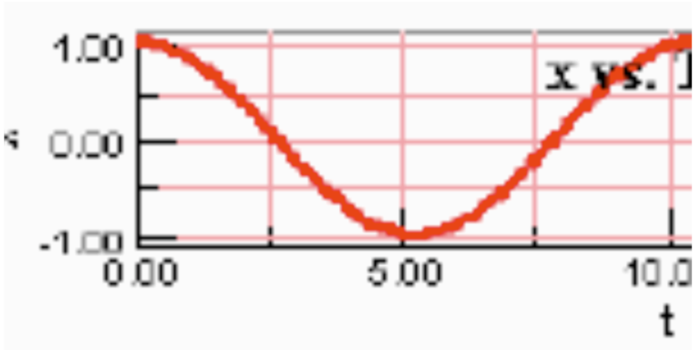
Write the motion equation for the block.



We pull a block away from equilibrium and release from rest. It takes T seconds to make one whole motion with amplitude A.

Sketch the graphs for velocity and acceleration as a function of time, as well as graphs for kinetic and potential energy as a function of time. Sketch graphs for F, K, U, E as a function of position. Compare x, v, a, K, U, at different times and distances, such as x = A, A/4, A/2; t = T, T/4, T/2.

The period for potential energy *U* is = C**T,* where T is the period of motion.

1. C = 0.25   2. C = 0.5   3. C = 1   4. C = 1.25   5. C = 1.5   6. C = 2

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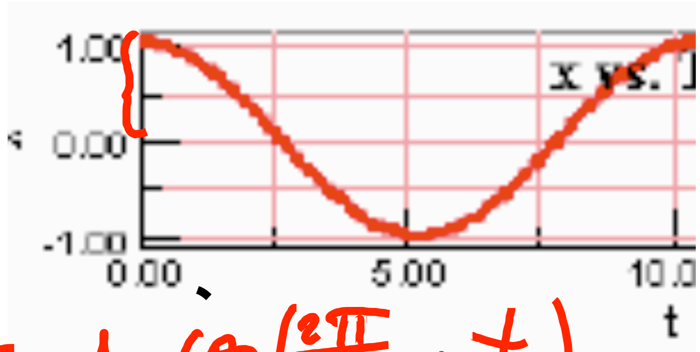

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The period for potential energy *U* is = C**T*, where T is the period of motion.

1. C = 0.25
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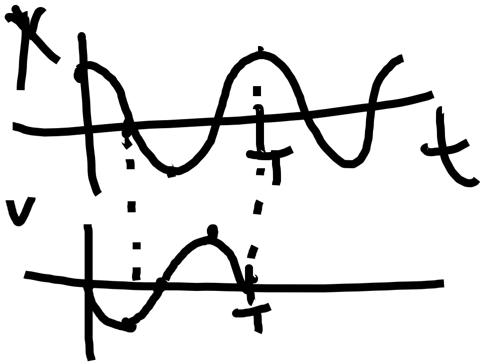
3. C = 1
4. C = 1.25
5. C = 1.5
6. C = 2

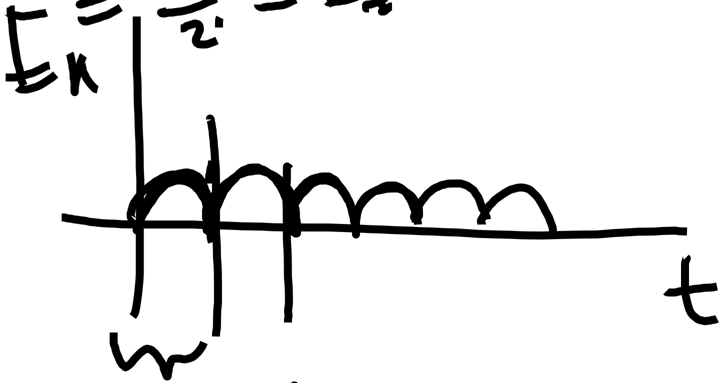
T = C·T



$x = A \cdot \cos \omega t$

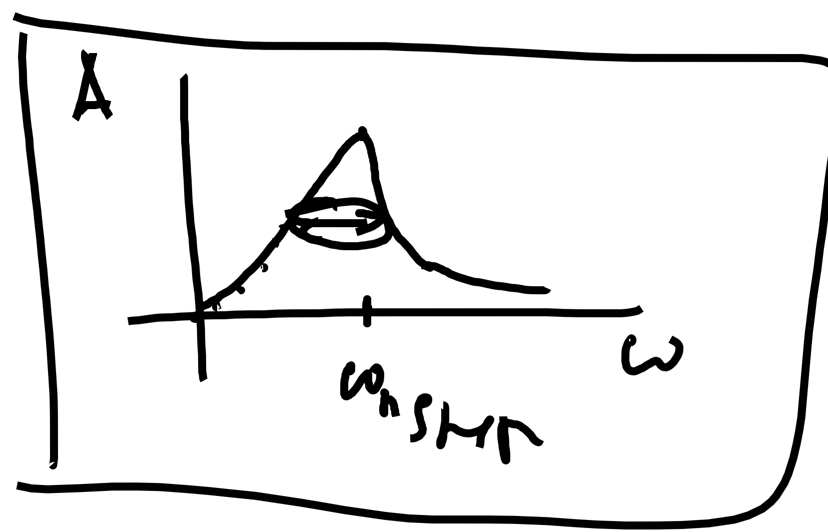
$v = -A\omega \sin \omega t$

$E_k = \frac{mv^2}{2} = E_t - V$



$T_{KE} = \frac{1}{2} T$

$V = E_t - E_{KE}$

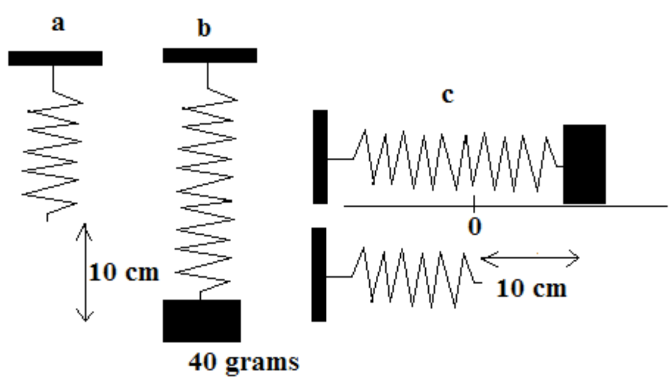


T = 1 2 3 4 5 6 7 8 9 10

$\frac{T}{4} = 2\,s$

$T = 2 \cdot 4 = 8$

Work Together!

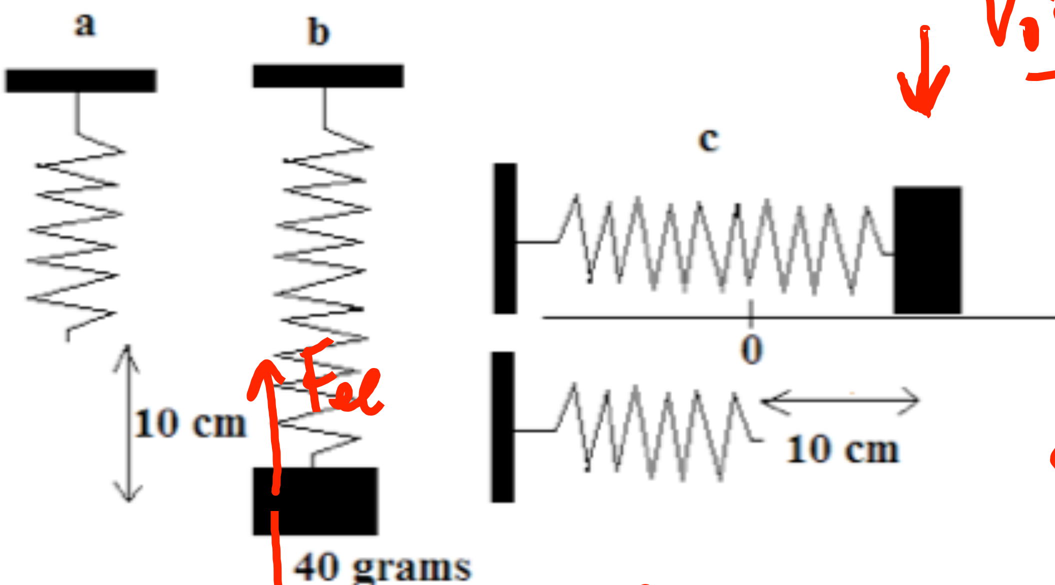


The picture (a) shows massless unloaded spring.

The picture (b) shows the same spring holding (at rest) the 40 grams weight.

Now we turn the picture (b), making the system horizontal and release the block (the picture (c); you can also see the same unloaded spring placed horizontally). Use $g = 10\ m/s^2$ Try to find: the spring constant, the maximum kinetic energy, the period of oscillations.

If the block in picture (c) is released from rest, plot the graphs for its displacement, velocity, acceleration as a function of time, plot the graphs for kinetic energy, potential energy, total energy as a function of time, and as a function of position of the block. Compare the values at different specific times and positions, such as $t = 0$, $T/4$, $T/2$; $x = A$, $A/4$, $A/2$.



$v_0 = 0$

1. $4\,\frac{N}{m}$
2. $-4\,\frac{N}{m}$
3. $6\,\frac{N}{m}$

$mg = 0.04\cdot 10$

$$|F_{el}| = k\cdot|\Delta x|$$

$$k = \frac{|F|}{|\Delta x|} = \frac{0.4}{|0.1|} = 4$$

$$K_{max} = \frac{m v_{max}^2}{2} = \frac{m(A\cdot\omega)^2}{2}$$

$$OFK_{max} = \frac{m(A\omega)^2}{2} = \frac{mA^2\omega^2}{2} = \frac{mA^2\frac{K}{m}}{2} = \frac{mA^2K}{2m} = \frac{KA^2}{2} = \frac{4\cdot(0.1)^2}{2} = 0.02\,J$$

$$U_{max} = \frac{Kx_{max}^2}{2} = \frac{K\cdot A^2}{2}$$

$0 \leftarrow U_{max}$

1. ~~V_{max}~~
2. U_{max}
3. ~~a_{max}~~

Where?

$$E_{kin} = U$$

$$\frac{mv^2}{2} = \frac{kx^2}{2}$$

$x =$

$v =$

$a =$

$$E_{tot} = \frac{mv_{max}^2}{2} = \frac{kA^2}{2} = \frac{mv^2}{2} + \frac{kx^2}{2} = \frac{kx^2}{2} + \frac{kx^2}{2}$$

$$X = \frac{A}{2}$$

1. Yes

~~2. No~~

$$\frac{k A^2}{2} = \frac{k x^2}{2} + \frac{k x^2}{2}$$

$$A^2 = x^2 + x^2 = 2 \cdot x^2$$

$$x^2 = \frac{A^2}{2}$$

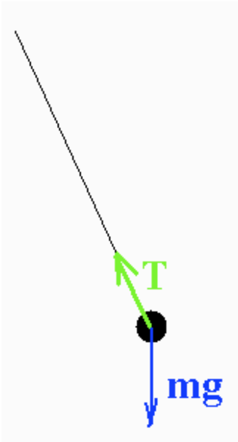
$$x = \sqrt{\frac{A^2}{2}} = \frac{A}{\sqrt{2}}$$

A vertical mass on a spring

If a mass is oscillating vertically at the end of a spring, you might think that we'd need to build in gravitational potential energy to handle the conservation of energy properly. You can do this if you want, measuring the elastic potential energy from the equilibrium length of the spring, but you don't have to.

If you measure the displacements of the spring about the equilibrium position with the mass attached to the spring, this is the point where the spring force cancels the force of gravity. Measured from this point, you just have to consider the kinetic energy and the elastic potential energy - don't worry about gravity.

A simple pendulum



Another simple harmonic motion system is a pendulum. A simple pendulum consists of a mass on a string.

The forces applied to the mass are the force of gravity and the tension in the string. A component of the force of gravity provides the restoring torque.

Applying Newton's second law for rotation:

$$\Sigma \boldsymbol{\tau} = I\boldsymbol{\alpha}$$

$$-mg\ L \sin(\theta) = I\alpha$$

The negative sign is because the torque is opposite to the angular displacement.

For small angles we can use the small-angle approximation:

$$\sin(\theta) = \theta$$

This gives, at small angles: $-mgL\theta = I\alpha$ or

$$\alpha = -\frac{m\,g\,L}{I}\,\theta$$

A hallmark of simple harmonic motion is that $\alpha = -\omega^2\theta$

$$\alpha = -\omega^2\theta$$

So, the angular frequency is $\omega = \left(\frac{m\,g\,L}{I}\right)^{1/2}$

For a simple pendulum the rotational inertia is given by:

$I = mL^2$

This gives $\omega = \left(\frac{g}{L}\right)^{1/2}$

Note that this is *independent of the mass* of the pendulum.

The general equation giving the position of the pendulum as a function of time is:

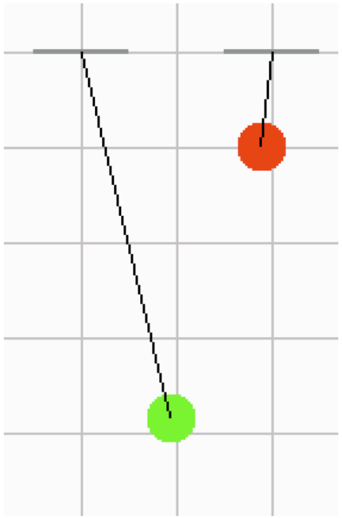
$$\theta(t) = \theta_{max} \cos(\omega t + \phi)$$

Pendulum Question

We have two simple pendula of equal lengths. One is a heavy mass on a string, the other is a light mass. Which has the longer period of oscillation?

1. the heavy one
2. the light one
3. neither, they're equal

Two pendula



What is the ratio of their periods?

1. $\frac{T_{green}}{T_{red}} = \frac{1}{4}$
2. $\frac{T_{green}}{T_{red}} = \frac{1}{2}$
3. $\frac{T_{green}}{T_{red}} = \frac{1}{1}$
4. $\frac{T_{green}}{T_{red}} = \frac{2}{1}$
5. $\frac{T_{green}}{T_{red}} = \frac{4}{1}$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$

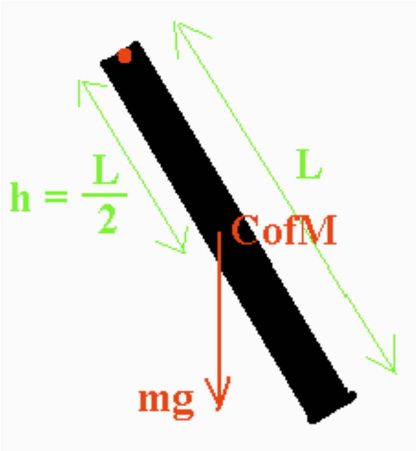
$T_1 = \ldots\sqrt{L_1}$

$T_2 = \ldots\sqrt{L_2}$

$L_2 = 4 \cdot L_1$

$T_2 = 2 \cdot T_1$

A physical pendulum



A physical pendulum is *any* pendulum where the mass is NOT all concentrated at one point.

For a physical pendulum $\omega = \left(\frac{m\,g\,h}{I}\right)^{½}$

Here h is not the length of the pendulum, but the distance from the point of rotation to the pendulum's center of gravity.

If the physical pendulum is a *rod* of length L supported at one end:

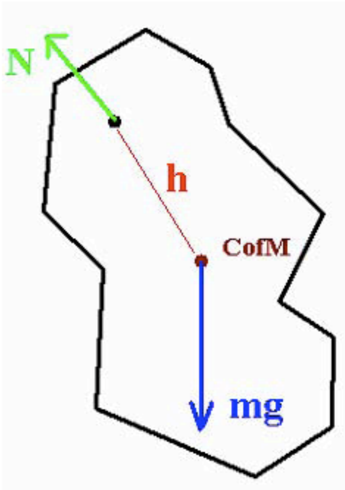
$h = \frac{L}{2}$ and $I = \frac{1}{3}mL^2$

This would give an angular frequency of:

$$\omega = \left(\frac{3g}{2L}\right)^{½}$$

This has the same frequency as a *simple pendulum* with a length of 2L/3.

A physical pendulum



A physical pendulum is any pendulum where the mass is NOT all concentrated at one point.

For a physical pendulum $\omega = \left(\frac{m\,g\,h}{I}\right)^{1/2}$

If we introduce an effective length as $\frac{1}{l_{\text{eff}}} = \frac{mh}{I}$

We can write it in the form of:

a "simple" pendulum $\omega = \left(\frac{g}{l_{\text{eff}}}\right)^{1/2}$

The *simple pendulum* of the length l_{eff} has the same frequency as a physical pendulum with the moment of inertia I.

Notice: h is the distance from the axis of rotation to the C of M;
I is the moment of inertia relative to the *actual* axis of rotation.

The pendulum with small and large amplitudes

Any pendulum undergoes simple harmonic motion when the amplitude of oscillation is small. What happens for large amplitudes?

The pendulum still oscillates, but the motion is no longer simple harmonic motion because the angular acceleration is not proportional to the negative of the angular displacement.

You should be able to see that for large angular displacements the graph is no longer sinusoidal.

Driven harmonic motion

In driven (or forced) harmonic motion, the system experiences a sinusoidally oscillating force. The response of the system depends on how the frequency of the force compares to the natural oscillation frequency of the system.

A good example of such a system is a playground swing. When you push a kid on a swing, you match the frequency of your pushes to the frequency of the swinging kid - this is the most effective way to get (and keep) the kid going.

That is known as resonance - when the driving frequency matches the natural frequency of the system (the resonance frequency) then the oscillation amplitude can grow to be quite impressive, even when the driver has a relatively small amplitude.

The phase relationship between the driving force and the oscillating mass is interesting to observe:

- Below resonance they're in phase
- At resonance they're 90 degrees out of phase
- Above resonance they're 180 degrees out of phase