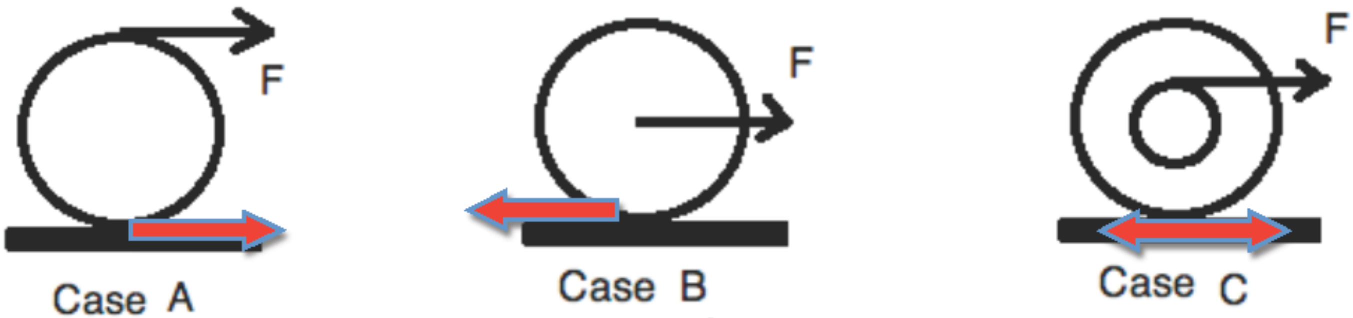
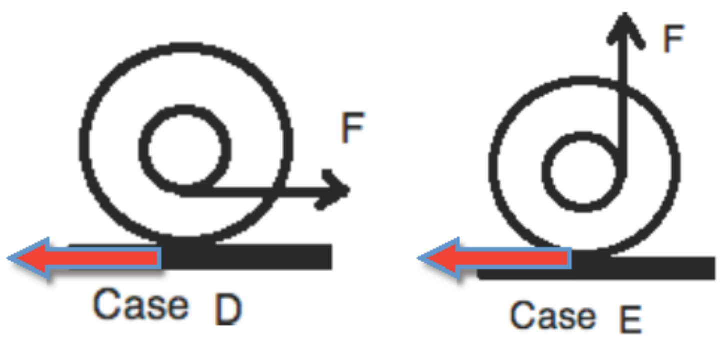
Pictures show a spool with a force F applied to it.
The spool rolls without slipping.





(the direction of friction depends on the ratio of the raddi)

The red arrow shows the direction of the force of friction.

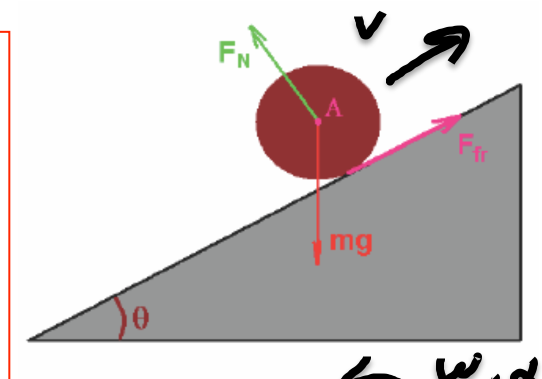
Example

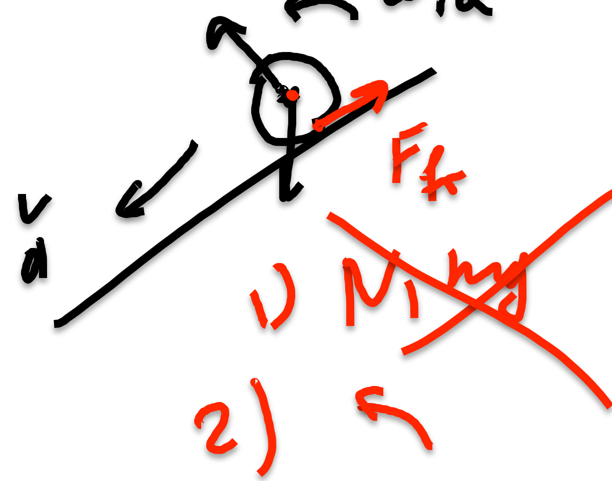
Does the sphere on the incline roll up the incline or down?

1. Up

2. Down

3. It can be both





I. Let's assume it rolls up the incline.

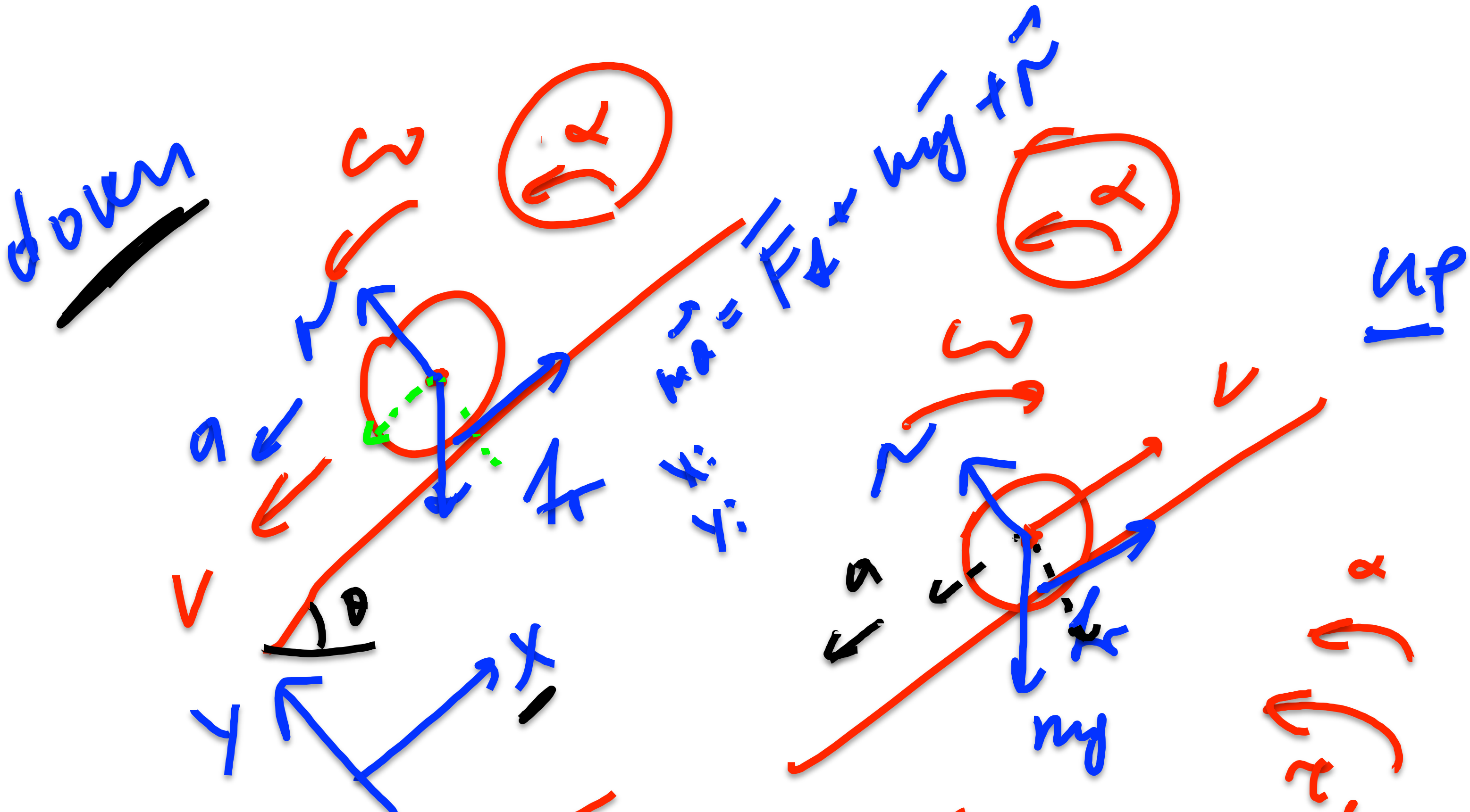
show linear velocity of the sphere

show angular velocity of the sphere

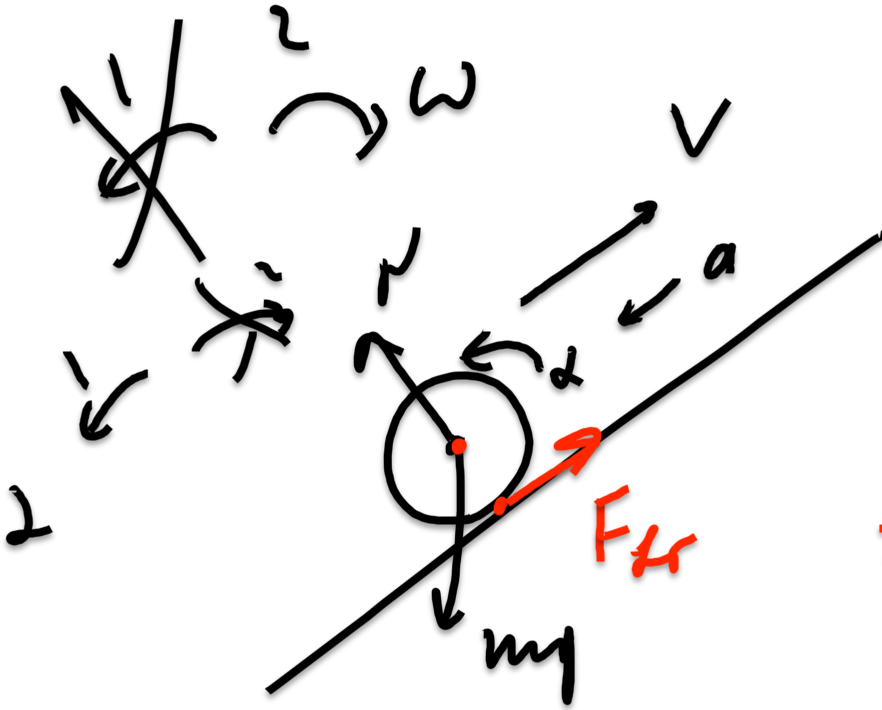
does the sphere accelerate or decelerate?

show angular acceleration

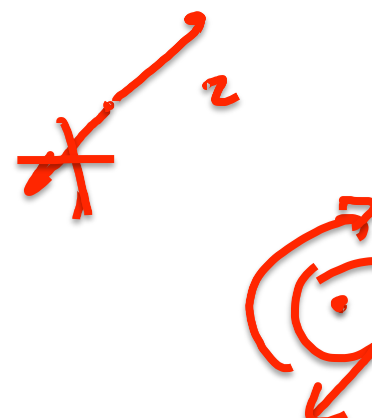
is angular acceleration consistent with the net torque acting on the sphere?



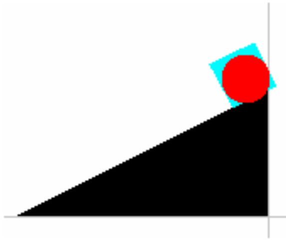
$-ma = F_{fr} - mg\sin\theta$ $\quad \tau_{net} = \tau_{fr} \quad$ $-ma = F_{fr} - mg\sin\theta$



$\tau = I\alpha$



A Race: Sliding and Rolling Down a Ramp



We have a solid sphere and a block of the same mass.

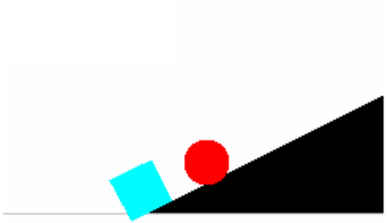
If we release them from rest at the top of an incline, and the sphere is rolling with no slipping, and the block is sliding with no friction; which object will win the race.

A) The sphere

B) The block

C) Two-way tie

A Race: Sliding and Rolling Down a Ramp



We have a solid sphere and a block of the same mass.

If we release them from rest at the top of an incline, and the sphere is rolling with no slipping, and the block is sliding with no friction; which object will win the race.

B) The block

We can say that the inertia of the sphere is grater then inertia of the block, because the sphere is rotation in addition to its translational motion.

"More inertia" => longer time!

A Race: Rolling Down a Ramp

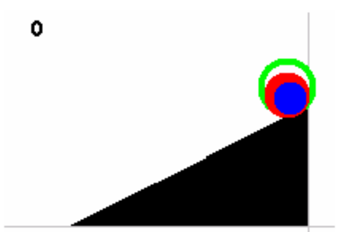
We have three objects of the same mass and radius; a solid disk, a ring, and a solid sphere.

The moments of inertia:

$I_{disk} = \{1/2\}mR^2$ $I_{ring} = mR^2$ $I_{spere} = \{2/5\}mR^2$

If we release them from rest at the top of an incline, which object will win the race (assume no slipping)?

(In the picture the radii are different for a visibility)



A) The sphere

B) The ring

C) The disk

D) Three-way tie

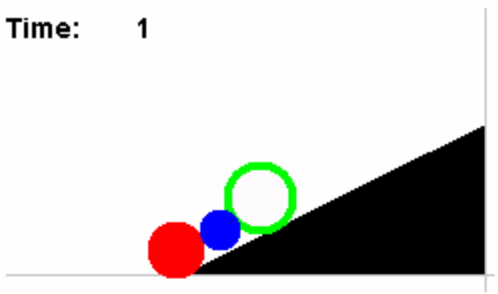
A Race: Rolling Down a Ramp (Sim 1)

We have three objects of the same mass and radius; a solid disk, a ring, and a solid sphere.

The moments of inertia:

$I_{disk} = \{1/2\} mR^2$ $\quad$ $I_{ring} = mR^2$ $\quad$ $I_{spere} = \{2/5\} mR^2$

If we release them from rest at the top of an incline, which object will win the race (assume no slipping).



"More inertia" => longer time!

A) The sphere wins (the disk is next; the ring comes last)

A Race: Rolling Down a Ramp (solution)

Let's take an object with a mass M and a radius R, and a moment of inertia of cMR^2.

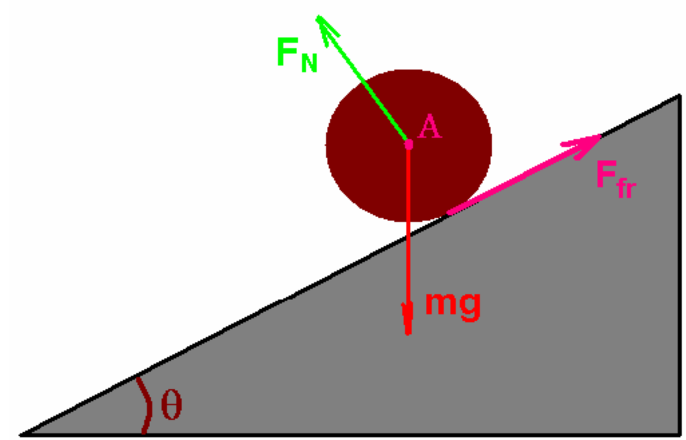
$I_{disk} = \{1/2\} mR^2$ $\qquad$ $I_{ring} = 1 mR^2$ $\qquad$ $I_{spere} = \{2/5\} mR^2$

Hence; $\qquad I = CmR^2$

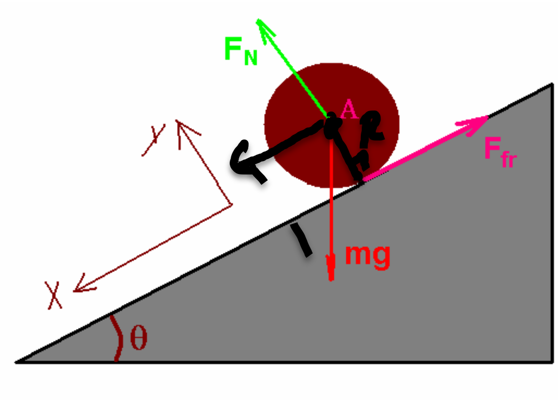
$C_{disk} = 1/2$ $\qquad$ $C_{ring} = 1$ $\qquad$ $C_{spere} = 2/5$

(for the sliding block C = 0; no ration!)

Let’s solve the same problem by the means of the Newton’s second law.



The free-body diagram of the object shows three forces.



Newton's second law for forces:

$$\Sigma F = ma$$

$N = mg \cdot \cos\theta$

The x-components is:

$$mg\sin(\theta) - f_s = ma \quad (1)$$

Newton's second law for rotation is: $\Sigma\tau = I\alpha$

Let's chose the axis of ration at the center of the object.

The only force which has the non-zero torque is friction.

$$f_s R = I\alpha \quad (2)$$

And we can write two more connections:

$$I = CmR^2 \qquad a = \alpha R \quad (3)$$

Combining the equations (1), (2) and (3),

we can solve them for the acceleration and find:

$$a = \frac{g \sin(\theta)}{1 + C}$$

For the motion with a constant acceleration from rest:

$$S = \tfrac{1}{2} a t^2$$

Three objects cover the same distance S, hence the object with the largest acceleration (i.e. the smallest C) reaches the end of the distance the first.

$$C\downarrow \;\; \Rightarrow \;\; t\downarrow$$

(“smaller inertia” => smaller time!)

Kinetic Energy

When and object is in a *translational* motion only:

$$K = \frac{1}{2} mv^2$$

When and object is in a *rotational* motion only:

$$K = \frac{1}{2} I\omega^2$$

When and object is *rolling*:

$$K = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$$

as the result of static friction acting on the rolling object!

Conservation of Energy

$$U_i + K_i + W_{nc} = U_f + K_f$$

Includes TWO works from static friction and = 0 (no slipping)!

$K = \frac{1}{2} mv^2$ if the object is moving but not rotating.

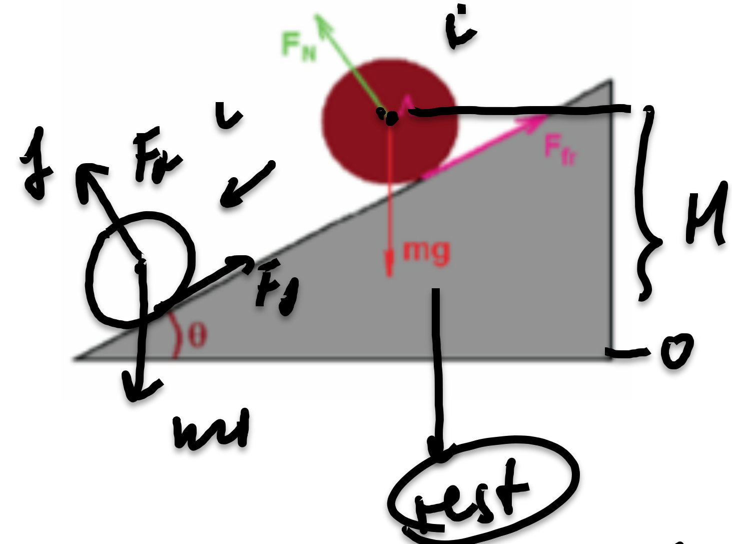
$K = \frac{1}{2} I\omega^2$ when an object is only spinning.

$K = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$ when an object is rolling

$K = \sum K_{all\text{-}objects}$ when there are many objects moving

Let's write the law of conservation of mechanical energy.

$$I = C \cdot m \cdot R^2$$



$$\frac{mv^2}{2} + \frac{I\omega^2}{2} + 0 = \frac{mv^2}{2} + \frac{I\omega^2}{2} + mgH$$

$$v = \omega \cdot R$$

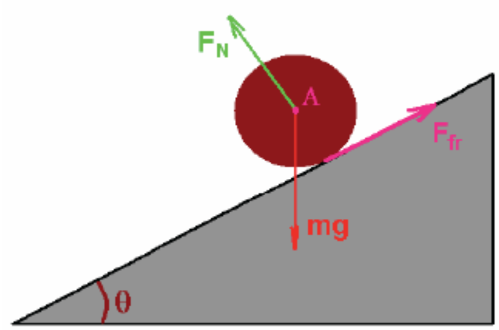
$$\omega = \frac{v}{R}$$

$$\frac{\cancel{m}v^2}{2} + \frac{c \cdot \cancel{m} \cdot \cancel{R^2} \cdot \left(\frac{v}{\cancel{R}}\right)^2}{2} = \cancel{m} g H$$

$$v^2 + c \cdot v^2 = 2gH$$

$$v = \sqrt{\frac{2gH}{1+c}}$$

Conservation of Energy



translational motion

$$\frac{mV_f^2}{2} = \frac{mV_i^2}{2} + W_{net} = \frac{mV_i^2}{2} + mg(h_i - h_f) + W_{fr\ trans}$$

rotational motion

$$\frac{I\omega_f^2}{2} = \frac{I\omega_i^2}{2} + W_{net} = \frac{I\omega_i^2}{2} + W_{fr\ rot}$$

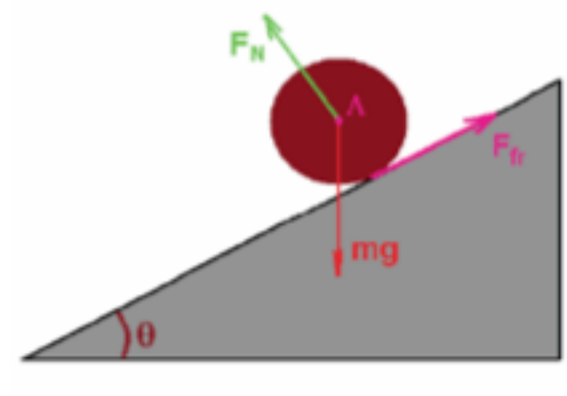
$$W_{fr\ rot} = -W_{fr\ trans}$$

(no slipping)

$$\Rightarrow \quad \boxed{\frac{mV_f^2}{2} + \frac{I\omega_f^2}{2} + mgh_f = \frac{mV_i^2}{2} + \frac{I\omega_i^2}{2} + mgh_i}$$

Let’s calculate the final speed of a rolling object (which is released from rest).

$$I = CmR^2$$



Angular Momentum

A spinning object has angular momentum,

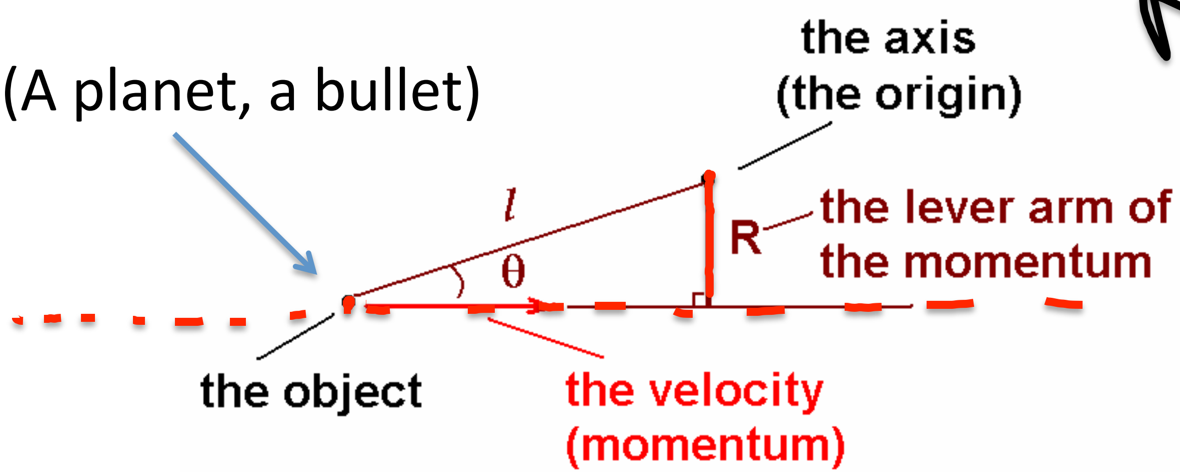
represented by L.

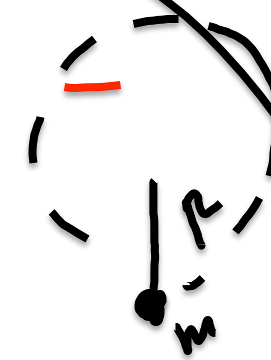
Four fast facts about angular momentum

1. $\mathbf{L} = I\boldsymbol{\omega}$
2. Angular momentum is a vector, pointing in the direction of the angular velocity.
3. If there is no net torque acting on a system, the system's angular momentum is conserved.
4. A net torque produces a change in angular momentum that is equal to the torque multiplied by the time interval during which the torque was applied.

Angular Momentum of a point-like object

$$L = pR = mv \cdot R$$





$$L = I \cdot \omega = mR^2 \cdot \frac{v}{R}$$

From the right triangle: $R = l \cdot \sin\theta$

l is the distance from the object to the axis

$$L = p \cdot l \cdot \sin\theta = mv \cdot l \cdot \sin\theta$$

Angular momentum

New variable => new name and new notation.

$$I\omega = L \qquad \textit{angular momentum}$$

General form of Newton's Second Law: $\sum \tau = \frac{\Delta L}{\Delta t}$ ⇒ $\Delta L = 0$

$L_i = L_f$

We can rewrite the law is the form:

$$\tau \Delta t = \Delta L$$

$\tau \Delta t$ is an ***angular impulse***.

Analogies

Second Law	$\Sigma \mathbf{F} = ma$	$\Sigma \boldsymbol{\tau} = I \boldsymbol{\alpha}$
Momentum	$\mathbf{p} = m\mathbf{v}$	$\mathbf{L} = I\boldsymbol{\omega}$
Impulse	$F\Delta t$	$\boldsymbol{\tau}\Delta t$
Second Law	$F_{Net}\Delta t = \Delta \mathbf{p}$	$\boldsymbol{\tau}_{Net}\Delta t = \Delta L$
Area	$F\Delta t = A_{\text{under the graph}}$	$\boldsymbol{\tau}\Delta t = A_{\text{under the graph}}$
Conservation	$F_{Net} = 0 \Rightarrow p = \text{const}$	$\boldsymbol{\tau}_{Net} = 0 \Rightarrow \mathbf{L} = \text{const}$

Newton's First Law for Rotation

Newton's first law: an object at rest tends to remain at rest, and an object that is spinning tends to spin with a constant angular velocity, *unless* it is acted on by a **nonzero net torque** or **there is a change in the way the object's mass is distributed**.

$$\tau_{Net} = 0 \quad \Rightarrow \quad L = \text{const}$$

$$L_{initial} = L_{final}$$

Figure Skater

A spinning figure skater starts spinning with her arms outstretched. When she moves her arms close to her body,

A) She spins faster.

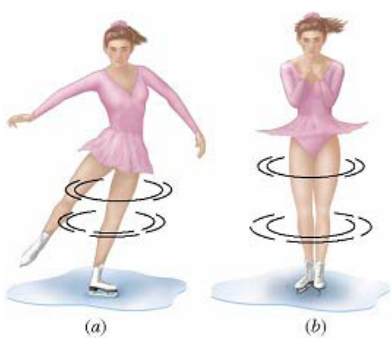
B) She spins slower.

C) The angular velocity stays the same.

<u>Figure Skater</u>

A spinning figure skater starts spinning with her arms outstretched. When she moves her arms close to her body, her moment of inertia decreases $I\downarrow$

Conserving angular momentum says:



$$L_i = L_f$$

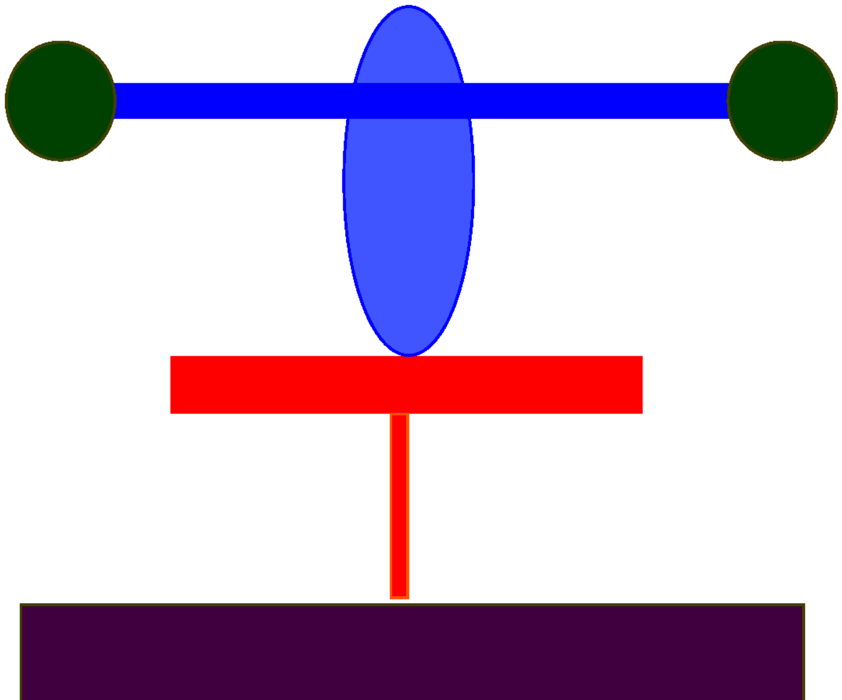
<u>or</u>

$$I_i\,\omega_i = I_f\,\omega_f$$

$$I_f < I_i \quad => \quad \omega_f > \omega_{fi}$$

A) She spins faster.

A person on the rotating stool



A spinning figure skater starts spinning with her arms outstretched. When she moves her arms close to her body,

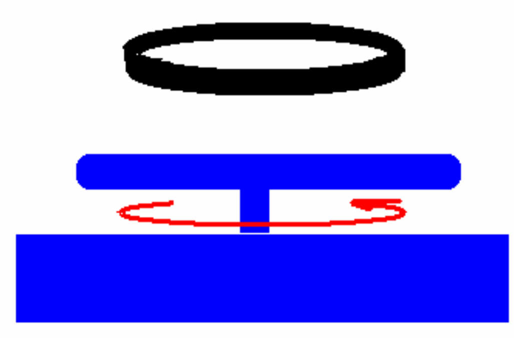
In this case the kinetic energy of the skater increases.

$$K_i = \tfrac{1}{2} I_i \omega_i^2 \qquad K_f = \tfrac{1}{2} I_f \omega_f^2$$

$$K_i < K_f$$

The figure skater does work on her arms and hands as she brings them closer to her body - that's where the extra energy comes from.

Disks – ring collision



A ring is falling down on the rotating turntable (a disk).

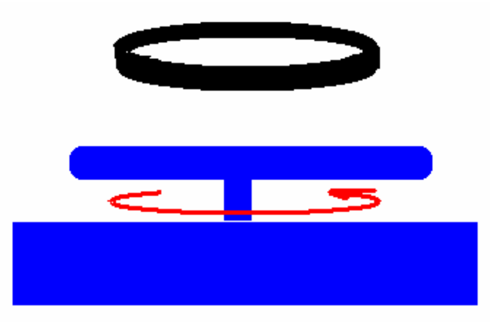
What is happening to the turntable?

A) It stops

B) It slows down

C) Nothing is happening

Disks – ring collision



A ring is falling down on the rotating turntable (a disk).

What is happening to the turntable?

B) It slows down

According to the law of conservation of angular momentum

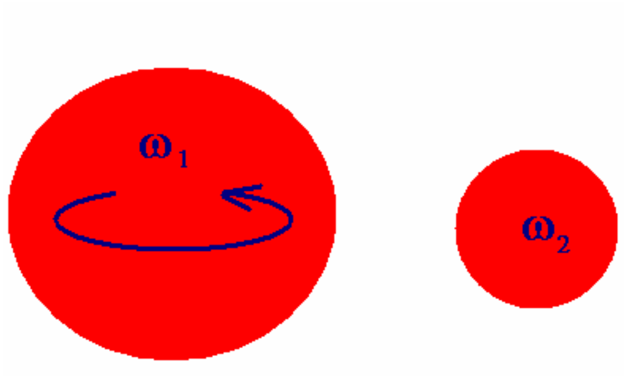
$$L_i = L_f$$

$$I_{turntable}\omega_{turntable-i} = (I_{turntable} + I_{ring})\omega_{turntable-f}$$

For this system: when $I\uparrow \Rightarrow \omega\downarrow$

(of course, we can drop just a sticky bal on the turntable, it is going to be the same situation)

An aging star



A star rotates making 50 revolutions a year.

After millions of years of shining its diameter becomes 50 times smaller but the mass still is almost the same.

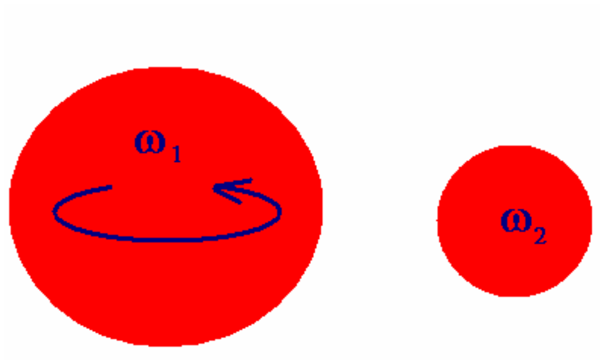
What is a new angular velocity of the star?

A) It is the same

B) It spins faster

C) It spins slower

An aging star



A star rotates making 50 revolutions a year.

After millions of years of shining its diameter becomes 50 times smaller but the mass still is almost the same.

What is a new angular velocity of the star?

Law of conservation of angular momentum $L_i = L_f$

$$I_{star-i}\,\omega_{star-i} = I_{star-f}\,\omega_{star-f}$$

$I_{star} = I_{spere} = \{2/5\}mR^2$ $m_i = m_f$ $R_i > R_f$

$I_{star-i} > I_{star-f}$ => $\omega_{star-i} <= \omega_{star-f}$

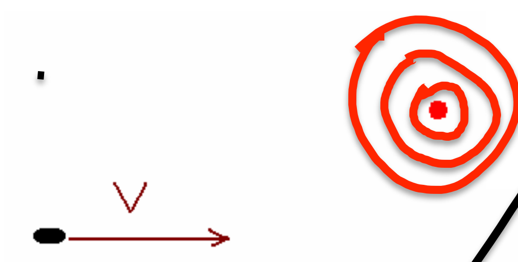
B) It spins faster

A bullet

A 9 gram bullet flying at the speed of 100 m/s missing a small target having the shortest distance between the target and the bullet of 10 cm.

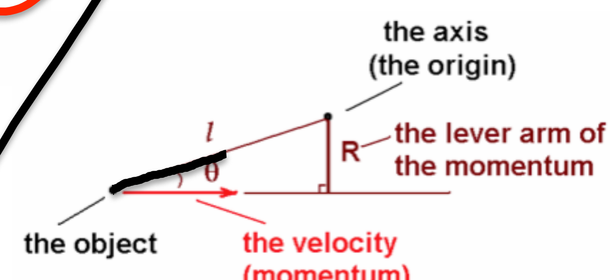
Find the angular momentum of the bullet relative to the target.

1. 0.09 kg m²/s
2. 0.9 kg m²/s
3. 9 kg m²/s
4. 90 kg m²/s



$$L = p \cdot l \cdot \sin\theta = mv \cdot l \cdot \sin\theta = m \cdot v \cdot R$$

$0.009 \cdot 100 \cdot 0.1$ (kg, m/s, m)



A bullet

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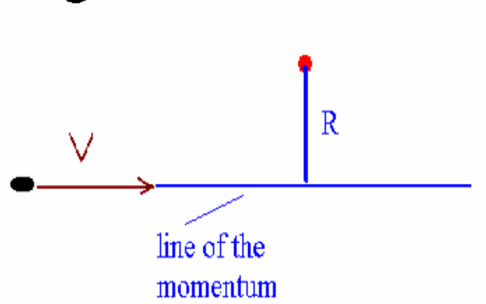


First, let's have all the variables converted into the Metric System:

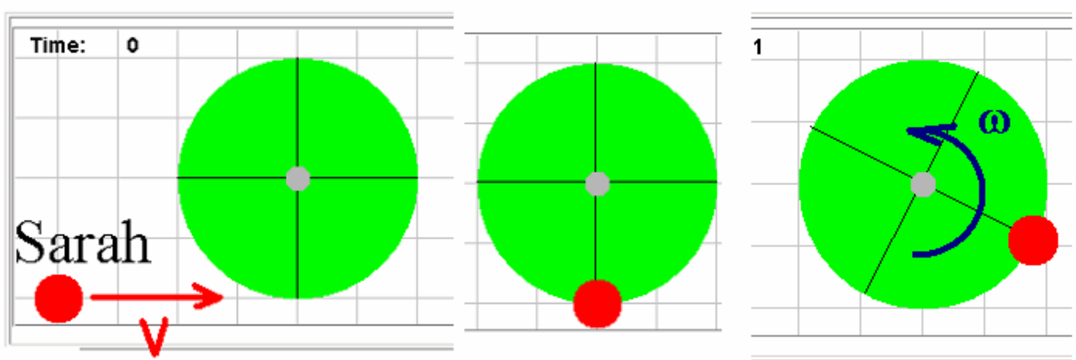
$m = 0.009$ kg, $R = 0.1$ m, $v = 100$ m/s

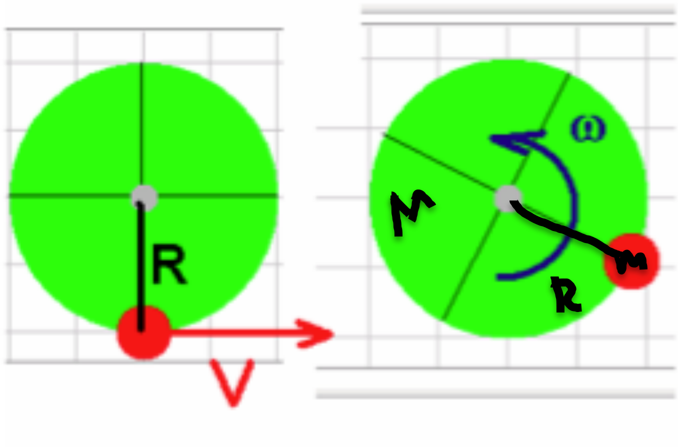
$$L = p \cdot l \cdot \sin\theta = mv \cdot l \cdot \sin\theta = mvR =$$
$$= 0.009 \cdot 100 \cdot 0.1 = 0.09 \text{ kgm}^2/\text{s}$$

The bullet has this angular momentum *at any* location!

<u>Jumping on a Merry-Go-Round</u>

Sarah, with mass m and velocity V, runs toward a playground merry-go-round, which is initially at rest, and jumps on at its edge. Sarah and the merry-go-round (mass M, radius R, and $I = cMR^2$) then spin together with a constant angular velocity ω_f. If Sarah's initial velocity is tangent to the circular merry-go-round, what is ω_f?





$$L = I \cdot \omega$$

$$m \cdot V \cdot R + 0 = m \cdot R^2 \cdot \omega + \frac{1}{2} M \cdot R^2 \cdot \omega =$$

$$= \left(m R^2 + \frac{1}{2} M R^2 \right) \omega$$

$$\omega_f = \frac{m\,v}{cMR + mR}$$

Numerical example:

Let's say: $m = 25$ kg; $v = 4$ m/s;

$M = 50$ kg; $R = 2$ m; $c = \frac{1}{2}$ (disk)

$$\omega_f = \frac{100}{50 + 50} = 1 \text{ rad/s}$$

$$\theta = \omega_0 \cdot t - \frac{\alpha t^2}{2}$$

$$0 = \omega_0 - \alpha t$$

If friction results in angular acceleration $a = -0.02$ rad/s^2, how much time passes until the disk stops?

$$\omega_f = \frac{100}{50 + 50} = 1 \text{ rad/s}$$

If friction results in angular acceleration a = -0.02 rad/s^2, how much time passes until the disk stops?

How many turns does it make before stops?

Analogies

Variable	Straight-line motion	Rotational motion	Connection
Displacement	x	θ	$\theta = x/r$
Velocity	v	ω	$\omega = v/r$
Acceleration	a	α	$\alpha = a_t/r$
Action	F	τ	$\tau = r F \sin\theta$
Inertia	m	I	$I = c\, mr^2$
Second Law	$\Sigma \mathbf{F} = ma$	$\Sigma \boldsymbol{\tau} = I\, \boldsymbol{\alpha}$	
Momentum	$\mathbf{p} = m\mathbf{v}$	$\mathbf{L} = I\boldsymbol{\omega}$	$L = r\, mv \sin\theta$
Impulse (II L)	$\Delta \mathbf{p} = \mathbf{F}_{Net}\, \Delta t$	$\Delta \mathbf{L} = \boldsymbol{\tau}_{Net}\, \Delta t$	
Kinetic Energy	$K = ½\, mv^2$	$K = ½\, I\omega^2$	
Work	$W = F\, d \cos\theta$	$W = \tau\, \Delta\theta \cos\theta$	
Power	$P = F\, v \cos\theta$	$P = \tau\, \omega \cos\theta$	
Area	$F\Delta t = A_{\text{under the graph}}$		$\boldsymbol{\tau}\Delta t = A_{\text{under the graph}}$
Conservation	$F_{Net} = 0 \;=>\; p = \text{const}$		$\boldsymbol{\tau}_{Net} = 0 \;=>\; \mathbf{L} = \text{const}$

Some tips on solving physics problems

1. Read the problem.
2. Read the problem again, slowly, make a picture when reading, reflect in the picture all the important pieces of information (objects, locations of the objects, states of the objects, processes, properties of the objects and processes, instants, distances, variables)
3. Choose the approach you want to try first:
   * Newton's second law for a translational or rotational motion.
   * Law of conservation of energy.
   * Law of conservation of linear or angular momentum.
4. Write the general equations for the chosen approach.
5. Write the same equations in terms of the variables specific for the problem (chose the reference frame, the axis of rotation, use specific notations for masses, forces, distances, times, velocities, accelerations, angles, etc)
6. Think of additional connections, write additional equations (definitions, kinematical equations for v = const or a = const, connections between linear and rotational variables, geometrical connections like sin or cos or tan, etc.)
7. Try to solve the system of your equations, treat all the variables as known.
8. If not succeed, go back to the # 4 and try another approach.
9. If not succeed, go back to # 2 and try a better picture.
10. If not succeed, use office hours, talk to a TF or a professor.