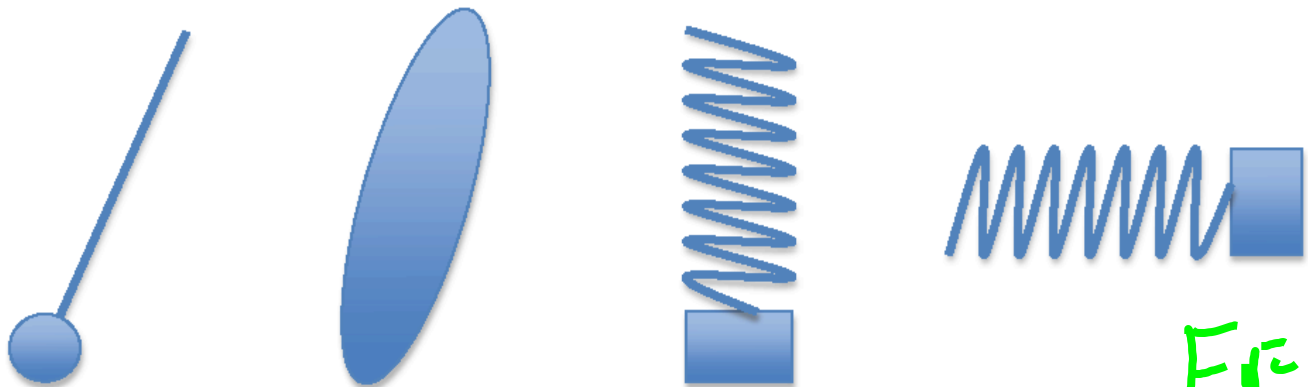
Oscillations !!



$\underline{F}^* = F_e = \underline{mg}$

$F_g = m \cdot g =$
$= 12 \cdot 10 =$
$= 0$

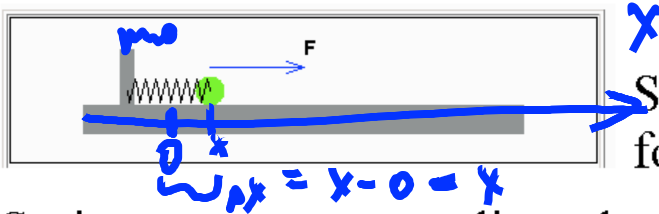
Always choose the origin at the equilibrium position !!!!

Restoring force always points at the equilibrium position !!!!

Does F_g act on the spring?

1. Yes

2 No



Springs

So far we've dealt mostly with constant forces.

Springs are more complicated - not only does the magnitude of the spring force vary, the direction of the force depends on whether the spring is being stretched or compressed.

Measuring all distances from the equilibrium length of the spring, the force from an ideal spring is given by Hooke's Law:

$$|\mathbf{F}| = k|\Delta \mathbf{x}|$$

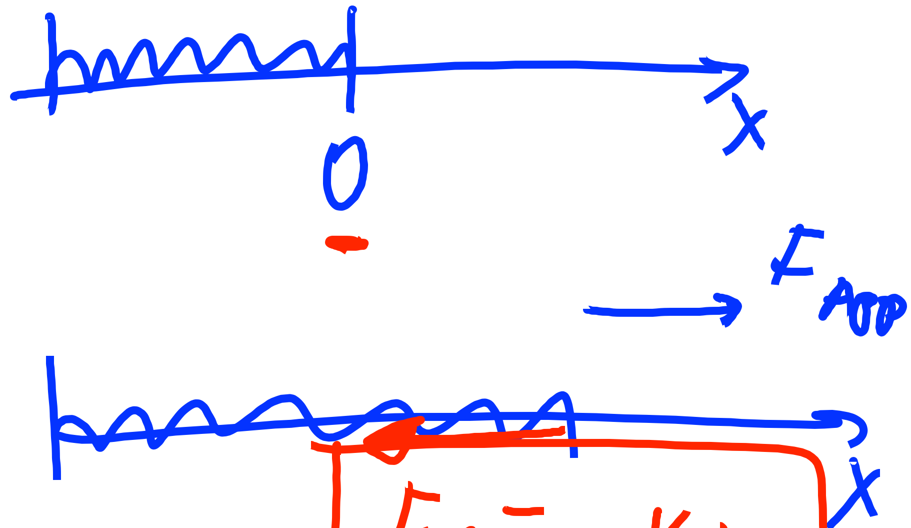
$|F| = k \cdot |x|$

k is the spring constant, a measure of the stiffness of the spring (N/m).

$|\mathbf{F}|$ is the absolute value of the elastic force

$|\Delta \mathbf{x}|$ is the absolute value of the displacement from the equilibrium position.

The spring force is *always* opposite to the direction to the displacement!!



$$\boxed{F_{el} = -k \cdot x}$$

$$|F_{el}| = k \cdot |x|$$

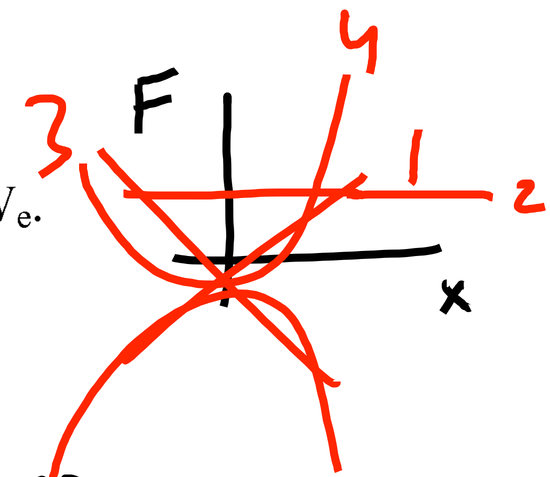
Let's say:

The spring stretches 10 cm when 100 g (about 1 N) is attached to it.

The spring constant is $$k = \frac{1\ \text{N}}{0.1\ \text{m}} = 10\ \text{N/m}$$

$F = -K \cdot x$

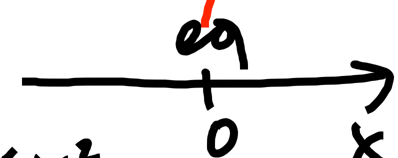
Spring Potential Energy



To stretch or compress a spring we have to do some *work* W_e.

Elastic force is a conservative force, it has potential energy:

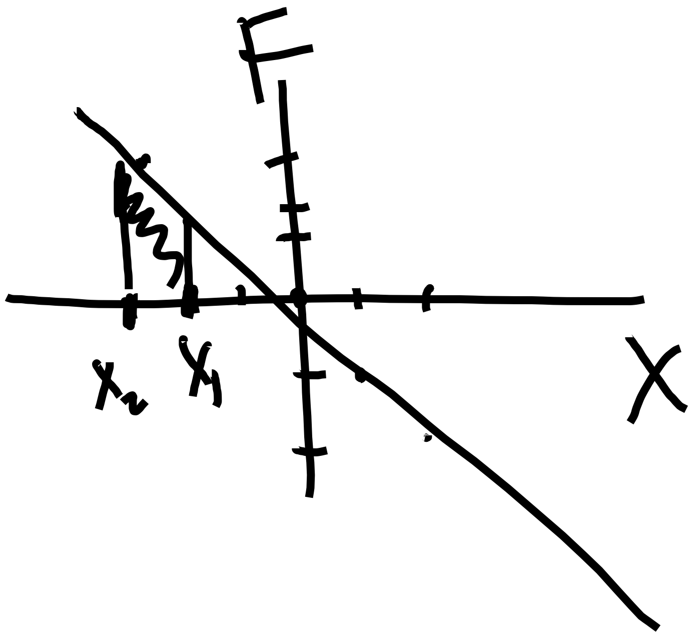
$$W_e = U_1 - U_2$$



The potential energy of a spring is given by:

$$U = \tfrac{1}{2}\, k(\Delta x)^2 = \frac{1}{2} K x^2$$

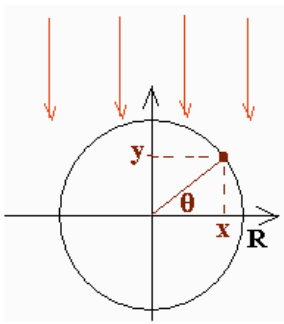
$$F = -k \cdot x \longrightarrow F = -x$$



x	F
0	0
1	-1
2	-2
-3	3

$x(t) = R\cos(\omega t)$ $\quad\quad$ $y(t) = R\sin(\omega t)$

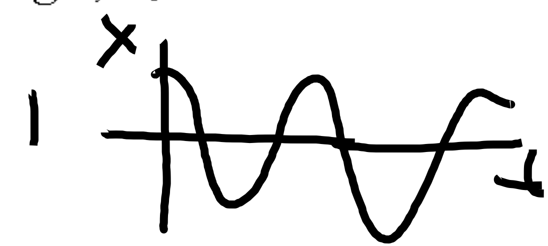
1. $\quad\quad$ 2.



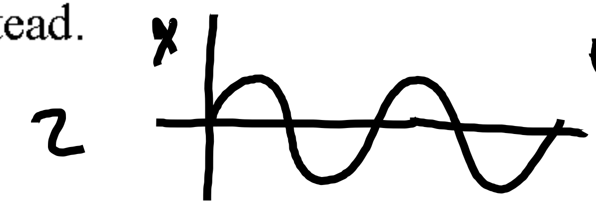
A light shining down in the y direction would cast a shadow of the circularly moving motion object – the motion of that shadow would *exactly* match the motion of the object undergoing simple harmonic motion to the left and to the right.

If, at $t = 0$, the objects are not at their extreme $+x$ position, we need to do modify our equation, shifting it by the appropriate **phase angle**, θ_o.

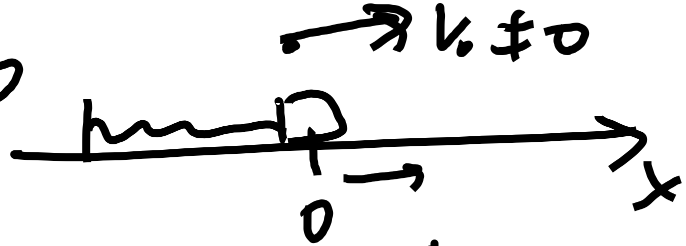
The equation for simple harmonic motion becomes:
$x(t) = A\cos(\omega t + \theta_o)$

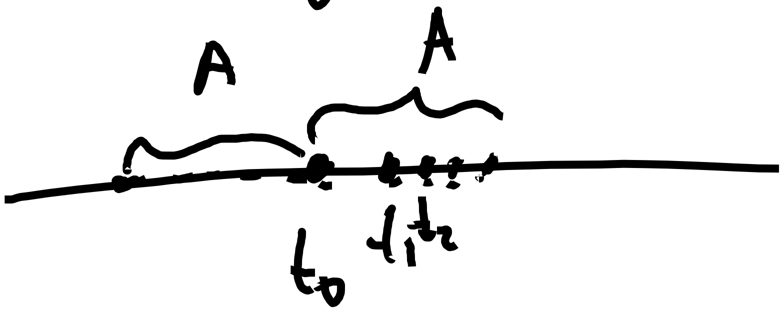


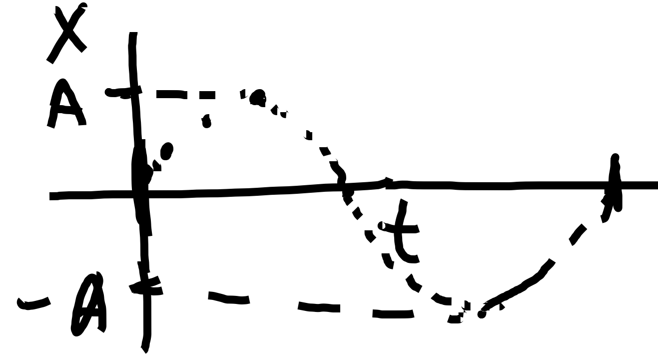
Instead of plugging in $\theta = \omega t$ we're using $\theta = \omega t + \theta_o$ instead.



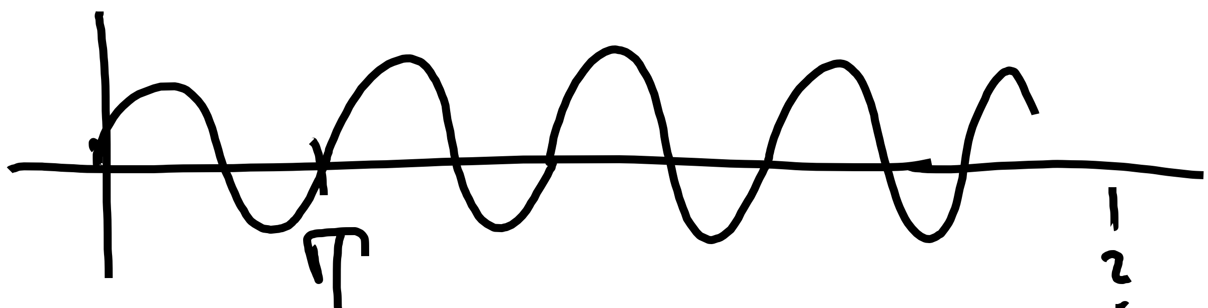
$t=0$





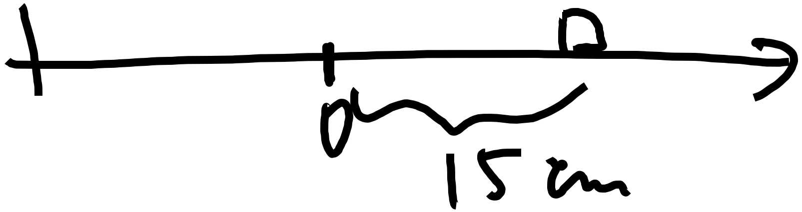


How many $\frac{1}{4}$ of T in 2 T?



1 5
2 6
3 7
4 8

$t = 0$ $\underline{\underline{V_0 = 0}}$



$L_{(t)} = ?$

$\underline{\underline{cm}}$

10
20
30
40
50
60
70
80
90

Dissecting the SHM equation

Our general equation for position in SHM is:

$$x(t) = A\cos(\omega t + \theta_o)$$

A is the amplitude, which is the maximum displacement from the equilibrium position.

$\boldsymbol{\omega}$ is an angular frequency, which is the number of oscillations per 2π seconds.

$\boldsymbol{\omega t + \theta_o}$ is a phase; $\boldsymbol{\theta_o}$ is an initial phase

$T = 2\pi/\omega$ is a period, which is the time for one complete oscillation (one complete oscillation has four similar parts)

$f = 1/T$ is the frequency, i.e. the number of oscillations per one second.

General features of simple harmonic motion

All simple harmonic motion systems have these two features:

No loss of mechanical energy.

A restoring force or torque that is proportional to the displacement from equilibrium. The force or torque is also opposite to the displacement - that's what "restoring" implies.

Implications of these facts

The motion of the system is described by an equation of the form

$$x(t) = A\cos(\omega t + \theta_o).$$

The relationship between the acceleration and the displacement is:

$\omega = (k/m)^{1/2}$.

$$a = -\omega^2 x$$

where ω is the angular frequency of the system.

The period of oscillation is $T = \dfrac{2\pi}{\omega}$

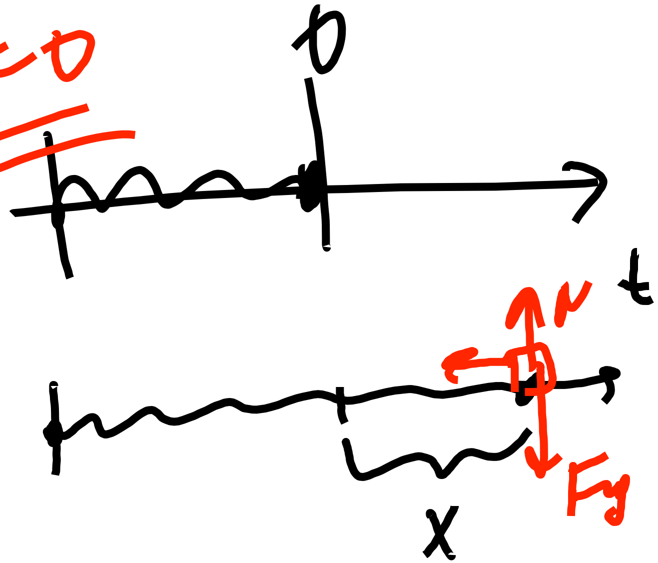
Because of ... law

1.N II L

2.LCLM

3.LCME

$F_x = 0$



$$F = -k \cdot x$$

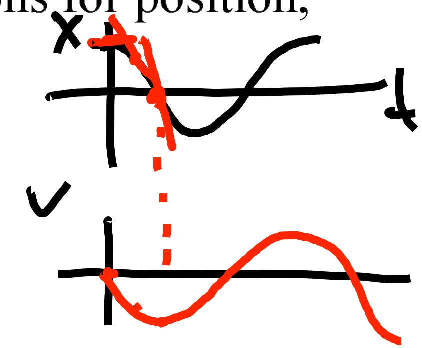
$$F_{ne} = ma$$

$$-kx = ma$$

$$a = -\frac{k}{m} \cdot x$$

Graphs of position, velocity, and acceleration

In SHM (simple harmonic motion), the general equations for position, velocity, and acceleration are:



$$x(t) = \underline{A} \cos(\omega t + \theta_o)$$

$$v(t) = -\underline{A\omega} \sin(\omega t + \theta_o)$$

$$a(t) = \underline{-A\omega^2} \cos(\omega t + \theta_o)$$

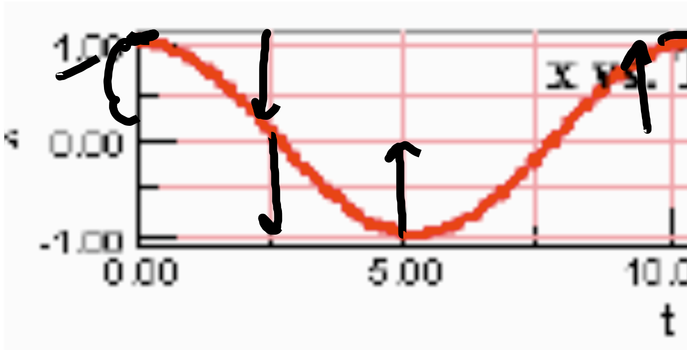
The phase angle θ_o is determined by the initial position and initial velocity.

The angular frequency for an object of mass m oscillating on a spring of spring constant k the angular frequency is given by:

$$\boxed{\omega^2 = \frac{k}{m}} \qquad T = 2\pi/\omega \qquad \omega = 2\pi f \qquad f = 1/T$$

Whatever is multiplying the sine or cosine represents the maximum value of the quantity.

Thus: $\underline{x_{max} = A \qquad v_{max} = A\omega \qquad a_{max} = A\omega^2}$



We pull a block away from equilibrium and release from rest. It takes T seconds to make one whole motion with amplitude A.

How much time does it take for the block to travel distance A?

1. T s
2. $T/2$ s
3. $T/3$ s
4. $T/4$ s
5. T/5 s

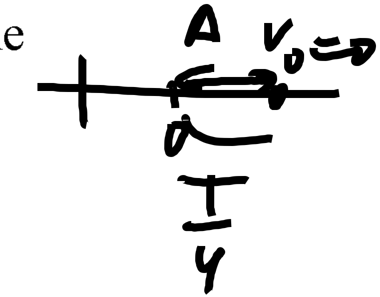
Question

An object attached to a spring is pulled a distance A from the equilibrium position and released from rest. It then experiences simple harmonic motion with a period T. The time taken to travel between the equilibrium position and a point A from equilibrium is T/4. How much time is taken to travel between points A/2 from equilibrium and A from equilibrium? Assume the points are on the same side of the equilibrium position

1. T/8
2. More than T/8
3. Less than T/8
4. It depends whether the object is moving toward or away from the equilibrium position

The answer

The further the object gets from equilibrium the slower it gets, so the average speed for moving between x = A and x = A/2 is *less* than the average speed between x = A/2 and x = 0.



Thus it takes *longer* than T/8 to move between x = A and x = A/2.

Let's figure out exactly how long it takes.

The equation of motion is $x = A\cos(\omega t)$, $\omega = 2\pi/T$

At t = 0 the object is at x = A.

What is the time when x = A/2? We have to sole the motion equation:

$$\cancel{A}/2 = \cancel{A}\cos(\omega t) \quad \text{or} \quad \tfrac{1}{2} = \cos(2\pi t/T)$$

$$\frac{2\pi \cdot t}{T} = 60^\circ = \frac{\pi}{3}$$

What angle has a cosine of 1/2? In ***radians*** this is $\pi/3$,

so $2\pi t/T = \pi/3$ Solving for time t that gives: $t = T/6$

T/6 > T/8

Energy in a spring system

A block connected to a horizontal spring sits on a frictionless table. The block is moved to compress the spring, and the system is released from rest. What happens to the energy initially stored in the spring? What is the maximum speed of the block?

The initial energy is $U_i = \frac{1}{2} k x_i^2$

The block reaches maximum speed when the spring reaches its equilibrium length - that's the point where all the energy stored in the spring is converted to kinetic energy.

To find the speed use the master energy equation, where the initial position is the point where the block is released and the final position is the equilibrium position:

$$U_i + K_i + W_{nc} = U_f + K_f$$

Energy in SHM

Conservation of mechanical energy applies to a simple harmonic motion system.

The sum of the potential and kinetic energies is constant (with no friction)

The total energy is equal to the maximum potential energy:

$$E = U_{max} = \tfrac{1}{2} kA^2$$

$$E = K + U = const$$

or the maximum kinetic energy

$$E = K_{max} = \tfrac{1}{2} mv^2$$

$$\tfrac{1}{2} m v_m^2 = E_1 = E_2 = \tfrac{1}{2} k A^2$$

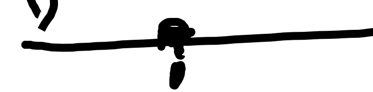
So, we can set them equal and find:

$$\tfrac{1}{2} m\, v_{max}^2 = \tfrac{1}{2} kA^2 \;\Rightarrow\; v_{max} = \sqrt{\frac{k}{m} A^2} = \sqrt{\frac{k}{m}} A = \omega A$$

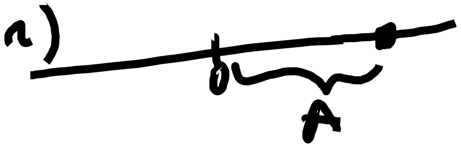
$$A = X_m$$

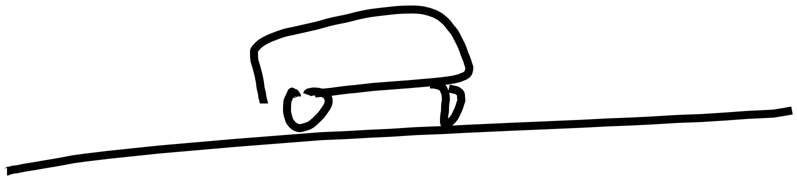
$$v_{max} = \omega A$$

1)



2)

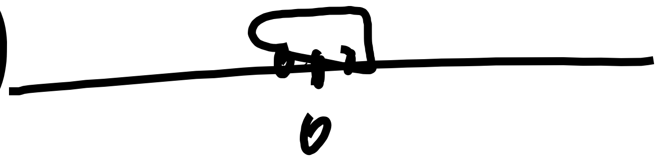




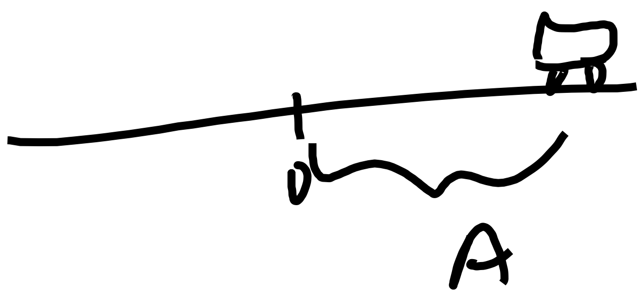
What $E_n \underline{\neq} 0$?

1. K
2. U

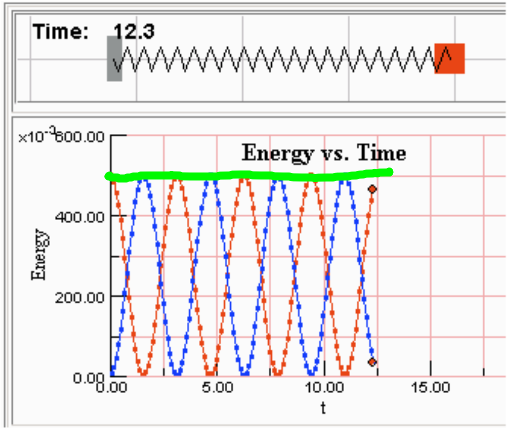
1)



2)



1. K
2. U



Graphs of energy in a spring system

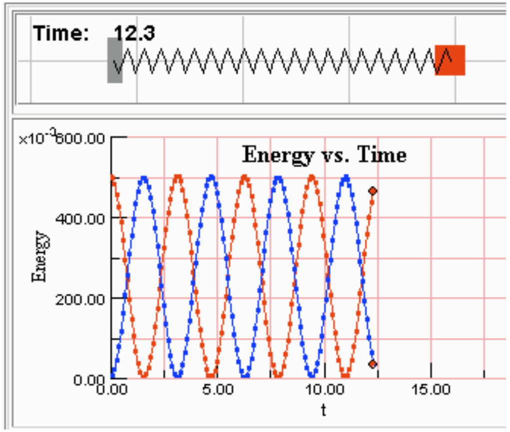
Two graphs are shown in the simulation, one showing the spring potential energy and the other showing the block's kinetic energy.

Which graph is which?

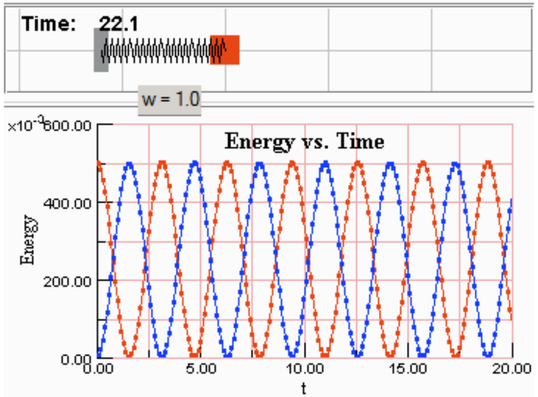
1. The red one is the kinetic energy; the blue one is the potential energy

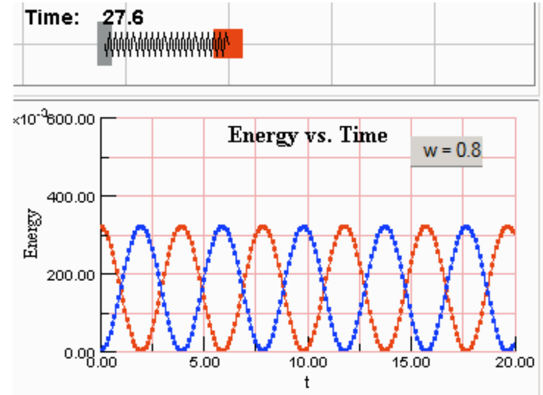
2. The blue one is the kinetic energy; the red one is the potential energy

3. The graphs are interchangable so you can't tell which is which



The blue graph is zero when the block turns around (v = 0) and it peaks when the block passes through the equilibrium position (maximum speed) so it must be the kinetic energy.





Graphs of potential and kinetic energy as a function of time show that the total energy is constant, and that energies go through two complete cycles for each oscillation of the object.

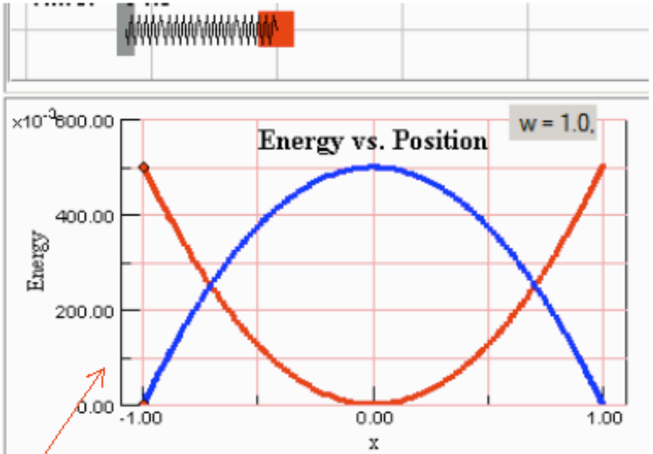
The first set of graphs is for an angular frequency $\omega = 1$ rad/s.

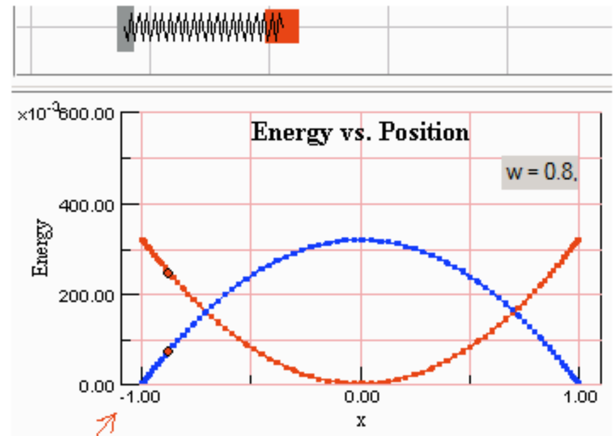
The second set of graphs is for $\omega = 0.8$ rad/s.

This change of ω is accomplished either by decreasing the spring constant or by increasing the mass. Which change did we make in this case?

1. We decreased the spring constant
2. We increased the mass
3. We could have done one or the other, you can't tell the difference

Graphing the energies as a function of position is also interesting.





$$\boxed{U = \frac{kx^2}{2}} = \frac{k[A\cos(\omega t)]^2}{2} = = U_{max}\cos^2(\omega t) = E_{total}\cos^2(\omega t)$$

$$\boxed{K = \frac{mv^2}{2}} = \frac{m[A\omega\sin(\omega t)]^2}{2} = = K_{max}\sin^2(\omega t) = E_{total}\sin^2(\omega t)$$

red or blue?
time or position?

$$U + K = E_{totla}[\cos^2(\omega t) + \sin^2(\omega t)] = E_{totla} = const$$

Understanding Oscillations

An object attached to a spring is pulled a distance A from the equilibrium position and released from rest. It then experiences simple harmonic motion. When the object is A/2 from the equilibrium position how is the energy divided between spring potential energy and the kinetic energy of the object? Assume mechanical energy is conserved.

1. The energy is 25% spring potential energy and 75% kinetic energy
2. The energy is 50% spring potential energy and 50% kinetic energy
3. The energy is 75% spring potential energy and 25% kinetic energy
4. One of the above, but it depends whether the object is moving toward or away from the equilibrium position
5. None of the above

$U_{max} = \frac{k \cdot A^2}{2}$ $\quad U = \frac{kx^2}{2}$; $\quad x = \frac{1}{4} A$

$U = \frac{k\left(\frac{A}{4}\right)^2}{2} = \frac{1}{16} \cdot \frac{kA^2}{2} = \frac{1}{16} U_{max}$

$U = \frac{1}{16} E_t$

Understanding Oscillations

An object attached to a spring is pulled a distance A from the equilibrium position and released from rest. It then experiences simple harmonic motion.

When the object is A/2 from the equilibrium position how is the energy divided between spring potential energy and the kinetic energy of the object? Assume mechanical energy is conserved.

The total energy is $\frac{1}{2} kA^2$.

At $x = A/2$ the spring potential energy is $\frac{1}{2} kx^2 = kA^2/8$

which is 1/8 = 25% of the total energy.