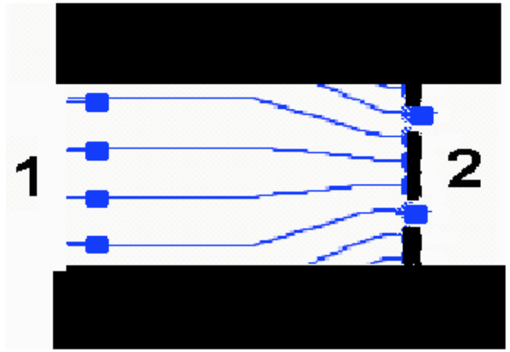
The continuity equation (Question 2)

Av= constant



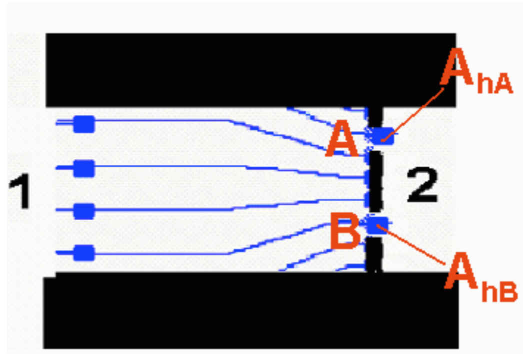
The fluid is flowing through the tube. At one end (#1) the tube is open.
At the other end (#2) the tube is covered by a plaster with two holes in it.

At which point the speed of the flow is greater?

1) Point 1 2) Point 2 3) the speed is the same

The continuity equation

$Av =$ constant or $A_1v_1 = A_2v_2$



The fluid is flowing through the tube. At one end (#1) the tube is open. At the other end (#2) the tube is covered by a plaster with two holes in it.

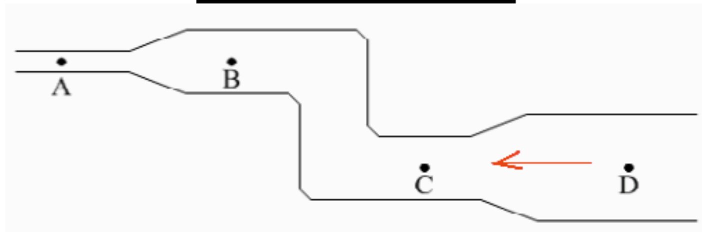
At which point the speed of the flow is greater?

The total cross-sectional area of two holes $A_2 = A_{hA} + A_{hB}$ is still **less** then the area at the open end!

$$A_1 > A_2 \quad \Rightarrow \quad v_1 < v_2$$

B) Point 2

Fluid in a Pipe



1. To connect the speed of the current at different places we use

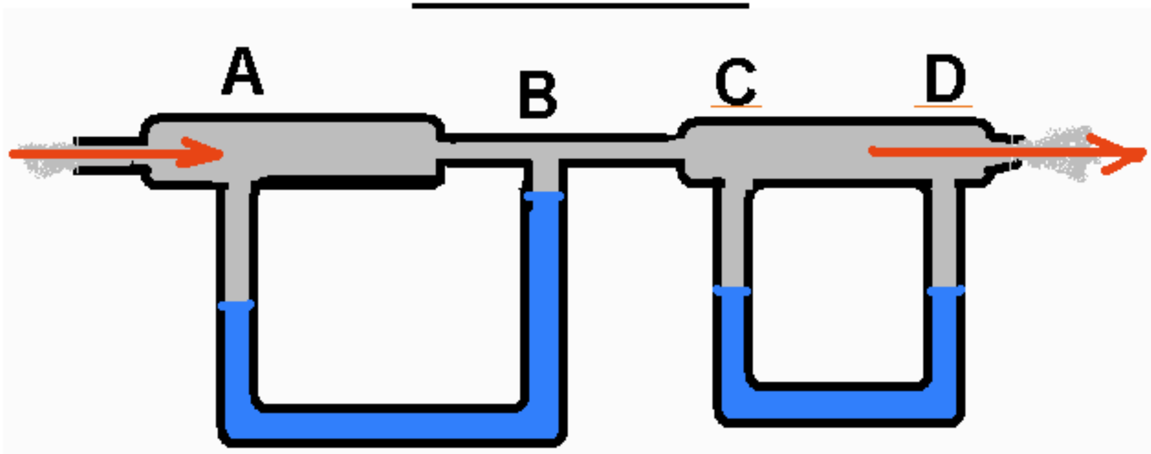
$$A_1 v_1 = A_2 v_2$$

2. To connect the pressure at different places and understand how the difference in the height and speed influences the pressure we use

Bernoulli's equation:

$$\rho g y_1 + \tfrac{1}{2}\rho v_1^2 + P_1 = \rho g y_2 + \tfrac{1}{2}\rho v_2^2 + P_2$$

Air flow demo

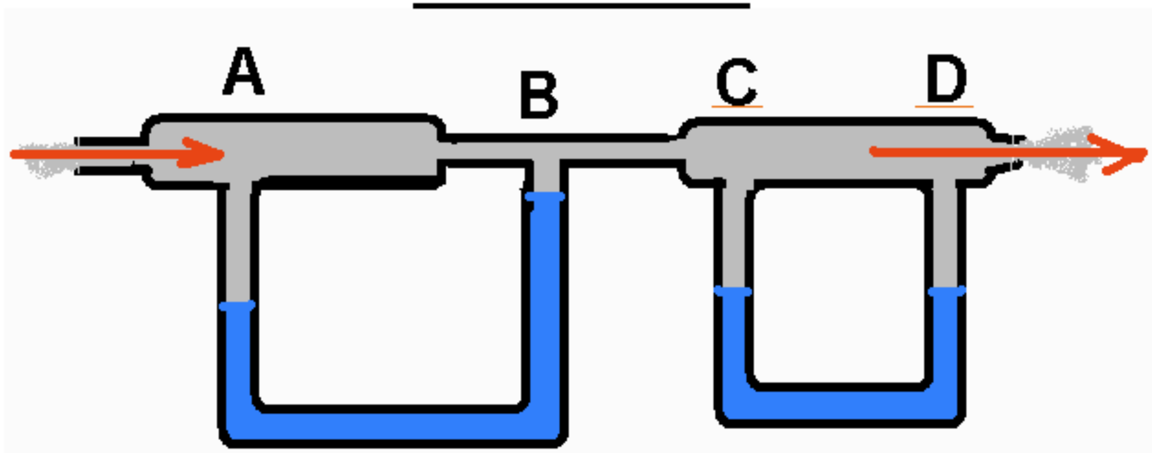


Question

At which point, C or D, the pressure is higher (if so)?

1) $P_C = P_D$ 2) $P_C < P_D$ 3) $P_C > P_D$

Air flow demo



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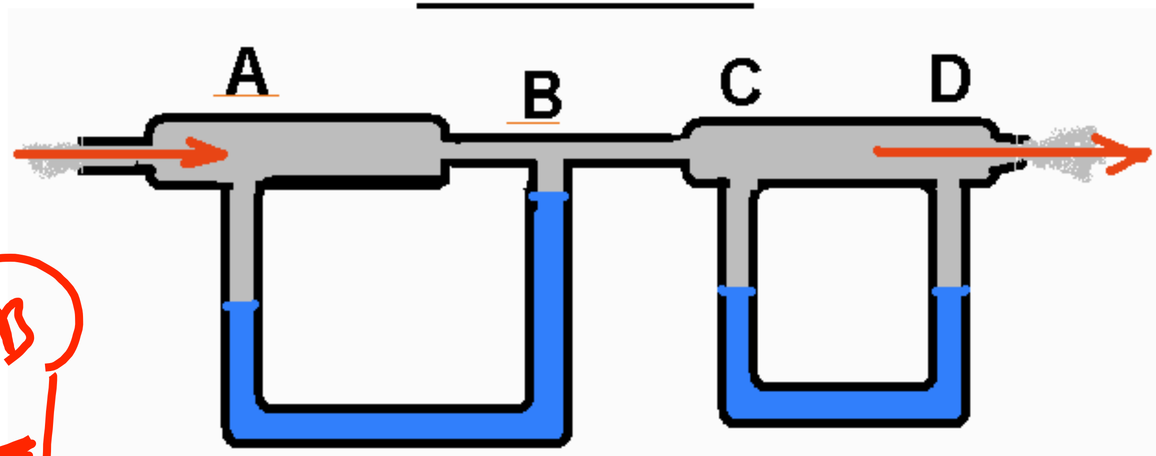
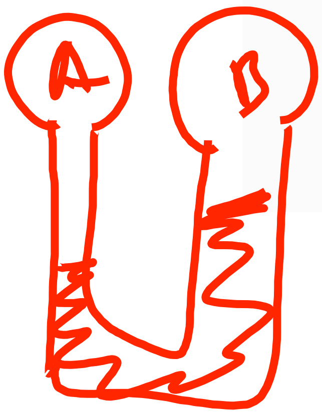
A) $P_C = P_D$

Air flow demo



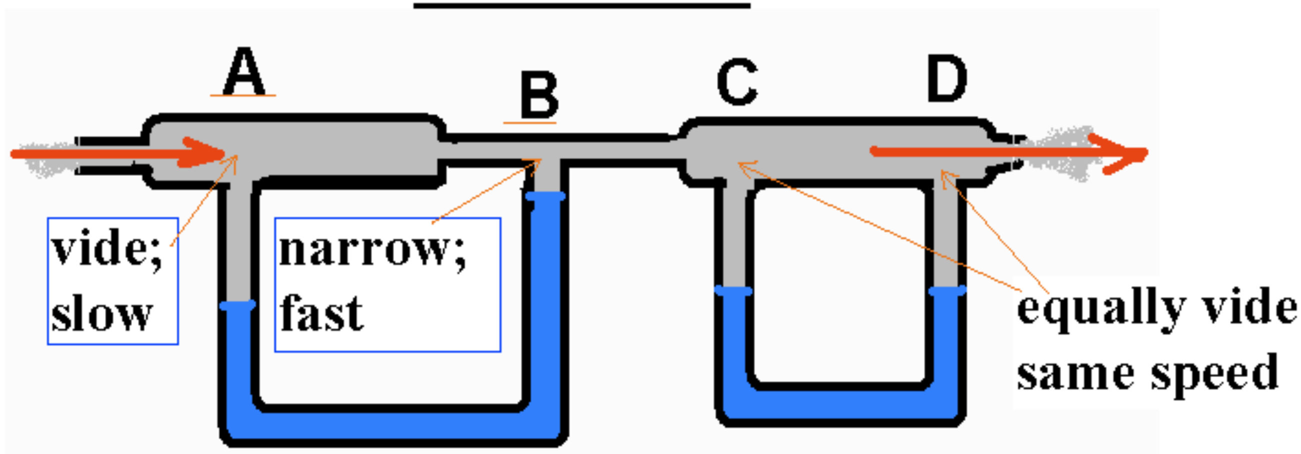


Question

At which point, A or B, the pressure is higher (if so)?

1) $P_A = P_B$ 2) $P_A < P_B$ 3) $P_A > P_B$

Air flow demo



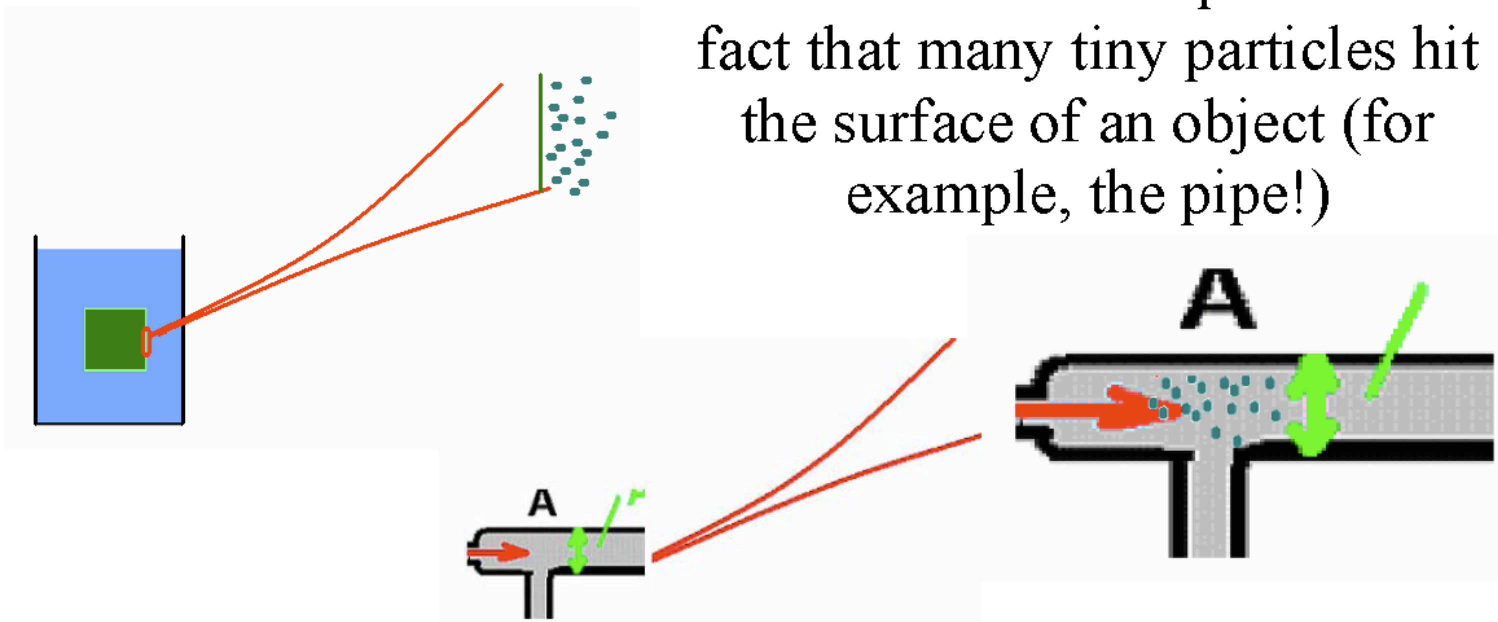
Question

At which point, A or B, the pressure is higher (if so)?

C) $P_A > P_B$

Remember?

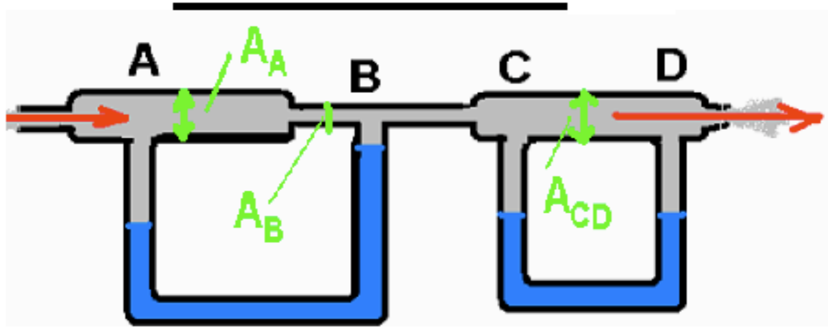
Pressure is the consequence of the fact that many tiny particles hit the surface of an object (for example, the pipe!)



When particles are moving TOO fast, they do not have time to make as many hits as they would have made if moving slower!

So: $v_A < v_B \quad \Rightarrow \quad P_A > P_B$

Air flow demo



$$A_1 v_1 = A_2 v_2$$

$$\rho g y_1 + \tfrac{1}{2}\rho v_1^2 + P_1 = \rho g y_2 + \tfrac{1}{2}\rho v_2^2 + P_2$$

For the points C and D: the level is the same, the area is the same (hence the speed is the same); and the result

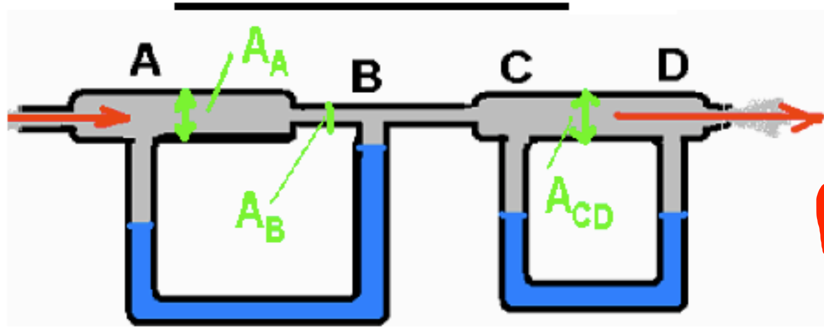
$$P_C = P_D$$

For the points A and B: the level is the same, but the area is different (hence the speed is different);

$A_A > A_B$ => $v_A < v_B$ and the result

$$P_A > P_B$$

Air flow demo



$$A_1 v_1 = A_2 v_2$$

$$\tfrac{1}{2}\rho v_A^2 + P_A = \tfrac{1}{2}\rho v_B^2 + P_B$$

$$P_A - P_B = \tfrac{1}{2}\rho v_B^2 - \tfrac{1}{2}\rho v_A^2 > 0$$

$$P_A - P_B > 0$$

$$P_A > P_B$$

For the points C and D: the level is the same, the area is the same (hence the speed is the same); and the result

$$P_C = P_D$$

For the points A and B: the level is the same, but the area is different (hence the speed is different);

$A_A > A_B$ => $v_A < v_B$ and the result

$$P_A > P_B$$

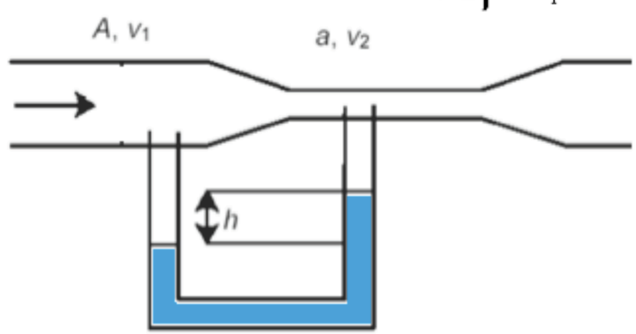
Application of Bernoulli's Principle

Venturi Meter

A *venturi meter* is a device for measuring the speed with which a fluid is flowing in a pipe.

$$A_1 v_1 = A_2 v_2$$

$$\tfrac{1}{2}\rho v_1^2 + P_1 = \tfrac{1}{2}\rho v_2^2 + P_2$$



Let's derive an expression for V_1.

Application of Bernoulli's Principle

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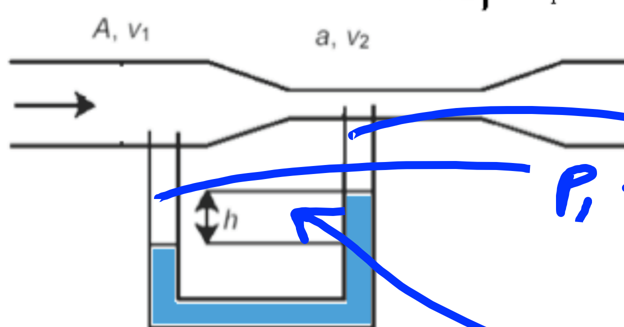


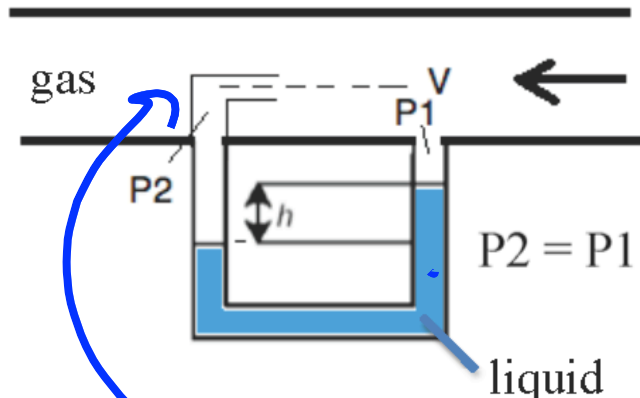
$$P_1 = P_2 + \rho_w \cdot g \cdot h$$

Let's derive an expression for V_1.

1. Yes
2. No

NO!

The densities are different!



$$P2 = P1 + \rho g h$$

$$P1 + \frac{\rho V^2}{2} = P2$$

Terminology:

Static (or absolute) pressure — Dynamic pressure

In a moving fluid the measured pressure depends on the orientation!

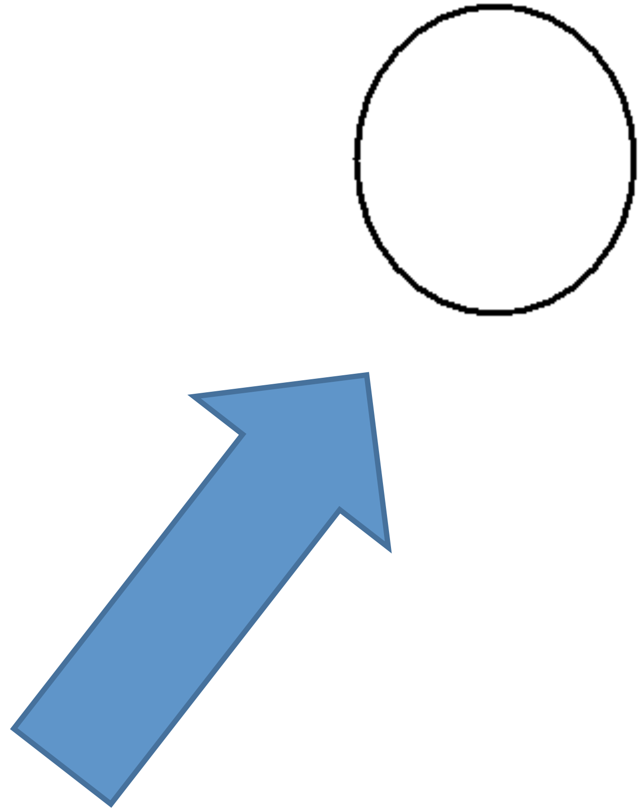
$P_2 - P_1 = \Delta P$

$\Delta P = \rho_{liq} g h$

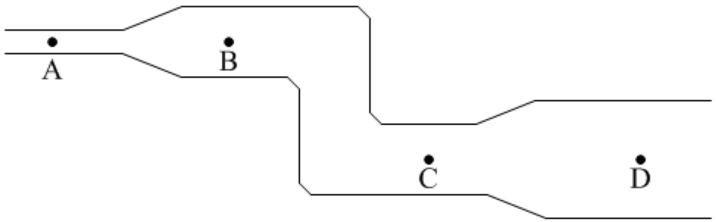
$\Delta P = \frac{\rho V^2}{2}$

$\rho g h = \frac{\rho V^2}{2}$

$V = \sqrt{2gh}$



Fluid in a Pipe



$$Av = \text{const}$$

$$\rho g y_1 + \tfrac{1}{2}\rho v_1^2 + P_1 =$$
$$= \rho g y_2 + \tfrac{1}{2}\rho v_2^2 + P_2$$

The picture above shows fluid in a section of pipe. The pipe is narrow where point A is located, widens out where points B and C are located, and widens again where point D is located.

Points C and D are vertically below points A and B.

If the fluid is *at rest* in the pipe, rank the four points based on their pressure.

1. Equal for all four
2. A>B=C>D
3. D>B=C>A
4. A=B>C=D
5. C=D>A=B
6. A>B>C>D
7. D>C>B>A
8. It's ambiguous - there are two possible answers
9. None of the above

"Rest" means v = 0

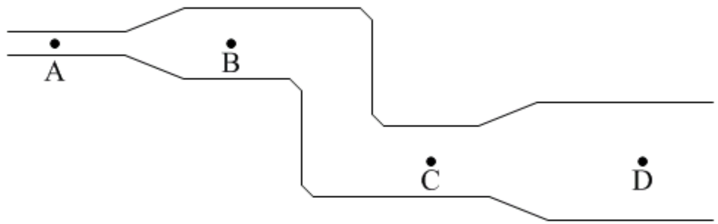
$$\boldsymbol{\rho g y_1 + P_1 = \rho g y_2 + P_2}$$

$y_A = y_B > y_C = y_D$

Lower points have higher pressure

C=D>A=B

Fluid in a Pipe



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$$\boldsymbol{\rho g y_1 + P_1 = \rho g y_2 + P_2}$$

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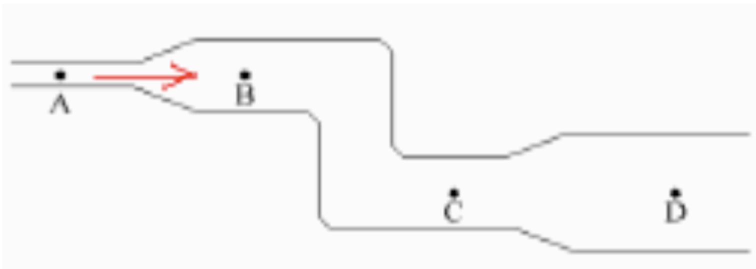
C=D>A=B

$P_2 - P_1 = \Delta P = \rho g (y_1 - y_2) = \rho g h$

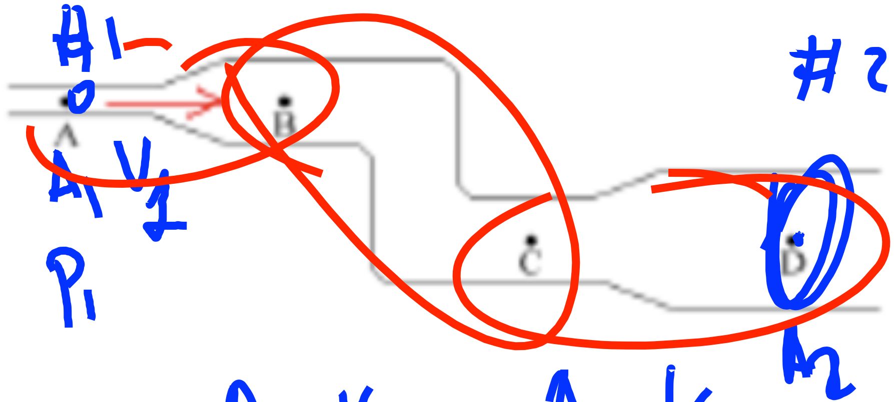
Work Together!

Fluid in a Pipe



If the fluid is *flowing from left to right* through the tube, rank the four points based on the speed and on the pressure of the fluid at their location.

1. Chose a pair of points
2. Write the continuity equation for the pair
3. Write the Bernoulli's equation for the pair
4. State the relation between the heights of the points
5. State the relation between the cross-sectional areas of the pipe a the points.
6. Use the continuity equation to relate the speed
7. Use the Bernoulli's equation to relate the pressure



$$A_1 V_1 = A_2 V_2$$

#2

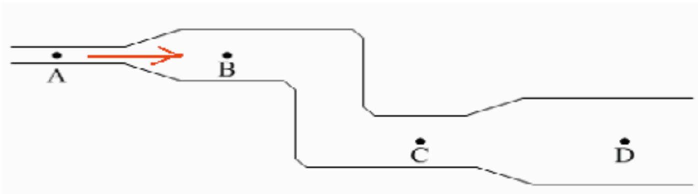
P_2

$A_1 < A_2$

$V_2 > V_1$

$$P_1 + \rho g y_1 + \frac{\rho v_1^2}{2} = P_2 + \rho g y_2 + \frac{\rho v_2^2}{2}$$

Fluid in a Pipe



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2. A>B=C>D
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4. A=B>C=D
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6. A>B>C>D
7. D>C>B>A
8. It's ambiguous - there are two possible answers
9. None of the above

For a moving fluid speed is directly related to the cross-sectional area.

$$A_A < A_B = A_C < A_D \quad \text{hence} \quad v_A > v_B = v_C > v_D$$

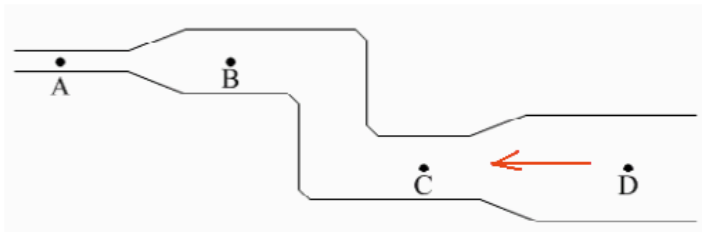
$$\text{and we know that} \quad y_A = y_B > y_C = y_D$$

$$\cancel{\rho g y_A} + \tfrac{1}{2}\rho v_A^2 + P_A = \cancel{\rho g y_B} + \tfrac{1}{2}\rho v_B^2 + P_B \qquad \mathbf{P_A < P_B}$$

$$\rho g y_B + \cancel{\tfrac{1}{2}\rho v_B^2} + P_B = \rho g y_C + \cancel{\tfrac{1}{2}\rho v_C^2} + P_C \qquad \mathbf{P_B < P_C}$$

$$\cancel{\rho g y_C} + \tfrac{1}{2}\rho v_C^2 + P_C = \cancel{\rho g y_D} + \tfrac{1}{2}\rho v_D^2 + P_D \qquad \mathbf{P_C < P_D}$$

Fluid in a Pipe (not a question)



$$Av = \text{const}$$

$$\rho g y_1 + \tfrac{1}{2}\rho v_1^2 + P_1 =$$
$$= \rho g y_2 + \tfrac{1}{2}\rho v_2^2 + P_2$$

If the fluid is *flowing from right to left* through the tube, rank the four points based on their pressure.

<u>The direction of the current does NOT involved in ANY of the equations, so it does not matter!</u>

For a moving fluid speed is directly related to the cross-sectional area.

$A_A < A_B = A_C < A_D$ hence $v_A > v_B = v_C > v_D$

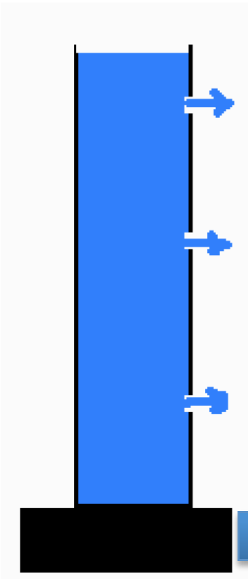
And we know that $y_A = y_B > y_C = y_D$

$$\cancel{\rho g y_A} + \tfrac{1}{2}\rho v_A^2 + P_A = \cancel{\rho g y_B} + \tfrac{1}{2}\rho v_B^2 + P_B \qquad \mathbf{P_A < P_B}$$

$$\rho g y_B + \cancel{\tfrac{1}{2}\rho v_B^2} + P_B = \rho g y_C + \cancel{\tfrac{1}{2}\rho v_C^2} + P_C \qquad \mathbf{P_B < P_C}$$

$$\cancel{\rho g y_C} + \tfrac{1}{2}\rho v_C^2 + P_C = \cancel{\rho g y_D} + \tfrac{1}{2}\rho v_D^2 + P_D \qquad \mathbf{P_C < P_D}$$

There's a hole in my bucket...

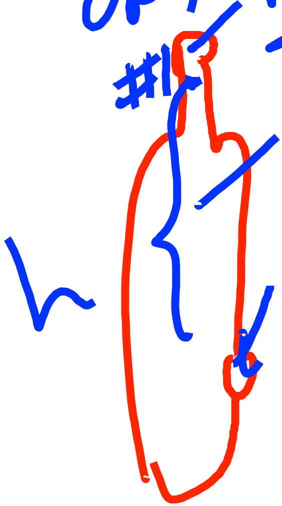


A cylinder is full of water. There are three holes in the side of the cylinder at the different height (initially covered).

When the holes are uncovered, water shoots out.

Which hole shoots the water furthest horizontally on the table?

1 The hole closest to the top.
2 The hole halfway down.
3 The hole closest to the bottom.
4 It's a three-way tie.



$p_1 = 0 \rightarrow p_1 \approx p_A$

ρ_w

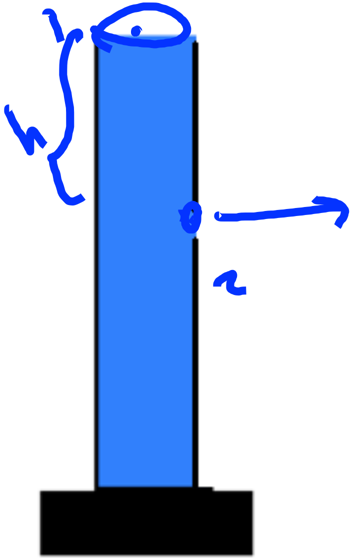
$p_2 = p_A = 10^5\, P_a$

1) Q

2) Nothing

$p_A + \rho_w g h = p_a + \rho_w g h_2 + \frac{\rho_w V^2}{2}$

$\sqrt{(\rho_w g h) \cdot \frac{2}{\rho_w}} = V$



$v_2 = ?$

$$\begin{cases} v_1 A_1 = A_2 v_2 \\ P_1 + \rho g h + \frac{\rho v_1^2}{2} = P_2 + \rho g \cdot 0 + \frac{\rho v_2^2}{2} \end{cases}$$

$P_1 \to P_A$ $\qquad$ $P_2 \to P_A$

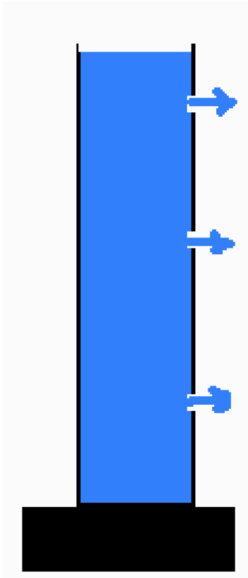
$$0 \sim \frac{A_2}{A_1} <<<<< 1$$

$$v_1 = v_2 \cdot \frac{A_2}{A_1} \approx 0$$

$$gh = \frac{v_2^2}{2}$$

$$v_2 = \sqrt{2gh}$$

There's a hole in my bucket...



A cylinder is full of water. There are three holes in the side of the cylinder at the different height (initially covered).

When the holes are uncovered, water shoots out.
Which hole shoots the water furthest horizontally on the table?

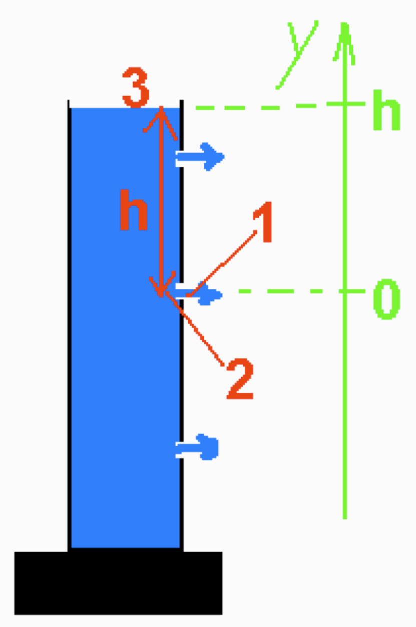
Water from the lowest hole has the highest pressure; hence the highest speed. But this hole is the closes to the surface.

Water from the highest hole has the lowest pressure; hence the lowest speed. But this hole is the furthest to the surface.

Probably, the hole in the middle has the best chance to win.

B) The hole halfway down.

There's a hole in my bucket...



Let's consider the hole at a distance h from the top of the cylinder.

According to the Bernoulli's equation:

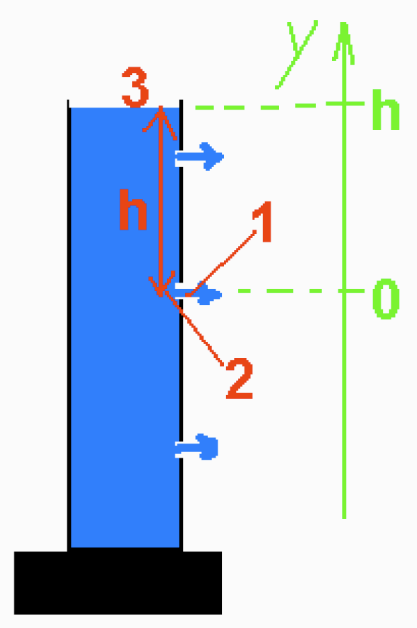
$\rho g y_1 + \frac{1}{2}\rho v_1^2 + P_1 = \rho g y_2 + \frac{1}{2}\rho v_2^2 + P_2$

We will apply this equation to two pairs of points.

The Point # 1 is right after the hole (we are looking at the speed of water at this point). ***The pressure right outside the hole is equal (always) to the atmospheric pressure***.

The Point # 2 is right before the hole.

The Point # 3 is at the surface of the water (hence, again, only atmospheric pressure is involved at this point).



There's a hole in my bucket...

Let's consider the hole at a distance h from the top of the cylinder.

The Bernoulli's equations:

$$\rho g y_1 + \tfrac{1}{2}\rho v_1^2 + P_1 = \rho g y_2 + \tfrac{1}{2}\rho v_2^2 + P_2$$

$$\rho g y_2 + \tfrac{1}{2}\rho v_2^2 + P_2 = \rho g y_3 + \tfrac{1}{2}\rho v_3^2 + P_3$$

We set the zero level at the level of the hole.

According to the Bernoulli's equations:

For the points # 1 and # 2

$$\tfrac{1}{2}\rho v_1^2 + P_{Atm} = \tfrac{1}{2}\rho v_2^2 + P_2$$

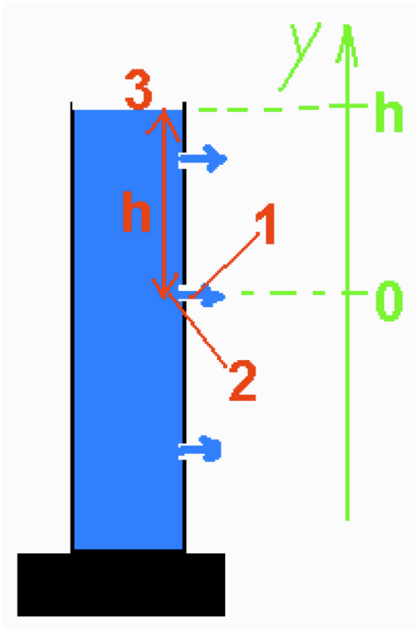
For the points # 2 and # 3

$$\tfrac{1}{2}\rho v_2^2 + P_2 = \rho g h + \tfrac{1}{2}\rho v_3^2 + P_{Atm}$$

Hence, for the points #1 and #3 we can write

$$\tfrac{1}{2}\rho v_1^2 + P_{Atm} = \rho g h + \tfrac{1}{2}\rho v_3^2 + P_{Atm}$$

(We could write this equation just by considering the points #1 and #3)



There's a hole in my bucket...

For the points #1 and #3 we have

$$\tfrac{1}{2}\rho v_1^{\,2} + \cancel{P_{Atm}} = \rho g h + \tfrac{1}{2}\rho v_3^{\,2} + \cancel{P_{Atm}}$$

Let's solve it for the speed v_1

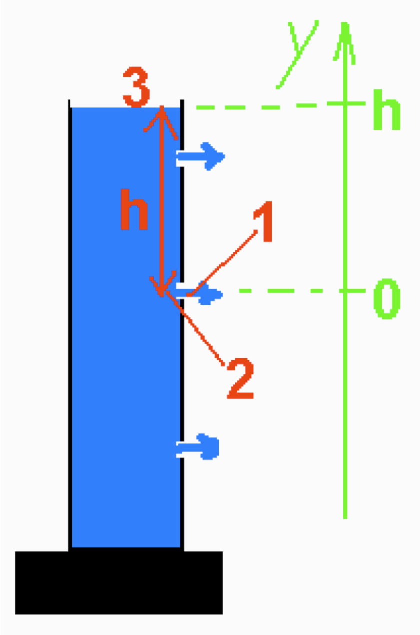
First, let's cancel out P_{Atm}

$$\tfrac{1}{2}\rho v_1^{\,2} + \cancel{P_{Atm}} = \rho g h + \tfrac{1}{2}\rho v_3^{\,2} + \cancel{P_{Atm}}$$

This gives $$\tfrac{1}{2}\rho v_1^{\,2} = \rho g h + \tfrac{1}{2}\rho v_3^{\,2}$$

Now we can cancel out the density $$\tfrac{1}{2}\cancel{\rho} v_1^{\,2} = \cancel{\rho} g h + \tfrac{1}{2}\cancel{\rho} v_3^{\,2}$$

This leads to $$\tfrac{1}{2} v_1^{\,2} = g h + \tfrac{1}{2} v_3^{\,2}$$



There's a hole in my bucket...

$$\frac{1}{2}v_1^2 = gh + \frac{1}{2}v_3^2$$

To connect the speeds v_1 and v_3
we apply the Continuity Equation

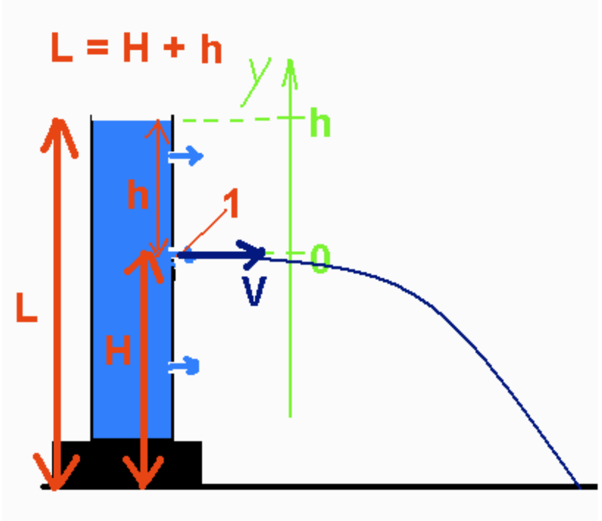
$$A_1 v_1 = A_3 v_3$$

Technically, we have the system of two equations, to solve which we need to know the values for the areas A_1 and A_3.

However (!), in this situation the area of the hole is so much smaller than the area of the cylinder $A_1 << A_3$

So, the speed of the water at the top of the cylinder is much smaller then the speed at the hole $V_3 << V_1$ and we can set $V_3 = 0$,

This gives the result: $\frac{1}{2}v_1^2 = gh$ or $v_1 = \sqrt{2gh}$ (!!)



$$V = v_1 = \sqrt{2gh}$$

Now, when we know the initial velocity, we can solve the problem as the we did it for a projectile launched horizontally from the height H.

$$H = \tfrac{1}{2}gt^2$$

The time it takes to reach the ground is:

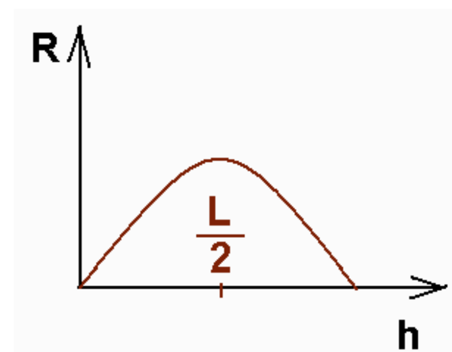
$$t = \left[\frac{2\,(H)}{g}\right]^{1/2}$$

The range R is:

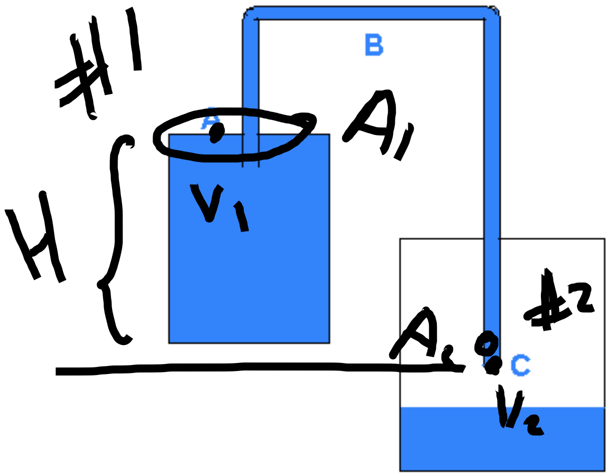
$$R = vt = (2gh)^{1/2}\left[\frac{2(H)}{g}\right]^{1/2} =$$

$$= 2\,[h(H)]^{1/2} = 2\,[h(L - h)]^{1/2}$$

$$\mathbf{R = 2\,[h(L - h)]}$$



Understanding a siphon



1. Choose a pair of points.

$A_1 > A_2$

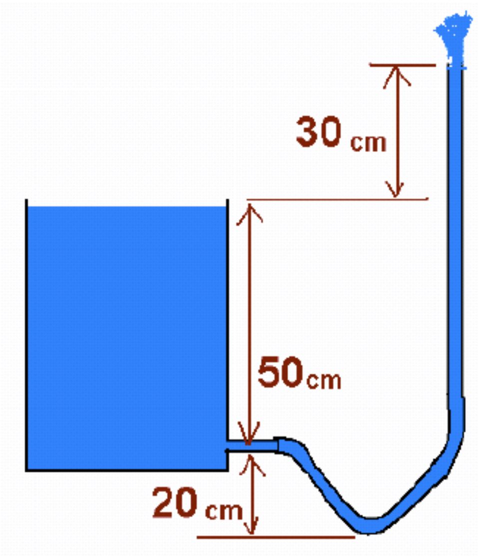
$v_1 = v_2 \cdot \frac{A_2}{A_1} \to 0$

2. Choose the zero level to measure potential energy; label areas, velocities, pressures and heights for two points of your choice.

3. Write the continuity equation; write the Bernoulli's equation for the chosen points.

4. Analyze the equations, derived the relations between the variables (=, or <, or >)

$$P_A + \rho g H + \frac{\rho v_1^2}{2} = P_A + \rho g \cdot 0 + \frac{\rho v^2}{2}$$



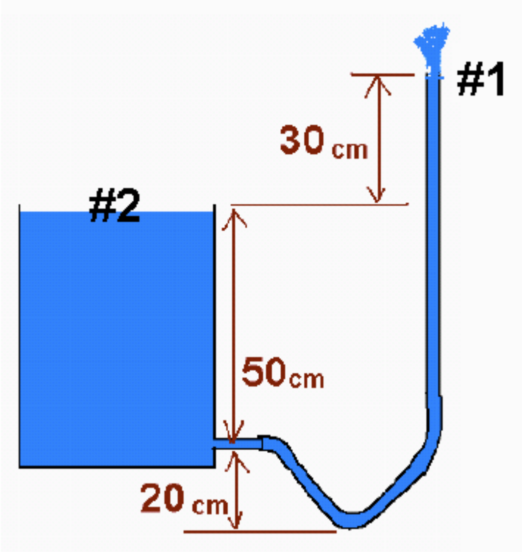
Building a Fountain

A student has a project to make a model of a fountain.

The student uses the large bucket and a hose attached to it.

This is (see the picture) how the student sees the project at first.

What is seriously wrong with this picture?



Building a Fountain

What is seriously wrong with this picture?

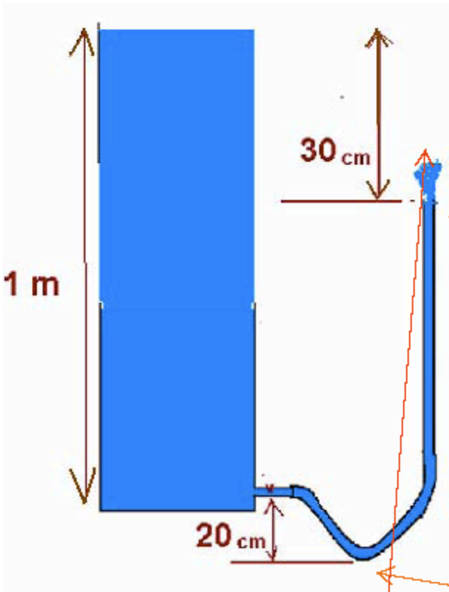
The end of the hose (point 1) is higher then surface of the water in the bucket (the point 2).

Without an additional pressure on the water in the bucket the water ***cannot*** reach the height higher then its original level!

But both points (#1 and #2) are open to the air around; hence have the same pressure $P_{Atm.}$

So, the student decides to make the fountain slightly differently

Building a Fountain



A student has a project to make a model of a fountain.

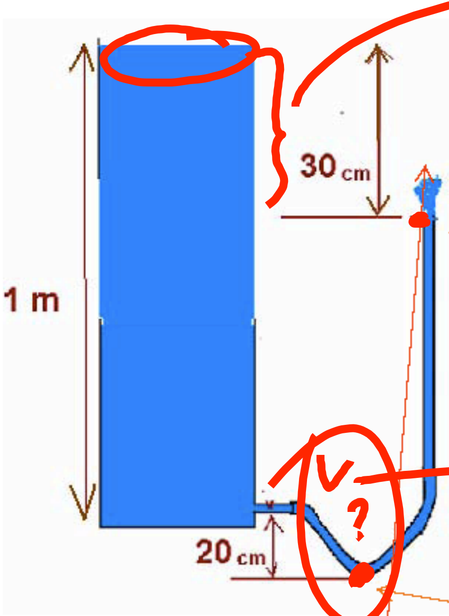
The student uses the large bucket and a hose attached to it (see the picture).

What is the speed of the water when it leaves the hose?

The atmospheric pressure is 100 kPa. Find the pressure at the lowest point of the hose.

What is the height reached by the water from the fountain?

Building a Fountain



A student has a project to make a model of a fountain.

The student uses the large bucket and a hose attached to it (see the picture).

$V = \sqrt{2gh} = \sqrt{2 \cdot g \cdot 0.3} = \sqrt{6} \approx 2.21 \, m/s$

$1 = 2.21$

$2 > 2.21$

$3 < 2.21$

What is the speed of the water when it leaves the hose?

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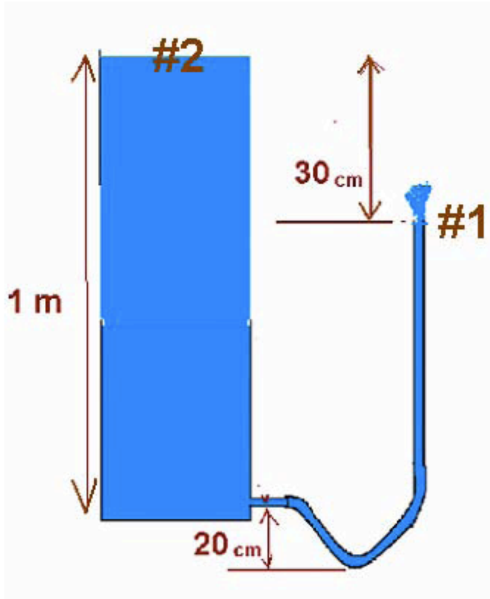
What is the height reached by the water from the fountain?



Building a Fountain

What is the speed of the water when it leaves the hose?

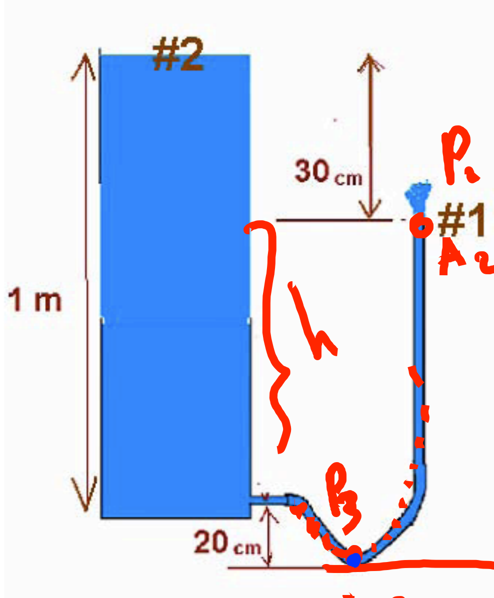
Speed is related directly to a cross-sectional area.

Along the hose the cross-sectional area is the same, so in the hose the speed of the flow is the same, and any point in the hose can be used for the analysis.

The very top point of the hose (where water leaves the hose) is the easiest because it is an open point (the hole), and the pressure at this point is just an atmospheric pressure $P_1 = P_{Atm}$.

The next most convenient point is at the surface of the water in the bucket, because a) the pressure at this point is also $P_2 = P_{Atm}$; and b) because the area of the surface is so large, we can set the speed at this point $v_2 = 0$.

Building a Fountain



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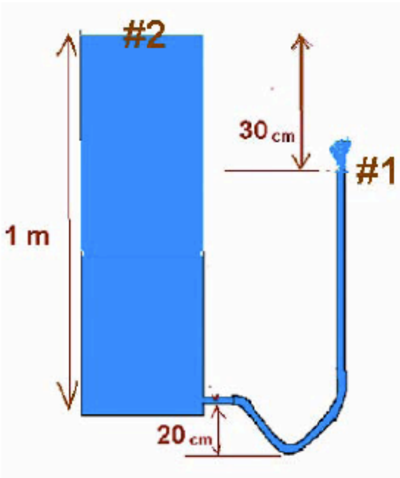
Work Together



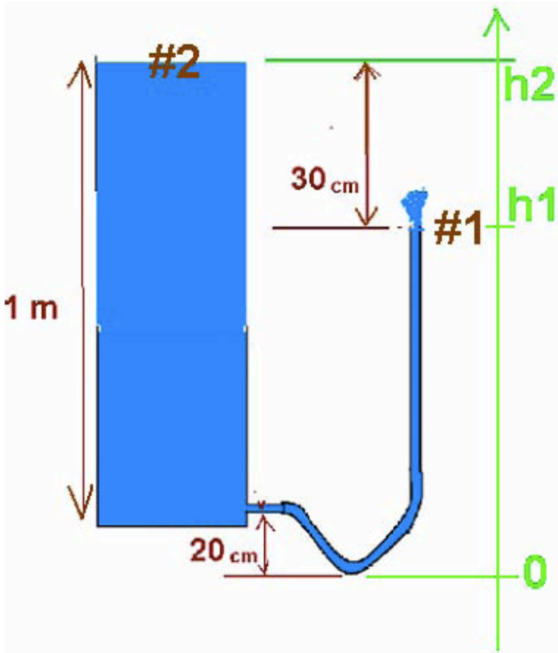
A big and very wide tank full of water has a hose attached to it.

What is the speed of the water when it leaves the hose?

Use the Bernoulli's equation.

$$\rho g y_1 + \tfrac{1}{2}\rho v_1^2 + P_1 = \rho g y_2 + \tfrac{1}{2}\rho v_2^2 + P_2$$

Notice that the points # 1 and # 2 are open to the atmosphere.



Building a Fountain

What is the speed of the water when it leaves the hose?

Bernoulli's equation

$$\rho g y_1 + \frac{1}{2}\rho v_1^2 + P_1 = \rho g y_2 + \frac{1}{2}\rho v_2^2 + P_2$$

our data

$\rho = 1000\ \mathrm{kg/m^3}$

$P_1 = P_{Atm}$ $\quad h_1 = 0.9$ m

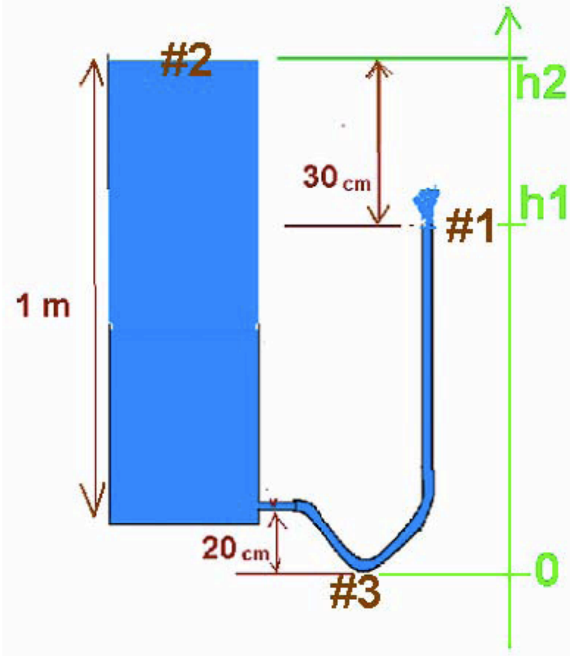
$P_2 = P_{Atm}$; $\quad v_2 = 0$; $\quad h_2 = 1.2$ m

so,

$$\rho * 9.8 * 0.9 + \frac{1}{2}\rho v_1^2 + \cancel{P_{Atm}} = \rho * 9.8 * 1.2 + \frac{1}{2}\rho * 0^2 + \cancel{P_{Atm}}$$

$$9.8 * 0.9 + \frac{1}{2}v_1^2 = 9.8 * 1.2$$

$$v_1 = 2.43\ \mathrm{m/s}$$



Building a Fountain

The atmospheric pressure is 100 kPa. *Find the pressure at the lowest point of the hose.*

Now we need to connect the point 3 and some other point, say 1.

The most important fact for this situation is that the speed of the water at the point 1 and 3 is the same, because the cross-sectional areas are the same.

$$\rho g y_1 + ½\rho v_1^2 + P_1 = \rho g y_3 + ½\rho v_3^2 + P_3$$

our data

$P_1 = P_{Atm}$ $h_1 = 0.7$ m $\rho = 1000\ kg/m^3$

$v_3 = v_1$; $h_2 = 0$ m

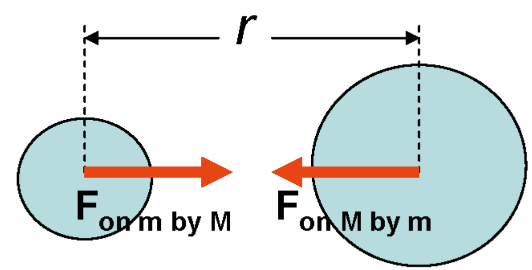
So, $1000*9.8*0.7 + \cancel{½\rho v_1^2} + 10^5 = \rho g*0 + \cancel{½\rho v_3^2} + P_3$

$P_3 = 106860$ Pa (this is the absolute pressure; $P_{gauge} = 6860$ Pa)

Today's Physics Topics

- Universal gravitation – an attractive inverse-square law force exerted between any two masses
- Gravitational force GMm/r^2 used with $F=ma$ for circular orbits – we can calculate orbital periods
- Gravitational potential energy $-GMm/r$ gives mgh for small changes h, and determines escape speeds required to get to large r
- Angular momentum is conserved for elliptical orbits

Newton's Law of Universal Gravitation



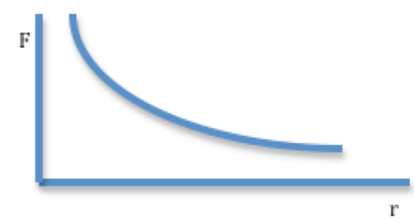
Two objects of mass *m* and *M*,
with their <u>centers of mass</u>
separated by a distance *r*,
exert attractive forces on one another.
(Equal magnitude but opposite direction, by Newton's Third Law)

The magnitude of this gravitational force is given by:

$$F_g = \frac{GmM}{r^2}$$

where G is the universal gravitational constant:

$$G = 6.67 \times 10^{-11}\ \text{N m}^2/\text{kg}^2$$



Calculating *g*

Newton's form of the equation for the force of gravity must be consistent with the *mg* we have been using up to this point in the course:

$$\frac{GmM}{r^2} = mg$$

For an object of mass *m* at the surface of the Earth, this tells us that:

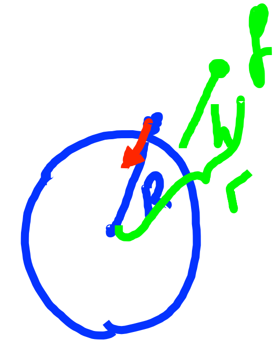
$$g = \frac{GM_E}{R_E^2} = \frac{(6.67 \times 10^{-11}\ \text{N m}^2/\text{kg}^2) \times (5.97 \times 10^{24}\ \text{kg})}{(6.38 \times 10^6\ \text{m})^2} = 9.8\ \text{m/s}^2$$

The radius of the Earth is so large compared to the heights of objects around us that we find that *g* does not vary significantly in our common experience.

Calculating g

Newton's form of the equation for the force of gravity must be consistent with the mg we have been using up to this point in the course:

$$m \cdot a = \frac{GmM}{r^2} = mg$$



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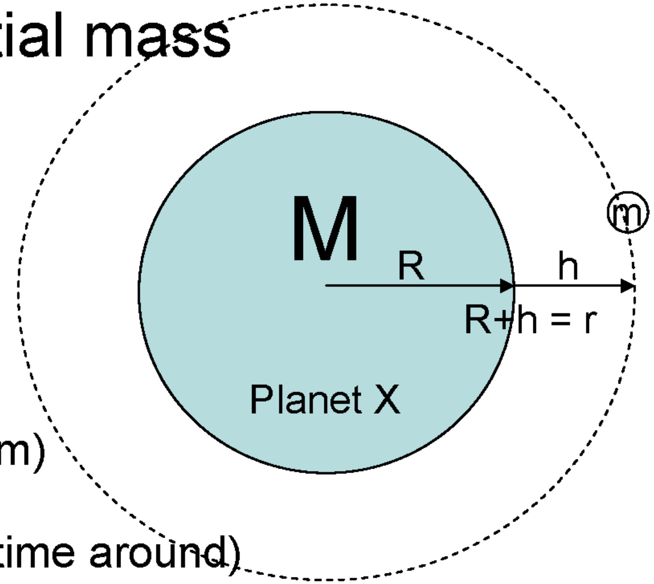
$$g = \frac{GM}{(R+h)^2}$$

The radius of the Earth is so large compared to the heights of objects around us that we find that g does not vary significantly in our common experience.

Circular Orbits

- Orbit radius r = Planet radius R + height h above
- Gravitational mass = inertial mass
- **F** = m**a**

$$\frac{GM\cancel{m}}{r^2} = \frac{\cancel{m}\, v^2}{r}$$



(1) $GM = v^2 r$ (independent of m)

(2) $v = 2\pi r/T$ (circumference)/(time around)

(3) $GM = 4\pi^2 r^3 / T^2$ [substitute (2) in (1)]

or $T^2 = (4\pi^2/GM)\, r^3$ (Kepler's 3rd Law)

Gravitational potential energy

The energy of interaction (that is, the gravitational potential energy) of two objects of mass *m* and *M* separated by a distance *r* is:

$$U_g = -\frac{GmM}{r}$$

The negative sign just tells us that the interaction is attractive.

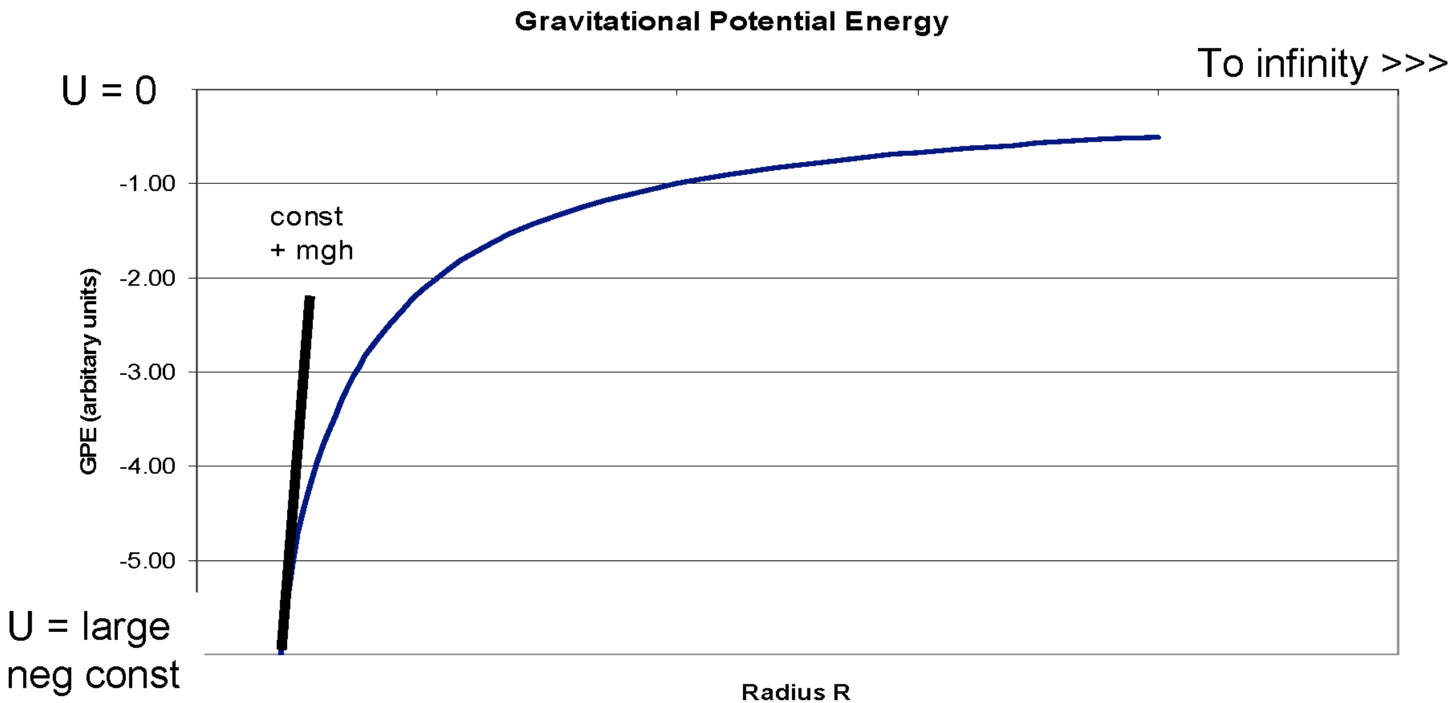
Note that with this equation the potential energy is defined to be zero when *r* = infinity. This expression is derived using calculus.

(If you are curios: prove that close to the ground level U = mgh + const)

Gravitational potential energy

$$U_g = -\frac{GmM}{r}$$

Substituting $r = R_E + h$ can be manipulated to give *mgh* + a large negative constant



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