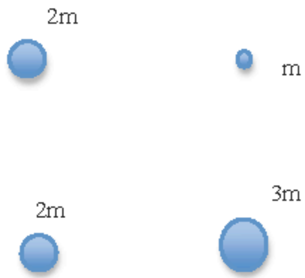




$$V_{24} = -G\frac{m \cdot M}{\sqrt{2}\,a}$$

$$U_{sys} = V_{12} + V_{13} + V_{14} + V_{23} + V_{24} + V_{34}$$

Total potential energy



Four balls of different mass are placed at the corners of a square with the side a.

Neglecting the size of the balls, find the total potential energy of the system.

How many different pairs should we consider?

1. 3
2. 4
3. 5
4. 6
5. 7

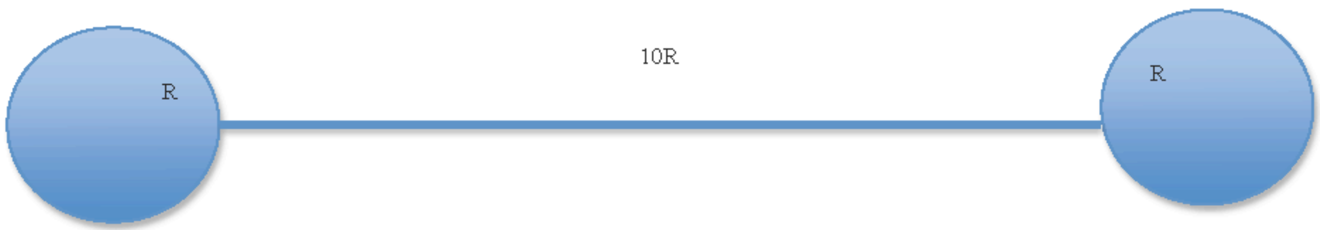
(P.S. it is a good exercise to find the net force acting on each ball)

$$U_g = -\frac{GmM}{r}$$

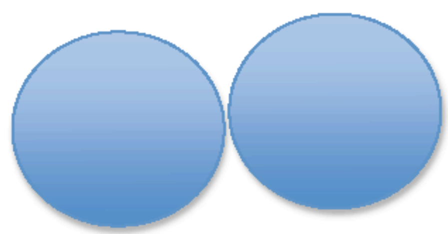
$$F_g = \frac{GmM}{r^2}$$

Colliding galaxies!

Two big identical galaxies with radius R and mass M at distance L between the galaxies. If galaxies are initially at rest, what is their speed at the moment when they start colliding?



$$U_g = -\frac{GmM}{r}$$



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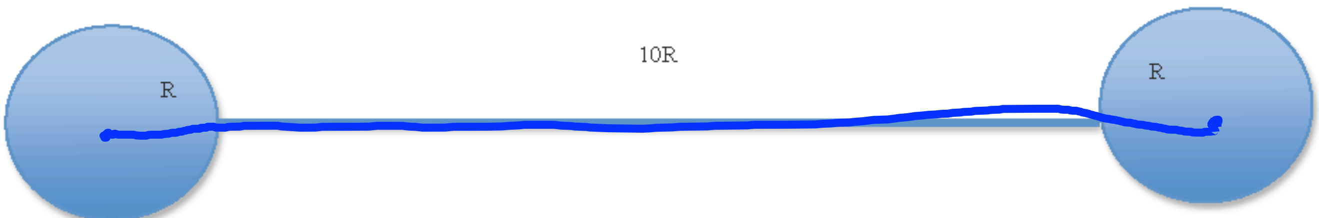
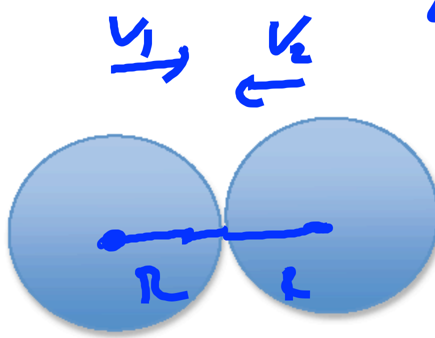


$$E_1 = 0 - G\frac{M \cdot M}{12R}$$

LCME: $E_1 = E_2$

$$U_g = -\frac{GmM}{r}$$



LCLM:

$$0 = M \cdot V_1 - M \cdot V_2 \Rightarrow V_1 = V_2$$

$$E_2 = \frac{MV^2}{2} + \frac{MV^2}{2} + \left(-G\frac{M \cdot M}{2R}\right)$$

Energy in a Circular Orbit

Imagine that we have an object of mass m in a circular orbit around an object of mass M. An example could be a satellite orbiting the Earth. What is the total energy associated with this object in its circular orbit?

As usual, $E = U + K$.

$$U = \frac{- G\, m\, M}{R} \qquad \text{and} \qquad K = \tfrac{1}{2} m v^2$$

The only force acting on the object is the force of gravity. Applying Newton's Second Law gives:

$$\Sigma \mathbf{F} = m\mathbf{a}$$

Energy in a Circular Orbit

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$r = R + h$ 5)

As usual, $E = U + K$. 1)

$$U = \frac{-\,G\,m\,M}{R+h} \quad 3) \qquad \text{and} \qquad K = \tfrac{1}{2} m v^2 \quad 2)$$

$$G\frac{mM}{r^2} = m \cdot \frac{v^2}{r} \quad 4)$$

The only force acting on the object is the force of gravity. Applying Newton's Second Law gives:

$\Sigma \mathbf{F} = m\mathbf{a}$

$\Sigma F = m\mathbf{a}$

$$\frac{G\,m\,M}{r^2} = \frac{mv^2}{r} \longrightarrow V = \sqrt{\frac{GM}{r}}$$

Therefore: $mv^2 = \dfrac{G\,m\,M}{r}$ and $K = \frac{1}{2} mv^2 = \dfrac{G\,m\,M}{2r}$

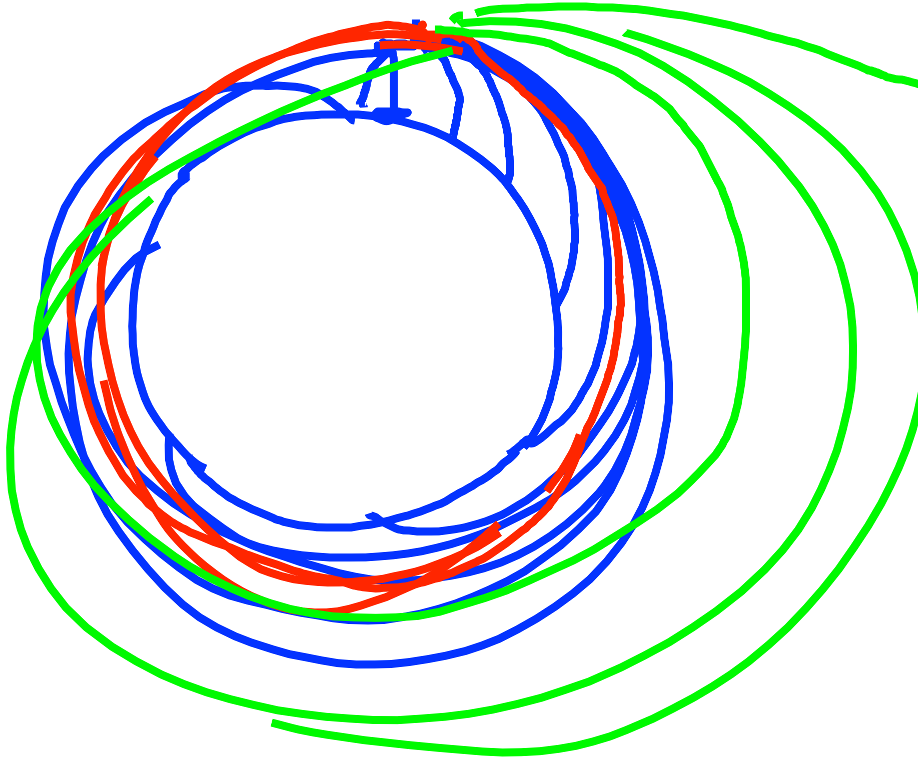
The kinetic energy is positive, and half the size of the potential energy.

$$E = \frac{-G\,m\,M}{r} + \frac{G\,m\,M}{2r} = \frac{-G\,m\,M}{2r} < 0$$

Total Energy!

A negative total energy tells us that this is a *bound* system. The satellite is bound to the Earth - energy would have to *be added* to the system to remove the satellite from the Earth.

$$E + W = E_{final} > 0 \quad \Rightarrow W > 0$$

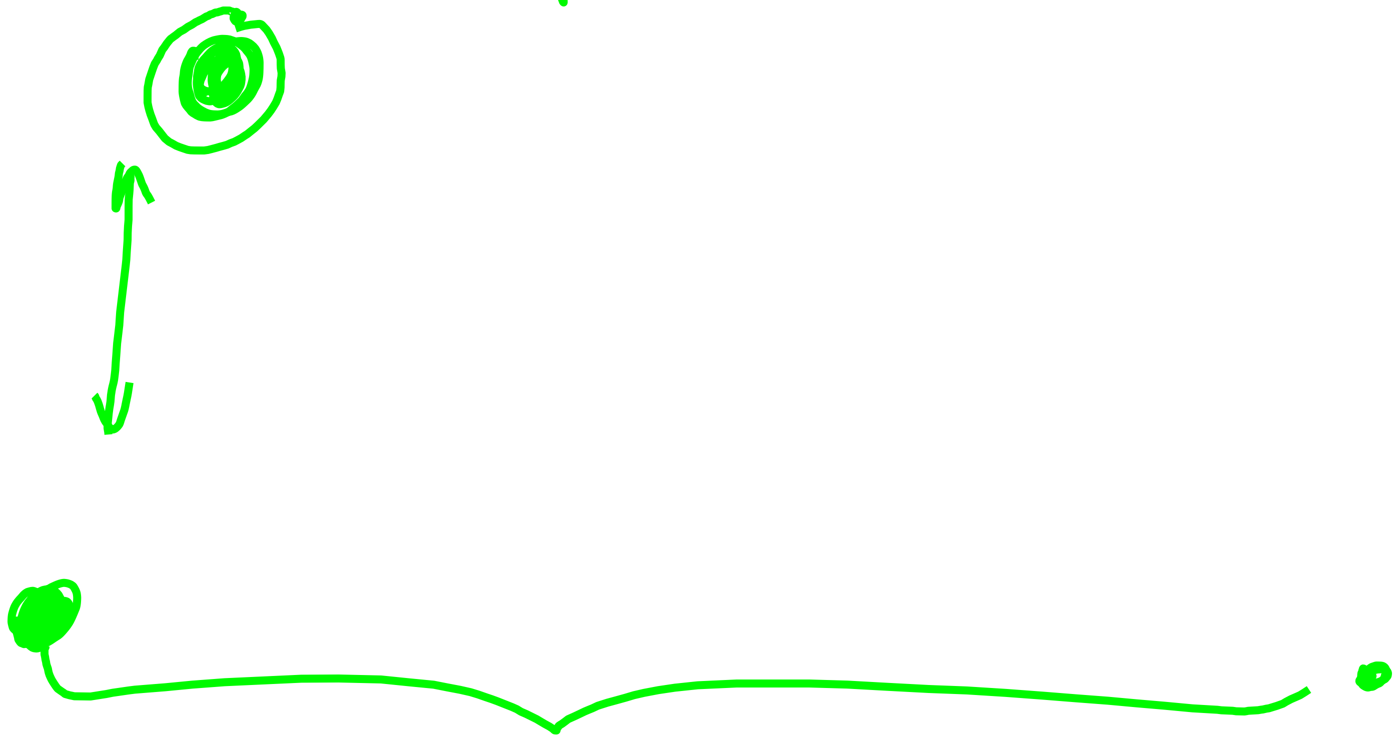


$$V = \sqrt{\frac{GM}{R}}$$

V_1

V_2

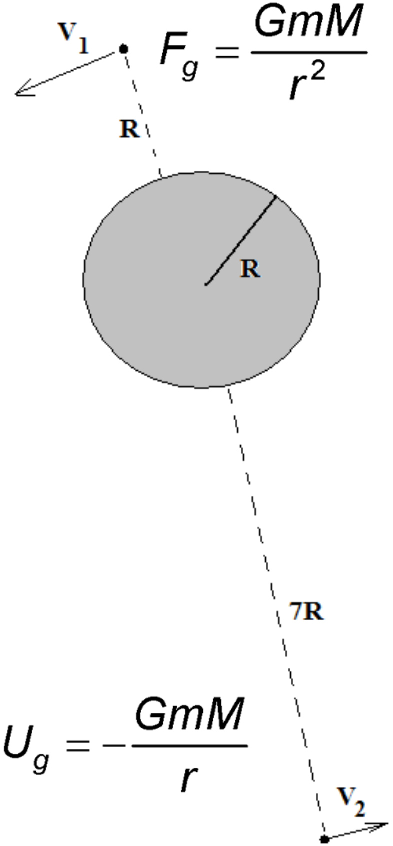
$$v_i = \sqrt{\frac{GM}{R}}$$



$$V_f = -\frac{GMm}{r} = 0$$

$$r \longrightarrow \infty$$

$$F_g = \frac{GmM}{r^2}$$



$$U_g = -\frac{GmM}{r}$$

A huge star of the radius R has two planets circulating about (assume the first planet has the mass twice the second one).

Try to find the ratio for orbital speeds, periods, kinetic energies, potential energies, total energies:

$$\frac{v_1}{v_2} = ? \quad \frac{T_1}{T_2} = ? \quad \frac{K_1}{K_2} = ? \quad \frac{U_1}{U_2} = ? \quad \frac{E_1}{E_2} = ?$$

Angular Momentum

Angular momentum is conserved as long as no net torque is applied.

For an object in orbit, does gravity apply a torque?

1. Yes
2. No

No - there is no torque because the force points toward the point around which rotation occurs.

This means we can apply angular momentum conservation.

At all points in the orbit angular momentum is conserved - for an elliptical orbit as r increases the speed must be reduced to compensate for that, and vice versa.

Escape speed

How fast would you have to throw an object so it never came back down? Ignore air resistance. Let's find the escape speed - the minimum speed required to escape from a planet's gravitational pull.

How should we try to figure this out?

Attack the problem from a force perspective?

From an energy perspective?

Forces are hard to work with here, because the size of the force changes as the object gets farther away. Energy is easier to work with in this case.

Escape speed

Let's start with the conservation of energy equation.

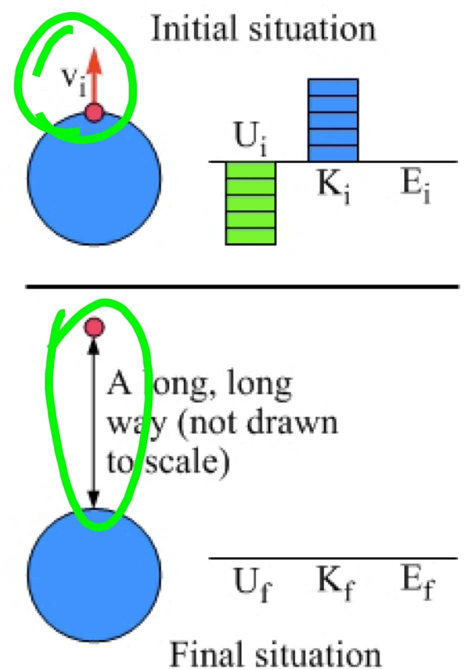
$$U_i + K_i + W_{nc} = U_f + K_f$$

Which terms can we cross out immediately?

Assume no resistive forces, so $W_{nc} = 0$

Assume the object barely makes it to infinity, so both U_f and K_f are zero.

This leaves: $U_i + K_i = 0$



Escape speed

$$U_i + K_i = 0$$

If the total mechanical energy is negative, the object comes back. If it is positive, it never comes back.

$$-\frac{GmM}{R} + \frac{1}{2}mv_{escape}^2 = 0$$

The mass of the object, *m*, does not matter. Solving for the escape speed gives:

$$v_{escape} = \sqrt{\frac{2GM}{R}}$$

M is the mass of the planet; *R* is the planet's radius.

For the Earth, we get v_{escape} = 11.2 km/s.

This week's topics

Temperature, temperature scales, thermal contact, thermal conduction, thermal equilibrium, measuring temperature, heat, internal energy, meaning of temperature, meaning of heat, thermal expansion, coefficient of thermal expansion (CTE), linear, areal, and volumetric CTE, heat capacity, specific heat (capacity), thermally insulated system, heat balance equation (an equation for thermal equilibrium), phase transition, critical temperature, latent heat (capacity), method for solving thermal equilibrium problems, convection, thermal radiation, thermal conduction, thermal conductivity, the ideal gas, absolute temperature, a mole, the Avogadro's number, the universal gas constant, RMS values, the ideal gas law, iso – laws, graphs for gas processes (PV, VT, PT diagrams), the Boltzmann's constant, the meaning of the absolute temperature, the meaning of the pressure, degree of freedom, the equipartition theorem, monatomic, diatomic, polyatomic gas, calculating internal energy, the first law of thermodynamics, work done by gas, calculating specific heat (Cv, Cp), isothermal process, adiabatic process, thermodynamic cycle, work done over a cycle, heat engine, entropy, second law of thermodynamics, heat engine efficiency, the Carnot cycle, maximum (ideal) heat engine efficiency, a heat pump and a refrigerator(***the last topic of test 3***)

Temperature

Temperature is a measure of the average kinetic energy per atom or molecule making up an object or a system.
(More on this next time)

For an isolated system, if all parts of it have the same temperature, then we say that the system has come to equilibrium.

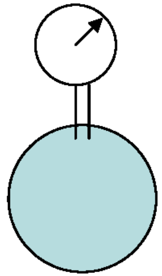
If systems "Hi" and "Lo" are brought into thermal contact, energy is transferred **from "Hi" to "Lo"** when $T_{Hi} > T_{Lo}$

This transferred energy is called heat Q.
The energy of "Hi" or "Lo" itself is called internal energy U.

$$\Delta U_{Lo} = Q_{in} \qquad \Delta U_{Hi} = -Q_{in} = Q_{out}$$

Measuring temperature

A device used to measure temperature is called a thermometer, and all thermometers exploit the fact that properties of a material depend on temperature. Examples of temperature-dependent properties include:



- the pressure in a sealed container of gas
- the volume occupied by a liquid
- the voltage generated across a junction of two different metals

"Thermocouple" – digital readout these days

All these effects, and plenty of others, can be used in thermometers.

Temperature scales

T = t + 273.15

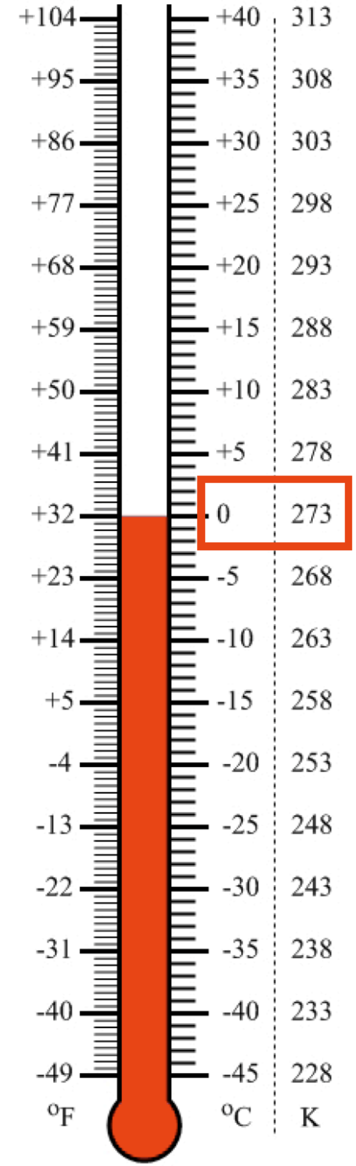
A change by 1°C is the same as a change by 1K. The Celsius and Kelvin scales are just offset by about 273.

A change by 1°C is the same as a change by 1.8°F. To convert between Celsius and Fahrenheit we use:

$$T_C = \left(\frac{5°C}{9°F}\right)(T_F - 32°F)$$

$$T_F = \left(\frac{9°F}{5°C}\right)(T_C) + 32°F$$

Fahrenheit is relevant for weather reports, not this course



Equations involving temperature

If the equation involves T, use an absolute temperature (we generally use a Kelvin temperature).

If the equation involves ΔT, we can use Celsius or Kelvin.

$$\mathbf{T = t + 273.15}$$

(never negative!)

Thermal expansion

Linear expansion

Most materials expand when heated. As long as the temperature change isn't too large, each dimension of an object experiences a change in length that is proportional to the change in temperature.

$$\Delta L = L_0 \, \alpha \, \Delta T \qquad \text{or, equivalently,} \qquad L = L_0 (1 + \alpha \, \Delta T)$$

where L_0 is the original length, and α is the coefficient of linear expansion, which depends on the material.

Material	α (× 10^{-6}/°C)	Material	α (× 10^{-6}/°C)
Aluminum	23	Glass	8.5
Copper	17	Iron	12

T K

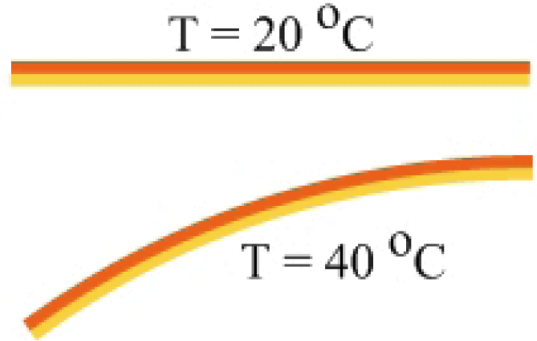
$\Delta T = \Delta t$

$= L_0 / (1 + \alpha \Delta t)$

Bimetallic strip

A bimetallic strip is made from two different metals that are bonded together. The strip is straight at room temperature, but it curves when it is heated. How does it work?

The metals have equal lengths at room temperature but different expansion coefficients, so they have different lengths when heated.



What is a common application of a bimetallic strip?

A bimetallic strip can be used as a switch in a thermostat. When the room is too cool the strip completes a circuit, turning on the furnace. The furnace goes off when the room (and the strip) warms up.

Thermal expansion

Volume expansion

For small temperature changes, we can find the new volume using:

$$\Delta V = V_0 (3\alpha)\Delta T \quad \text{or, equivalently,} \quad V = V_0(1 + 3\alpha\,\Delta T)$$

where V_0 is the original volume.

Why? $(1 + \alpha\Delta T)^3 = 1 + 3\alpha\Delta T + 3\alpha^2(\Delta T)^2 + \alpha^3(\Delta T)^3$

small — Really small — Really, really small

What happens to holes?

When an object is heated and expands, what happens to any holes in the object? Do they get larger or smaller?

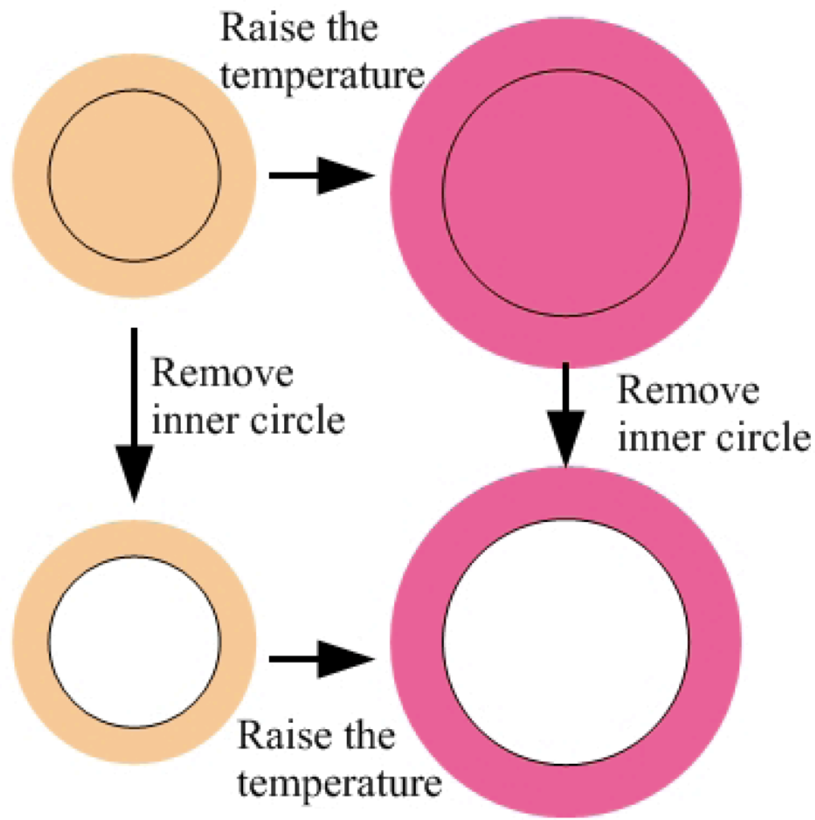
1. The holes get smaller
2. The holes stay the same size
3. The holes get larger

Holes expand, too

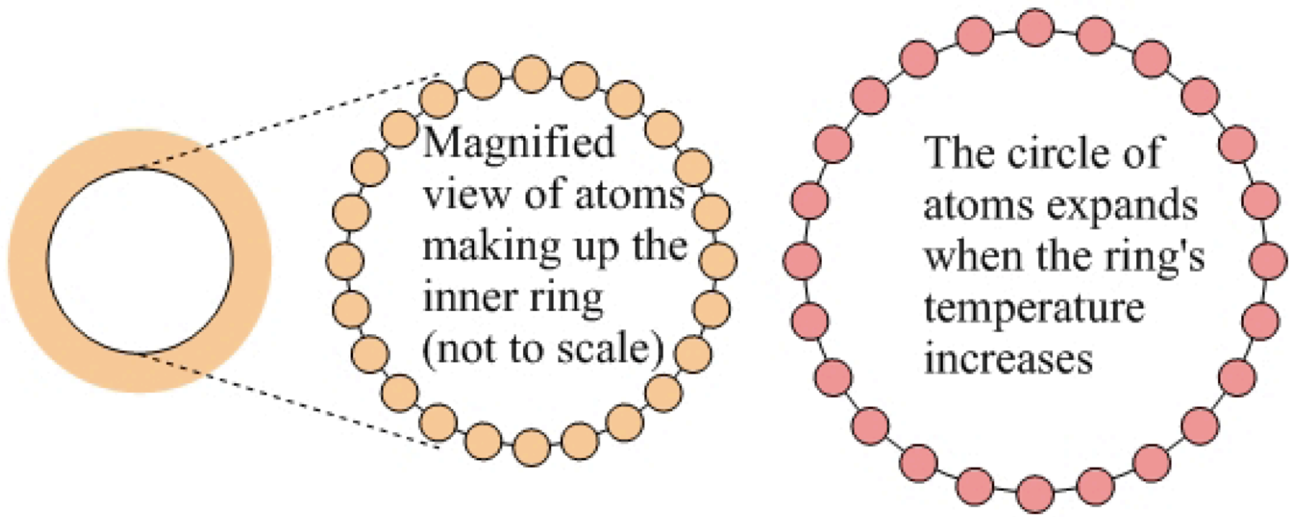
Holes expand as if they were filled with the surrounding material.

If you draw a circle on a disk and then heat the disk, the whole circle expands.

Removing the material inside the circle before heating produces the same result – the hole expands.



Holes expand, too



“Heat”

“Heat” is energy transferred between a system and its surroundings because of a temperature difference between them.

An isolated system does not change the energy, hence it does not exchange heat with its surrounding; but the parts of the system can exchange the energy/heat.

When an object accepts energy we assign a positive value to heat ($Q > 0$), when an object gives energy away we assign a negative value to heat ($Q < 0$).

For an isolated system $\Sigma Q = 0$ **HBE (!)**

"Heat" is energy transferred between a system and its surroundings because of a temperature difference between them.

The heat capacity of an object tells us how much heat is required *to raise a temperature of a s certain amount of the substance by one degree.*

In general: The heat capacity of an object ***C*** is defined by

$$Q = \boldsymbol{C}\Delta T \quad (\text{or } \boldsymbol{C} = Q/DT \text{ – the definition})$$

For gases (and all other substances, too) a *molar* heat capacity C can be introduced - the heat required to increase the temperature of *1 mole* of the gas by 1 K.

$$Q = Cn\Delta T \text{ (n is the number of moles)}$$

In addition, a mass heat capacity c (a little c), or *specific heat*, can be useful.

$$c = C/m \quad \Rightarrow \quad Q = cm(T_f - T_i)$$ (m is the mass of the object)

Specific heat

The specific heat of a material is the amount of heat required to raise the temperature of 1 kg of the material by 1°C.

The symbol for specific heat is *c*.

Heat lost or gained by an object is given by: $Q = mc\Delta T$

Material	c (J/(kg °C))	Material	c (J/(kg °C))
Aluminum	900	Water (gas)	1850
Copper	385	Water (liquid)	4186
Gold	128	Water (ice)	2060

A change of state (a phase transition)

Changes of state occur at particular temperatures, so the heat associated with the process is given by:

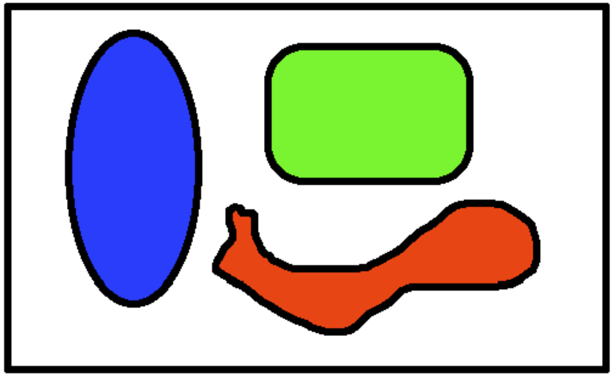
Freezing or melting: $|Q| = mL_f$
where L_f is the latent heat of fusion

Boiling or condensing: $|Q| = mL_v$
where L_v is the latent heat of vaporization

For water the values are:

L_f = 333 kJ/kg = 80 cal/g (T = 0 ^{0} C)
L_v = 2256 kJ/kg = 540 cal/g (T = 100 ^{0} C)
c_{liquid} = 4.186 kJ/(kg °C) = 1 cal/g°C
($c_{ice} \approx c_{steam} \approx$ 0.5 cal/g °C)

Isolated system



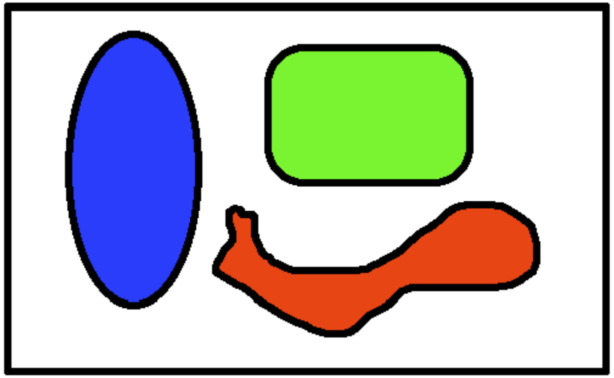
When a system is isolated, there is no energy exchange with an environment =>

Energy does not change =>

Energy can transfer only within the system.

Hot objects get cooler, cold objects get wormer, and/or some objects can change the state. If we account for ALL possible changes and assign an amount of heat to each, the *total* amount of the heat transferred within the system must be equal to **ZERO** (since there is NO energy exchange).

$$\sum Q = \mathbf{Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + \ldots = 0}$$



When an object absorbs some heat (internal energy increases) => Q > 0

When an object emits (looses) some heat (internal energy decreases) => Q < 0

Heating, cooling $Q = cm(T_f - T_i)$ (you get the right sign automatically)

1. List ALL possible processes
2. Write ALL Qs
3. Set HBE
4. Solve HBE

Melting $Q = mL_f$ Freezing $Q = -mL_f$

Boiling $Q = mL_v$ Condensing $Q = -mL_v$

$$\sum Q = Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + \ldots = 0$$