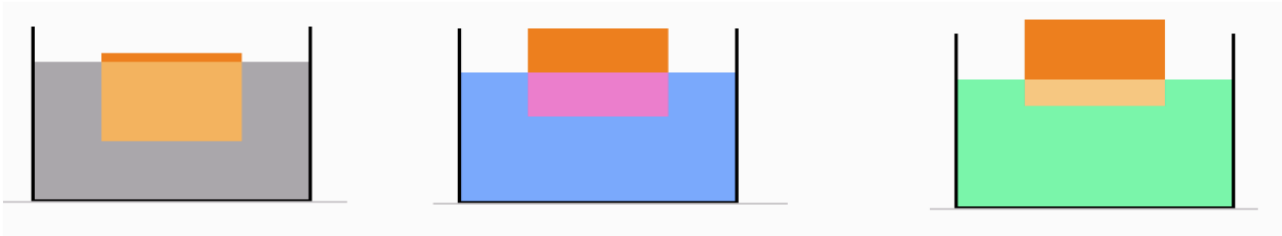
A floating object I

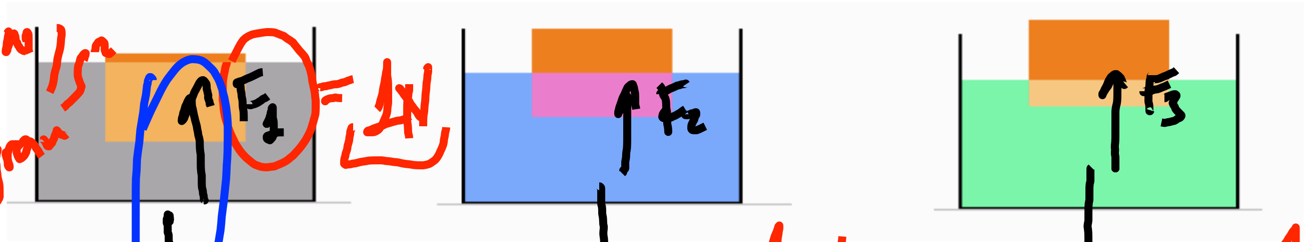


The same object was floating in three different liquids (see the pictures).

Which of the three liquids exerts the greatest buoyant force on the object?

1 The gray one
2 The blue one
3 The green one
4 All exert the same buoyant force

A floating object I



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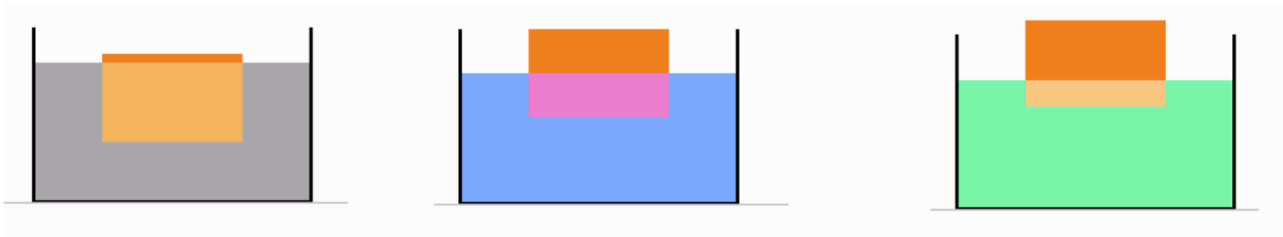


The same object was floating in three different liquids (see the pictures).

Which of the three liquids exerts the greatest buoyant force on the object?

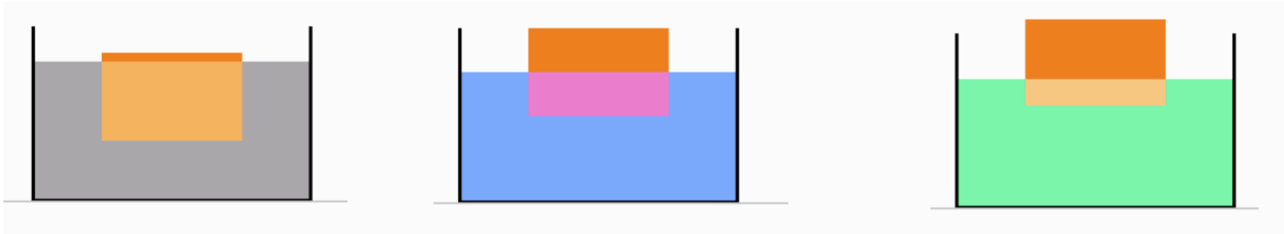
The object is in equilibrium in all cases.

In all cases the buoyant force cancels out the weight of the object.

The same object => the same weight => the same F_b

D) All exert the same buoyant force.

A floating object II

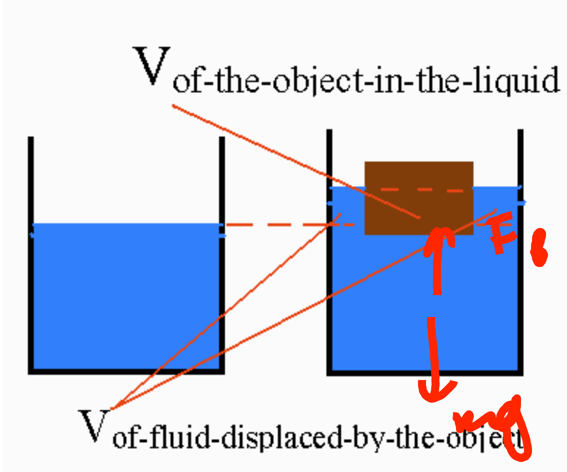


The same object was floating in three different liquids (see the pictures).

Which of the three liquids has the greatest density?

1 The gray one
2 The blue one
3 The green one
4 All have the same density

Another expression for buoyant force



When an object is placed into a liquid (or gas) it *displaces* the part of the liquid (or gas).

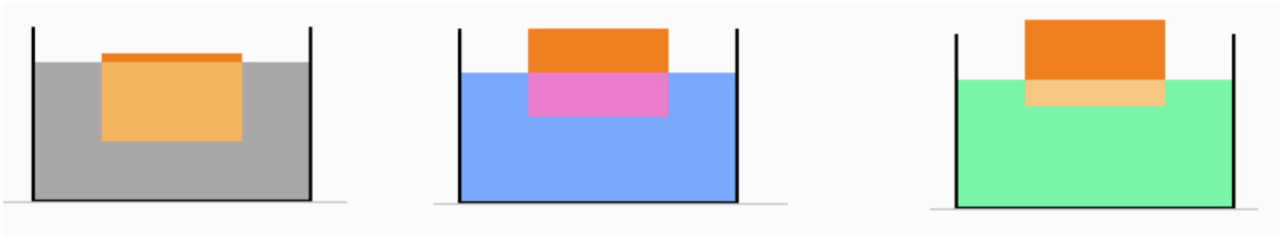
$$V_{\text{of-the-object-in-the-liquid}} = V_{\text{of-fluid-displaced-by-the-object}}$$

Hence (**Archimedes' principle**)

$$\mathbf{F_b = \rho_{liquid} g V_{\text{of-fluid-displaced-by-the-object}}}$$

(= weight of liquid displaced by the object!)

A floating object II



The same object was floating in three different liquids (see the pictures).

Which of the three liquids has the greatest density?

The same object => the same weight => the same F_b

All liquids exert the same buoyant force!!!

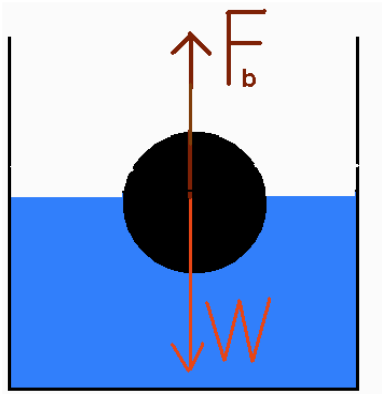
$$F_b = \rho g V_{object\text{-}in\text{-}liquid}$$

$F_b = const$ => $\rho g V_{object\text{-}in\text{-}liquid} = const$ => $\boldsymbol{\rho V_{object\text{-}in\text{-}liquid} = const}$

The liquid which needs the smallest volume to support the force has the greatest density.

C) The green one

Density of a floating object



When an object in a fluid is in equilibrium, that means the net force acting on the object is 0.

When two forces only act on the object in equilibrium, the magnitudes of the forces are equal.

$$W = F_b$$

or

$$m_{obj}\, g = \rho_{\text{fluid}}\, g\, V_{\text{object-in- fluid}}$$

Mass, density and the volume are related by $m = \rho V$

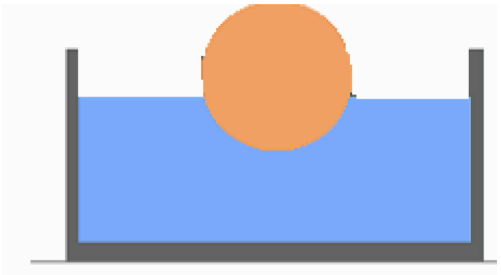
Hence, we can write (g is cancelled out)

$$\rho_{\text{objec}}\, V_{\text{object}} = \rho_{\text{fluid}}\, V_{\text{object-in- fluid}}$$

or

$$\rho_{obj} = \rho_{\text{fluid}}\, V_{\text{obj-in-the-liquid}} / V_{obj} \quad (V_{\text{obj-in-the-liquid}} = V_{disp})$$

Example



A basketball floats in a bathtub of water.

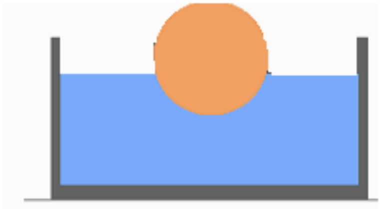
The ball has a mass of 0.5 kg and a diameter of 22 cm (0.22 m).

(a) What is the buoyant force?

(b) What is the volume of water displaced by the ball?

(c) What is the average density of the basketball?

Example



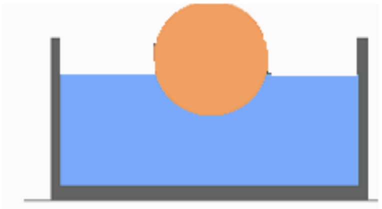
A basketball floats in a bathtub of water. The ball has a mass of 0.5 kg and a diameter of 22 cm (0.22 m).

(a) What is the buoyant force?

The ball is in equilibrium, hence

$$F_b = W = mg = 4.9\ \text{N}$$

Example



A basketball floats in a bathtub of water.
The ball has a mass of 0.5 kg and
a diameter of 22 cm (0.22 m).

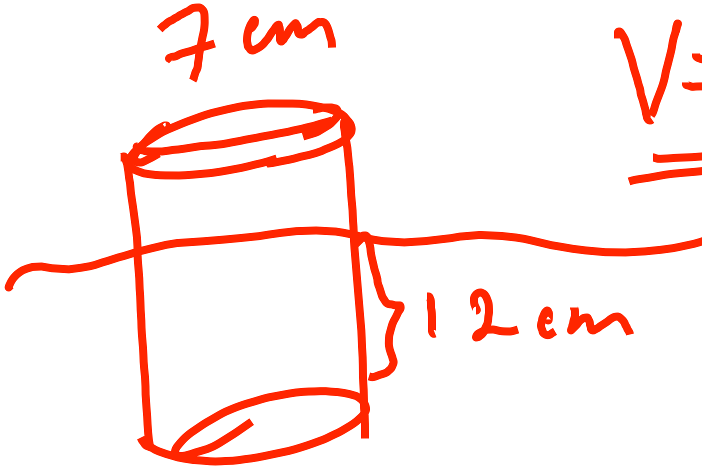
(b) What is the volume of water displaced by the ball?

By Archimedes' principle, the buoyant force
is equal to the weight of fluid displaced.

$$F_b = \rho V_{disp} g$$

$$V_{disp} = \frac{F_b}{\rho g} = \frac{4.9}{1000 * 9.8} = 5 \times 10^{-4}\ m^3$$

(remember, $V_{disp} = V_{ball\text{-}in\text{-}water}$, if we need to know it)



$V = \pi r^2 H =$

$= 3.1415 \cdot (3.5)^2 \cdot 12 =$

$= 461.8\ cm^3$

$V \sim 7 \cdot 7 \cdot 12 \sim 50 \cdot 10 = 500\ cm^3$

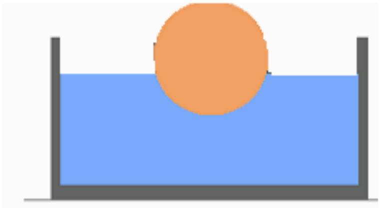
$g = 10\ m/s^2;\quad F_B = 1, 2, 3, 4, 5, 6, 7, 8, 9$

$$F_B = 500\,cm^3 \cdot 1000 \cdot 10 =$$

$$= 5 \cdot 10^2 (10^{-2})^3 \cdot 1000 \cdot 10 =$$

$$= 5\,N$$

Example



A basketball floats in a bathtub of water. The ball has a mass of 0.5 kg and a diameter of 22 cm (0.22 m).

(c) What is the average density of the basketball?

The density of the ball

$$\rho = \frac{m}{V}$$

, we need to determine its volume.
First, we need to find the volume of the ball (i.e. a sphere):

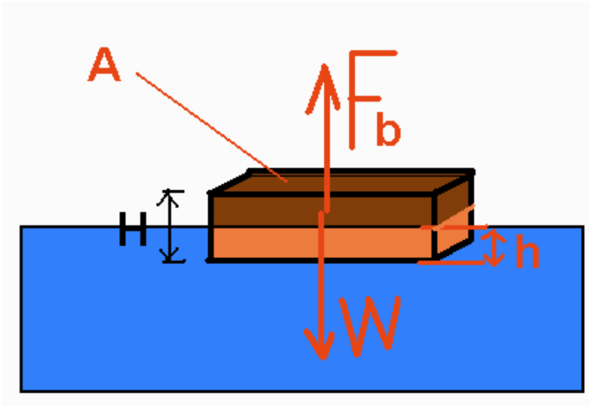
$$V = \frac{4}{3}\pi r^3$$

$r = d/2 = 0.11$ m; $\qquad V = 5.58 \times 10^{-3}\ m^3$

The density is

$$\rho = \frac{m}{V} = \frac{0.5}{5.58 \times 10^{-3}} = 90\ kg/m^3 \qquad \text{it} < 1000\ kg/m^3$$

PROBLEM



A block is floating in a liquid. What part of the block is in the liquid if the density of the block is 25 % less then the density of the liquid?

Solution

Again, we start from the equilibrium condition (two forces cancel out each other, hence, have equal magnitudes).

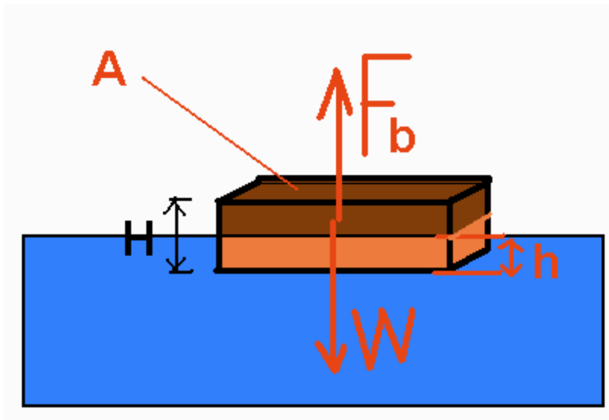
$$\mathbf{W = F_b}$$

(The buoyant force) $F_b = \rho_l g V_{ob\text{-}in\text{-}liquid} = \rho_l g h A$

(The weight) $W = mg = \rho_{ob} g V_{ob} = \rho_{ob} g H A$

$$\rho_{ob} g H A = \rho_l g h A$$

A block is floating in a liquid. What part of the block is in the liquid if the density of the block is 25 % less then the density of the liquid?



$$\rho_{ob} gHA = \rho_l ghA$$

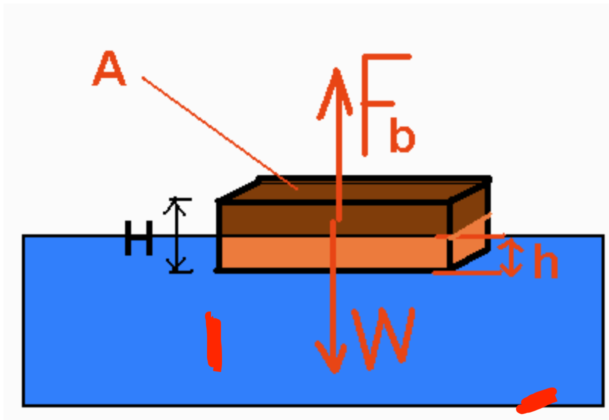
$$\rho_{ob} H = \rho_l h$$

$$\rho_{ob} = \rho_l - 0.25\rho_l$$

$$h = \frac{\rho_{ob}}{\rho_l} H = \frac{\rho_l - 0.25\rho_l}{\rho_l} H = \frac{0.75\rho_l}{\rho_l} H = 0.75H$$

Answer: 75 % of the block is in the liquid.

A block is floating in a liquid. What part of the block is in the liquid if the density of the block is 25 % less then the density of the liquid?



$$\rho_{ob} gHA = \rho_l ghA$$

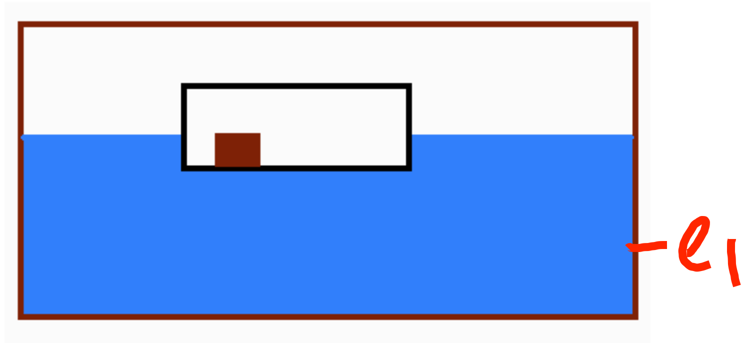
$$\rho_{ob} H = \rho_l h$$

$$\rho_{ob} = \rho_l - 0.25\rho_l$$

$$h = \frac{\rho_{ob}}{\rho_l} H = \frac{\rho_l - 0.25\rho_l}{\rho_l} H = \frac{0.75\rho_l}{\rho_l} H = 0.75H$$

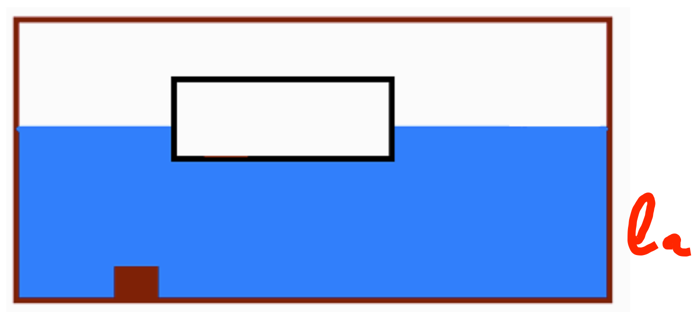
Answer: 75 % of the block is in the liquid.

Anchors aweigh



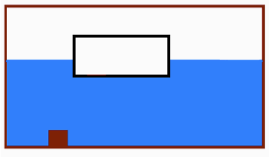
A boat contain an iron anchor with the mass of 12 kg.

1. $\ell_2 = \ell_1$
2. $\ell_2 > \ell_1$
3. $\ell_2 < \ell_1$



If the anchor is thrown overboard and is completely submerged, what is the normal force acting on the anchor?

Anchors aweigh



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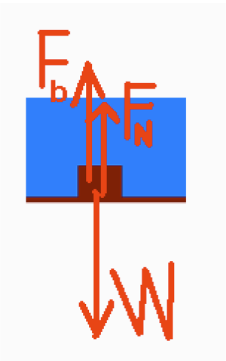


In the equilibrium, according to the FBD, $W = F_b + F_N$

or $m_{anc}g = \rho_{water} g V_{anc} + F_N$

Density of water (as well as iron) is known.

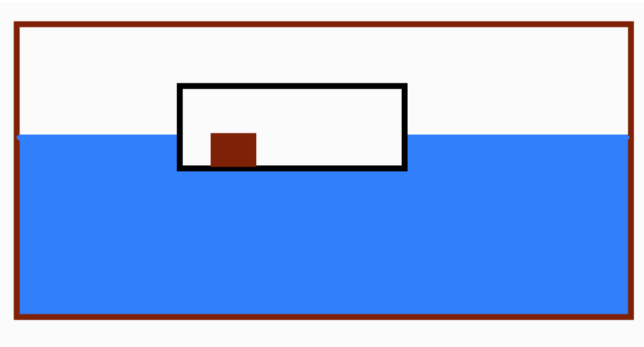
The volume V_{anc} can be found as $V_{anc} = m_{anc}/\rho_{iron}$

Finally:

$$F_N = m_{anc}g - \rho_{water} g\, m_{anc}/\rho_{iron} = m_{anc}g\,(1 - \rho_{water}/\rho_{iron})$$

As we can see, the apparent weight of the anchor $W_{app} = F_N$ in water is less than its weight.

Anchors aweigh



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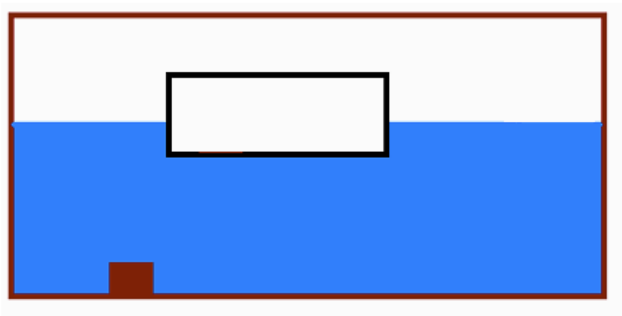
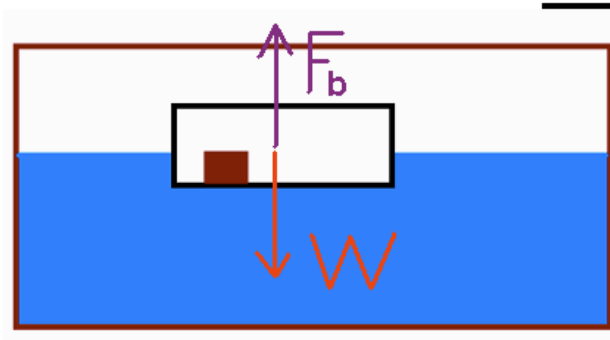
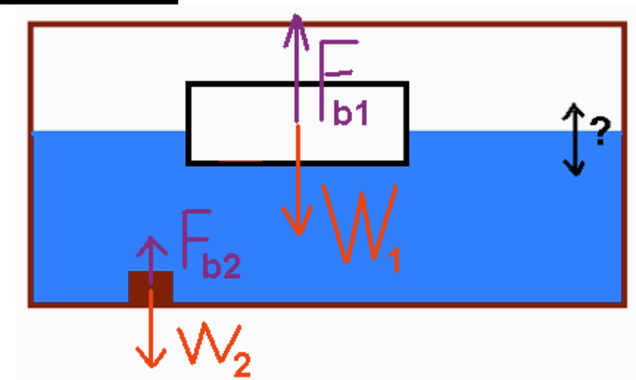


If the anchor is thrown overboard and is completely submerged, what is Happening to the level of the water? 1. nothing 2. it falls 3. It rises

Anchors aweigh





At first, the total weight of the boat and with the anchor W is balanced with the buoyant F_b: $W = F_b$

After the anchor was thrown overboard and completely submerged, we still have the forces acting on the boat balanced: $W_1 = F_{b1}$

But for the anchor the weight is grater than the buoyant force

$$W_2 > F_{b2}$$

(otherwise, the anchor would not get at the bottom of the reservoir!)

This leads to the conclusion: $F_b = W = W_1 + W_2 > F_{b1} + F_{b2}$

so $$F_b > F_{b1} + F_{b2}$$

or

$$\rho g V_{\text{of-fluid-displaced-1-and-2}} > \rho g V_{\text{of-fluid-displaced-1}} + \rho g V_{\text{of-fluid-displaced-2}}$$

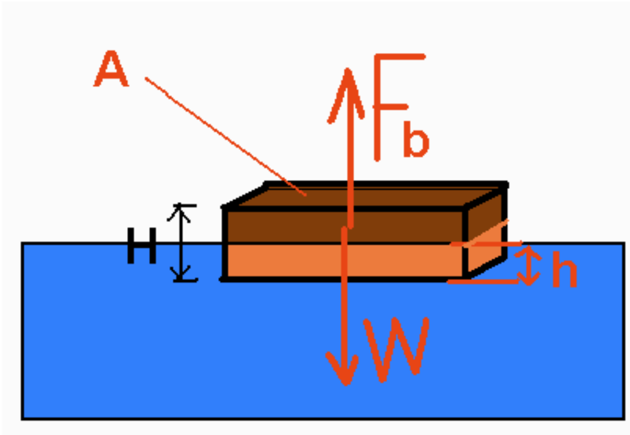
or

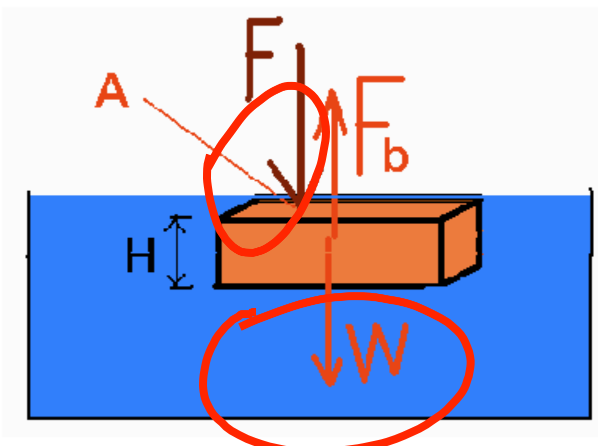
$$V_{\text{of-fluid-displaced1-and-2}} > V_{\text{of-fluid-displaced-1}} + V_{\text{of-fluid-displaced-2}}$$

The boat with the anchor on it displaces MORE water than the boat and the anchor by themselves.

Hence,

A) The water level falls





Problem

A block is floating in a liquid. The density of the block is 25 % less then the density of the liquid.

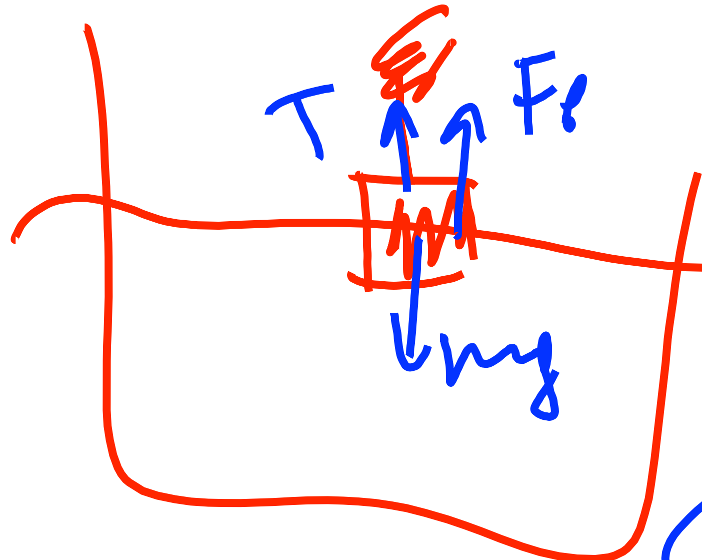
If the liquid is water and the total height of the block is 75 cm, what pressure should be applied to the block to immerse it in the water completely?

In equilibrium (two forces are acting down and one is acting up, and the net force is zero).

$$W + F = F_b$$

$$(V_{ob\text{-}in\text{-}liq} = V_{total} = V) \quad F = F_B - mg$$

$$F = F_b - W = \rho_l g V_{ob\text{-}in\text{-}liq} - \rho_{ob} g V = \rho_l g V - \rho_{ob} g V$$



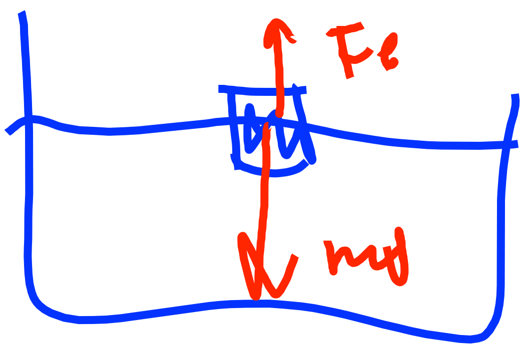
$$mg = F_b + T$$

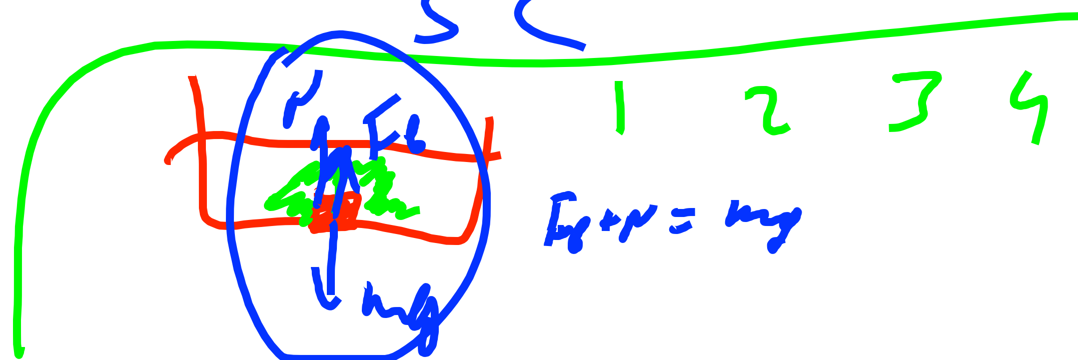
1 =

$$mg > F_b$$

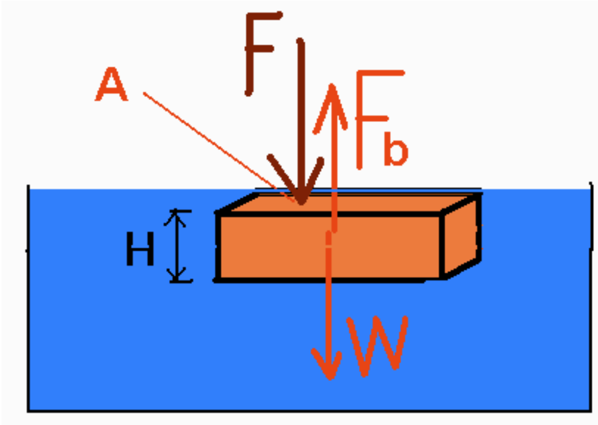
2 >

3 <





$$F_b + N = mg$$

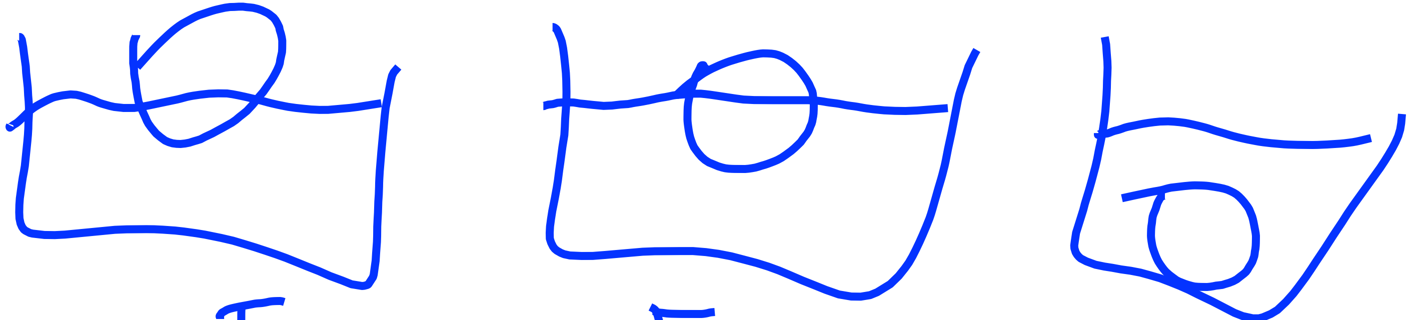


$$F = \rho_l g V - \rho_{ob} g V =$$

$$= (\rho_l - \rho_{ob}) g H A$$

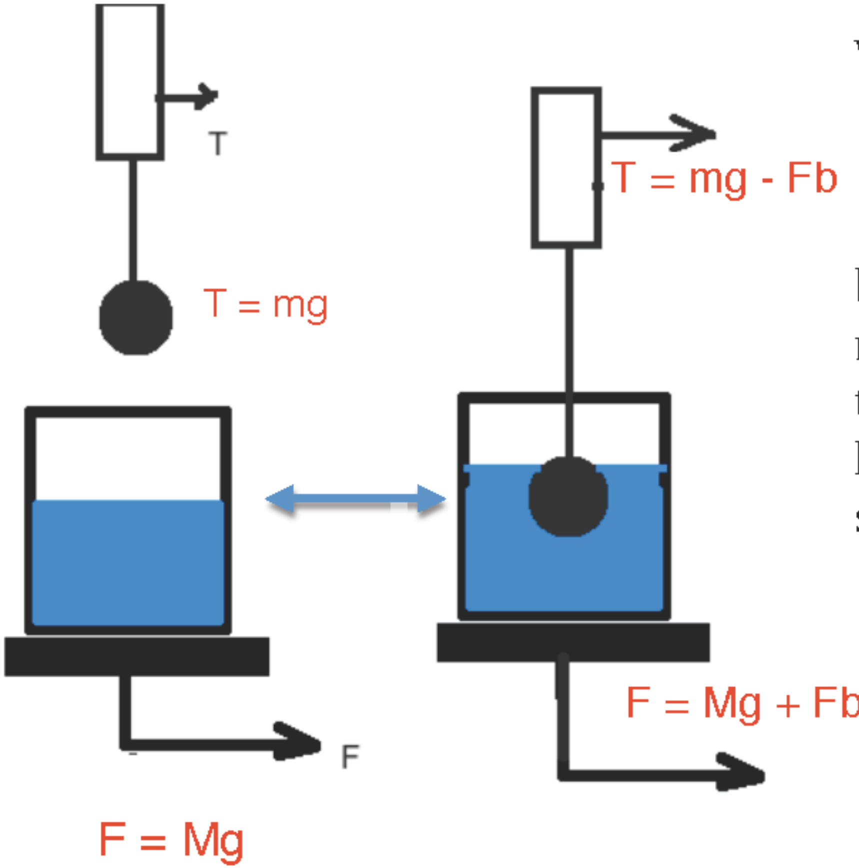
$$P = F/A = (\rho_l - \rho_{ob}) g H$$

$$P = (\rho_l - \rho_{ob}) g H = (1000 - 750) * 9.8 * 0.75 = 1837.5 \text{ Pa}$$



F_1 F_2 F_3

1. $F_1 = F_2 = F_3$
2. $F_1 > F_2 > F_3$
3. $F_1 < F_2 < F_3$

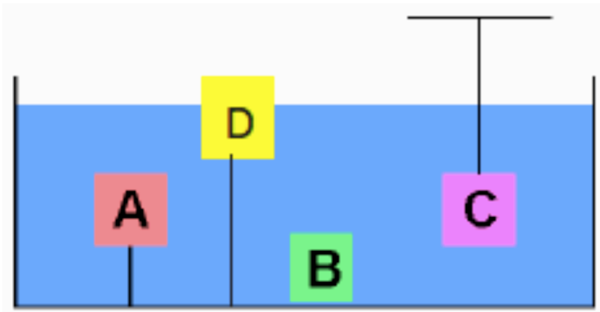


Why the change?

When we immerse th ball in water, the leve rises and the pressure o the bottom increase hence the force on th scale increases.

If x is the percentage of the total volume which is in the liquid, can we plot the graphs for Fb and T as a function of x?

Buoyant force



Find the tension in the string attached to boxes A, C, and D; fin[d] the normal force acting on box B[;] find the apparent weight of eac[h] box.

Assume, you know all th[e] parameters you need.

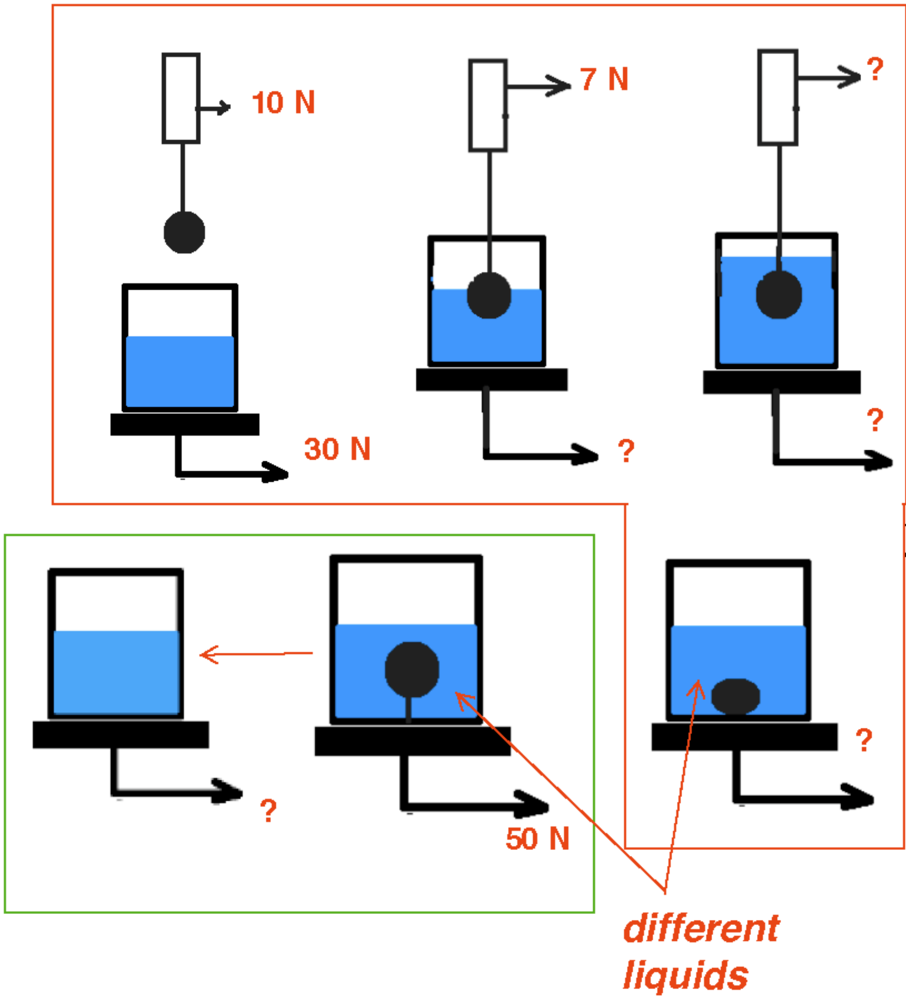
1. Draw FBD for a box. Balance the forces.

2. Write expressions for force of gravity and for buoyan[t] force.

3. Solve for tension, apparent weight.

Work Together

It is a good exercise to draw FBD for different situations and relat forces acting on the ball and on the scale.



For each situation find the tension in the string (or the apparent weight) and the reading on the scale.

Assume, you know all the masses, densities, volume you need.

Assuming the ball is the same for ALL cases, find missing forces. (N)

1. **2** 2. **4** 3. **7** 4. **28**
5. **33** 6. **36** 7. **40** 8. **52**

Work Together

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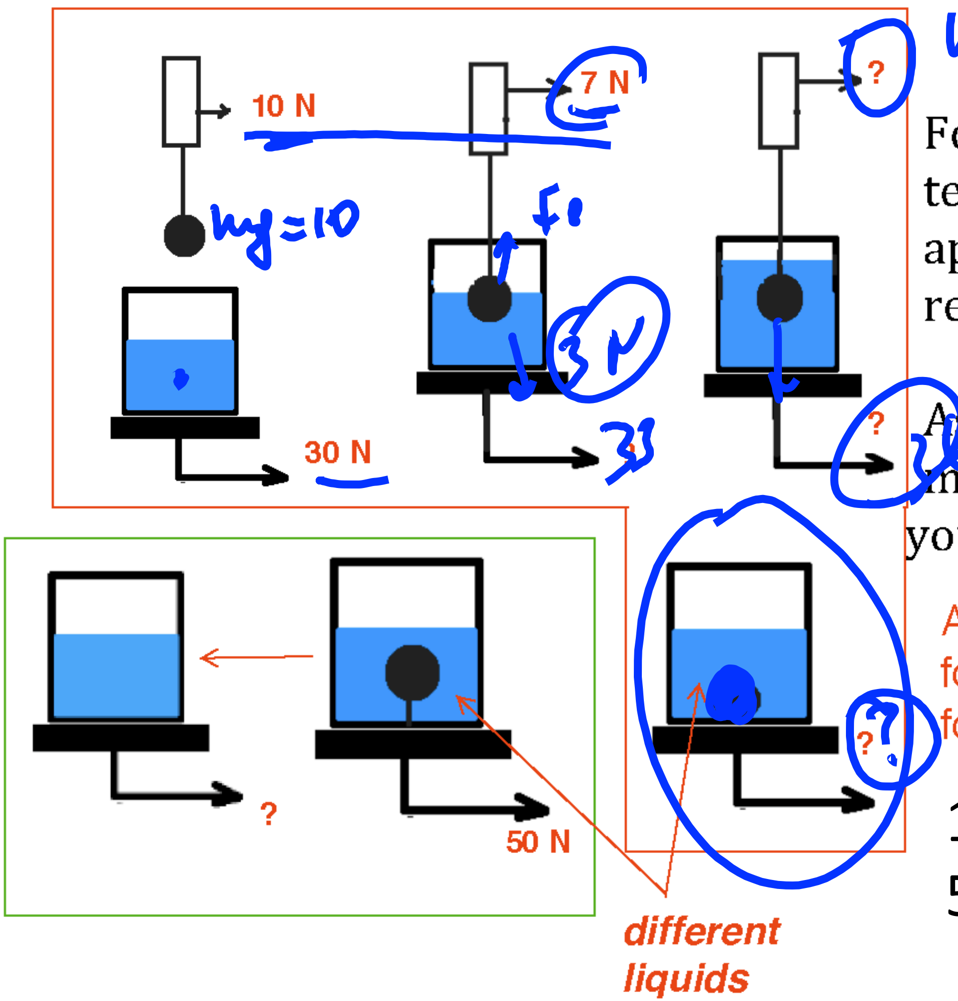

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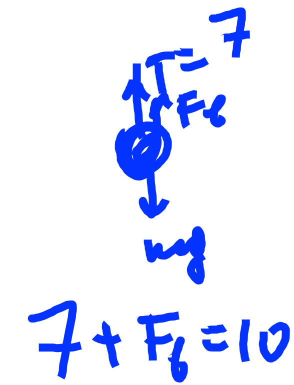


Assuming the ball is the same for ALL cases, find missing forces. (N)

1. **2** 2. **4** 3. **7** 4. **28**

5. **33** 6. **36** 7. **40** 8. **52**

Fluid Dynamics

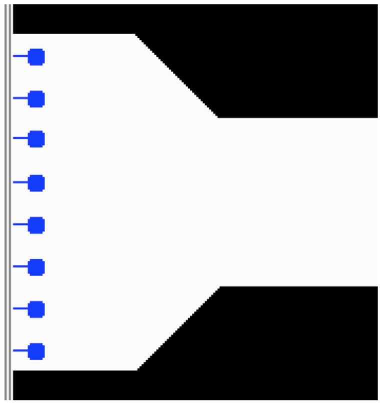
Ideal Fluid

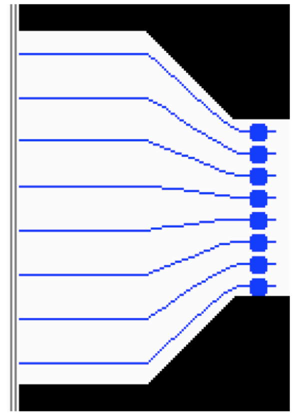
Fluid Dynamics deals with *moving* fluids.

An ideal fluid:

1. **Steady** – the velocity of the fluid at a point remains constant with respect to time.
2. **Laminar -** no turbulence, no disconnections in the current, the flow is smooth and uniform.
3. **Incompressible -** the density of the fluid does not change.
4. **Non-viscous (inviscid) -** no resistive force from objects or pipe walls.
5. **Irrotational -** the fluid won't make an object spin about its own axis.

Continuity Equation

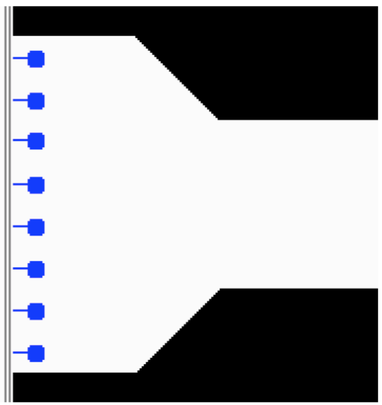


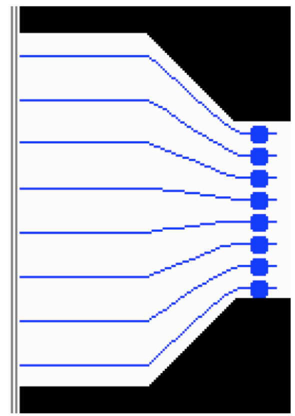


When an *incompressible* fluid flows through a tube and there are *no* sinks or faucets => the total amount of the fluid crossing up the tube cannot change (the fluid cannot disappear or materialize without a reason).

The amount of fluid passing through the tube each second is the same at ANY cross-section!

The mass flow rate (MFR) is the total mass flowing past a point in a given time interval, divided by that time interval.





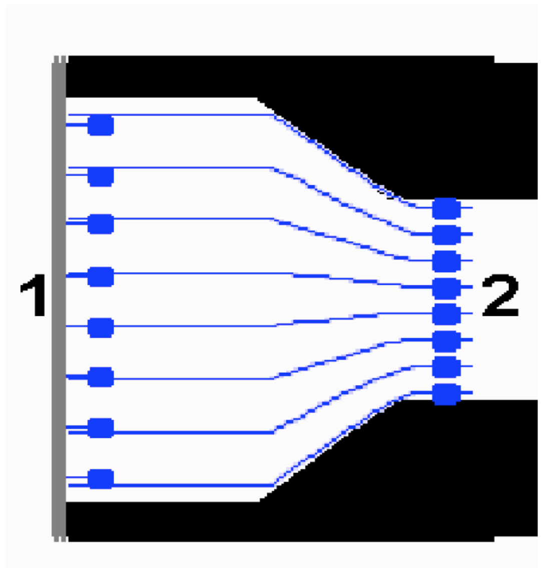
The amount of fluid passing through the tube each second (MFR !) is the same at ANY cross-section!

MFR = constant

The continuity equation (Question 1)

Av= constant

SIM

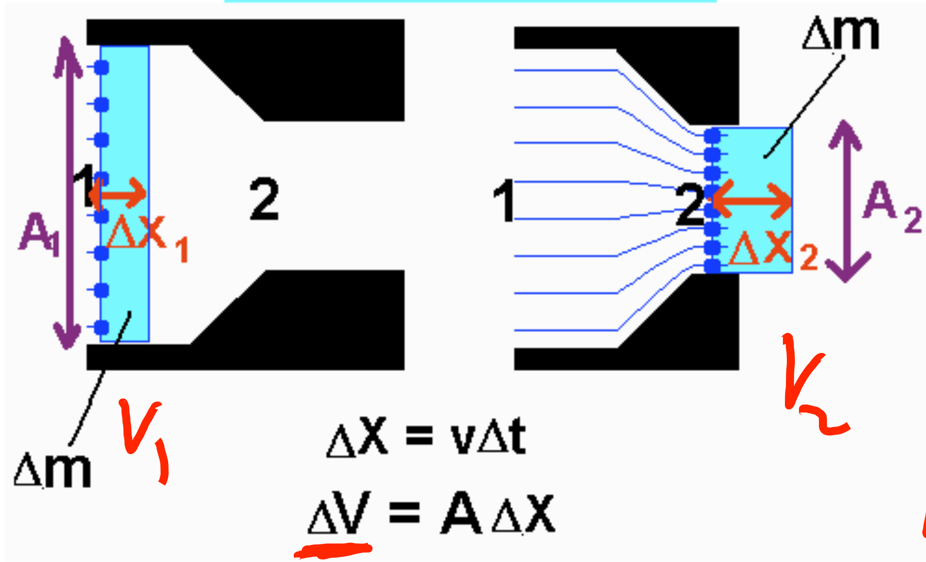


The fluid is flowing through the tube.
At which point the speed of the flow is greater?

1) Point 1 2) Point 2 3) the speed is the same

MFR = constant

SIM



$V_1 = V_2$

At any point (1 or 2): if

v is the velocity of the flow;

A is a cross-sectional area;

ρ is the density of the fluid:

$$\text{MFR (mass flow rate)} = \frac{\Delta m}{\Delta t} = \frac{\rho\ \Delta V}{\Delta t} = \frac{\rho A\ \Delta x}{\Delta t} = \rho A v$$

$$MFR = \rho A v$$

and

MFR = constant or $MFR_1 = MFR_2$

Hence,

$\rho A v$ = constant or $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

In *an incompressible fluid* the density is constant, $\rho_1 = \rho_2 = \rho$,

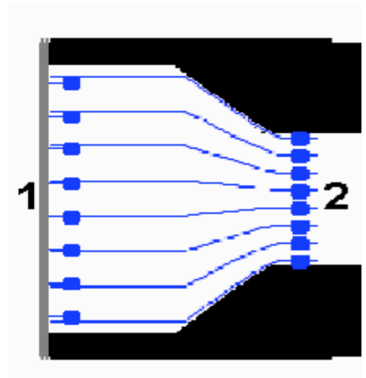
so

The continuity equation

Av= constant or $A_1 v_1 = A_2 v_2$

The continuity equation

$$Av = \text{constant} \quad \text{or} \quad A_1 v_1 = A_2 v_2$$



The fluid is flowing through the tube.
At which point the speed of the flow is greater?

$$A_1 > A_2 \quad \Rightarrow \quad v_1 < v_2$$

At the *narrower* cross-section the fluid flows *faster*!

B) Point 2

VFR

The volume flow rate (VFR) is the total volume of fluid flowing past a point in a given time interval, divided by that time interval.

$$\text{VFR (volume flow rate)} = \frac{\Delta V}{\Delta t} = \frac{A\ \Delta x}{\Delta t} = Av$$

The continuity equation

Av= constant

The volume of fluid passing through the tube each second (VFR !) is the same at ANY cross-section!

#1

The VRF (and MRF) = const;

Continuity equation

$$Av = const$$

#2

There are two reasons for fluid being into a motion and having its kinetic energy changing:

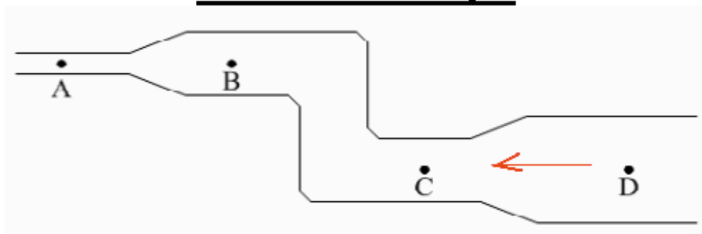
1) Force of gravity (when the current changes its height)

2) Difference in the pressure between different points in the fluid

Bernoulli's equation:

$$\frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 = P_1 - P_2 + \rho g(y_1 - y_2)$$

Fluid in a Pipe



1. To connect the speed of the current at different places we use

$$A_1 v_1 = A_2 v_2$$

2. To connect the pressure at different places and understand how the difference in the height and speed influences the pressure we use

Bernoulli's equation:

$$\rho g y_1 + \tfrac{1}{2}\rho v_1^2 + P_1 = \rho g y_2 + \tfrac{1}{2}\rho v_2^2 + P_2$$