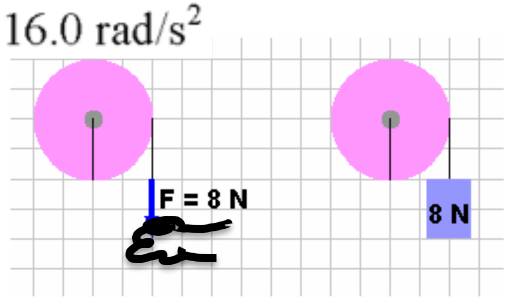
Change....Or Not?



We take two identical pulleys (M = 2.0 kg and radius R = 0.50 m), both with string wrapped around them.

On the one on the left we apply an 8 N force to the string.

On the one on the right we hang an object with a weight of 8 N.

Which pulley has the larger angular acceleration?

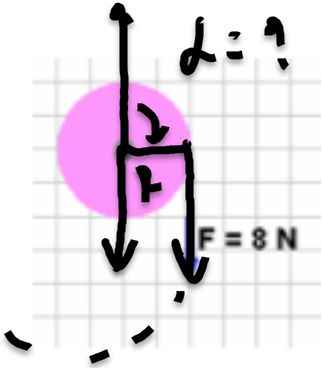
A) The one on the left

B) The one on the right

C) Neither, they're equal

1. Yes

2. No



$R = 0.50\ m = l$

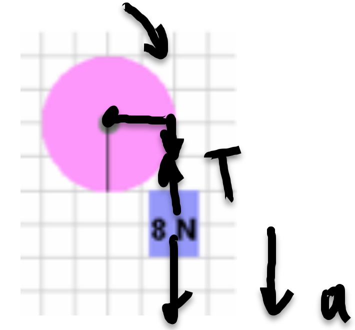
$M = 2.0\ kg$

$-\tau_{net} = -I \cdot \alpha$

$8 \cdot 0.5 = \frac{1}{2} \cdot 2 \cdot 0.5^2 \cdot \alpha$

$16 \frac{rad}{s^2} = \alpha$

$x^2 + y^2 = z^2$



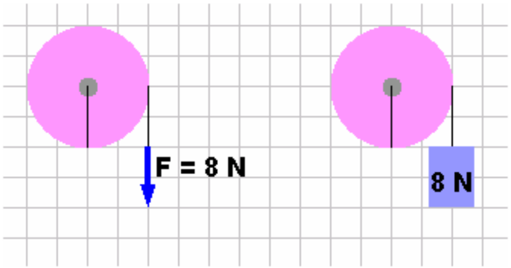
$mg = 8\ N$

$y: \ m \cdot a = mg - T$

$T = mg - ma = 8 - ma$

$-T \cdot R = -I \cdot \alpha$

Change....Or Not?



The one on the left has the angular acceleration of 16.0 rad/s^2 (we just found it). The right pulley has the tension in the string *less* then 8 N, hence will have the smaller angular acceleration (the rest of the parameters is the same!).

A) The one on the left

For the right pulley we can write: $\alpha = \tau/I = rT/(\frac{1}{2} Mr^2)$

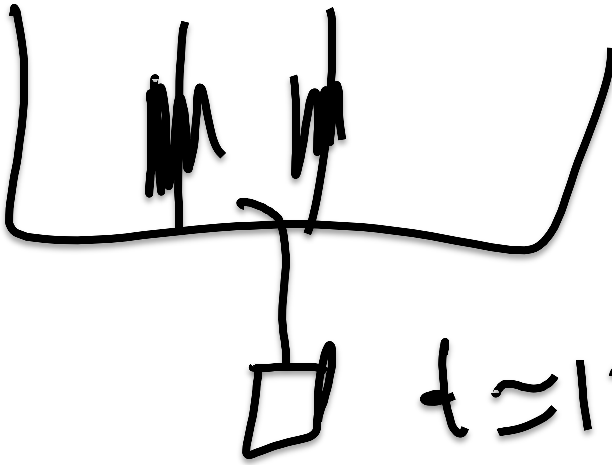
And for the block: mg - T = ma or T = mg - ma = 8 - ma

<u>We can even solve it for α!</u>

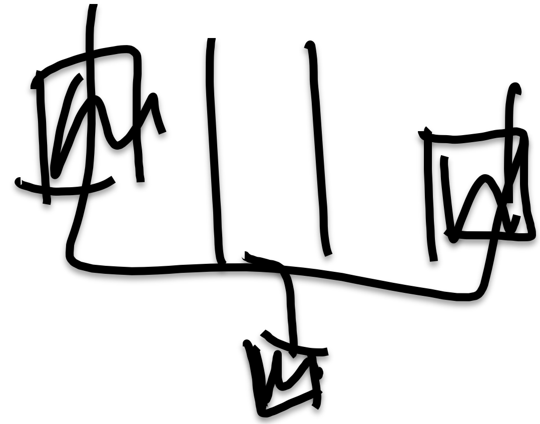
$W = mg \qquad a = \alpha r \qquad T = mg - ma = mg - m\alpha r$

$$\alpha = \tau/I = rT/(\tfrac{1}{2} Mr^2) = r(mg - m\alpha r)/(\tfrac{1}{2} Mr^2)$$

$$\alpha = \frac{1}{1+2\frac{m}{M}} \bullet \frac{2mg}{Mr} = \frac{1}{1+2\frac{W}{Mg}} \bullet \frac{2W}{Mr} = \frac{1}{1+2\frac{8}{2\cdot 9.8}} \bullet \frac{2\cdot 8}{2\cdot 0.5} = \frac{16}{1.8163} = 8.81 \text{ rad/s}^2$$



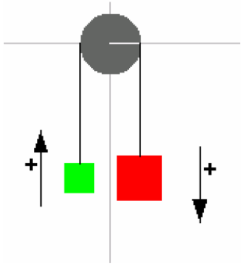
$t \simeq 13s$



$$t_2 \;\begin{matrix} > \\ = \\ < \end{matrix}\; t_1$$

1,
2,
3,

<u>Atwood's machine re-visited</u>



Atwood's machine is a device where two masses, M and m, are connected by a string passing over a pulley. Assume that M > m.

The pulley is *a solid disk* of mass m_p and radius r.

What is the acceleration of the two masses?

Newton's second law for *each* object.

For mass M:	For mass m:	For the pulley:
$\Sigma \mathbf{F}_y = M\mathbf{a}_y$	$\Sigma \mathbf{F}_y = m\mathbf{a}_y$	$\Sigma \boldsymbol{\tau} = I\, \boldsymbol{\alpha}$
$Mg - T_1 = Ma$	$T_2 - mg = ma$	$rT_1 - rT_2 = \frac{1}{2} m_p r^2 \boldsymbol{\alpha}$

The kinematical connection: $\alpha = a/r$

From:

$$Mg - T_1 = Ma \quad | \quad T_2 - mg = ma \quad | \quad rT_1 - rT_2 = \tfrac{1}{2} m_p r^2 \boldsymbol{\alpha}$$

And: $\alpha = a/r$

We have:

$$Mg - T_1 = Ma \quad | \quad T_2 - mg = ma \quad | \quad T_1 - T_2 = \tfrac{1}{2} m_p a$$

Combining the three equations to eliminate the two tensions gives:

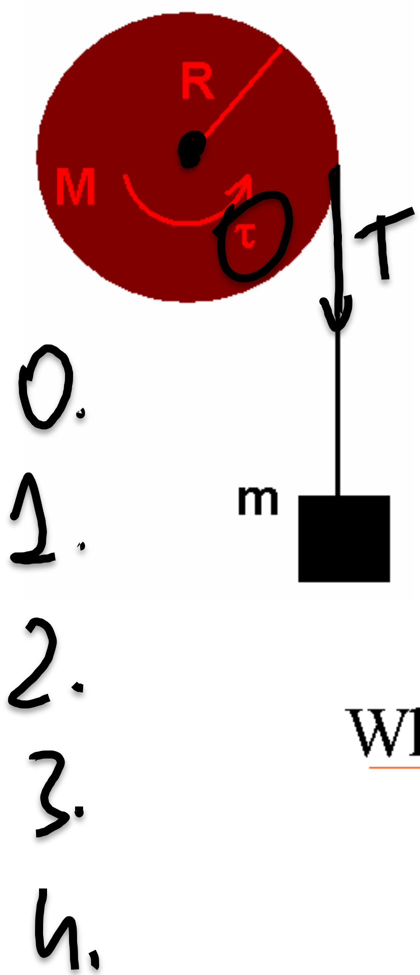
$(Mg - Ma) - (mg + ma) = \tfrac{1}{2} m_p a$

$Mg - mg = Ma + ma + \tfrac{1}{2} m_p a = (M + m + \tfrac{1}{2} m_p)a$

$$a = \frac{g\,(M - m)}{M + m + \tfrac{1}{2} m_p}$$

This is smaller then if the pulley were massless ($m_p = 0$)

A motor, a disk and a crate

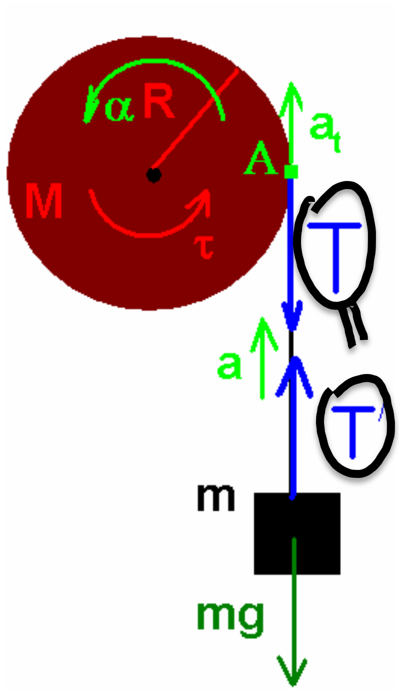


A motor applies a torque τ to a pulley, which is a solid disk of the mass M and the radius R.

A massless cord is wrapped up around the pulley and the crate of the mass m is attached to its end.

What is the acceleration of the crate?

A motor, a disk and a crate



A motor applies a torque τ to a pulley, which is a solid disk of the mass M and the radius R.
A massless cord is wrapped up around the pulley and the crate of the mass m is attached to its end.
What is the acceleration of the crate?

Two massive objects are moving:
We need to write two Newton's laws.

For the crate: $ma = T' - mg$

For the pulley: $I\alpha = \tau - TR$

(remember, there is a motor hiding behind the pulley which makes the pulley rotating!)

$T = T'$

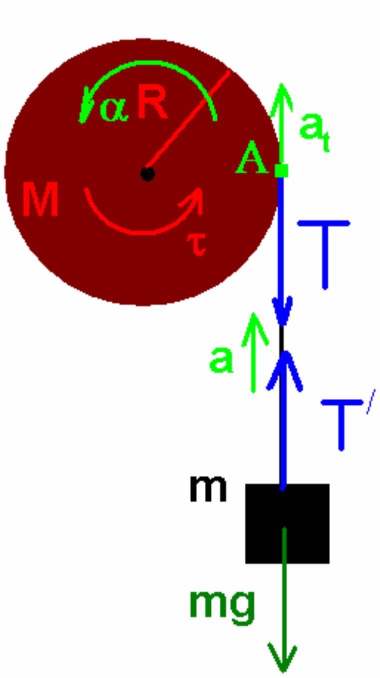
$ma = T - mg$

$I \cdot \frac{a}{R} = \tau - T \cdot R$

Now, either we can solve the equations and find the unknown, or not.

If not, that means we need more connections, more equations.

A motor, a disk and a crate



$ma = T' - mg$

$I\alpha = \tau - TR$

For a solid cylinder: $I = \frac{1}{2}MR^2$

For the magnitudes of the tension forces: $T = T'$

Linear acceleration of the crate is equal to the tangential (linear) acceleration of the cord (at the point A, or any other) $a = a_t$

For rotational and tangential accelerations: $a_t = \alpha R$

Now the system is complete and solvable!

$$a = \frac{\tau - mgR}{0.5MR + mR}$$

Rotational Work

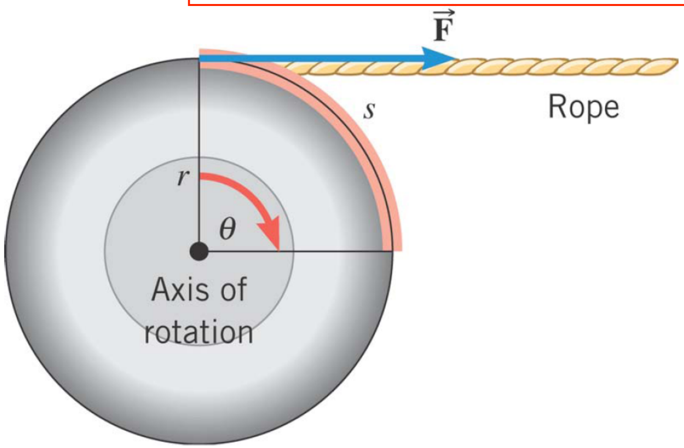
$$s = r\,\Delta\theta$$

$$W = Fs = Fr\,\Delta\theta$$

$$\tau = Fr$$

$$W = \tau\,\Delta\theta$$

A force does a work,
So does a torque.
© VV



Requirement: The angle must be expressed in radians.

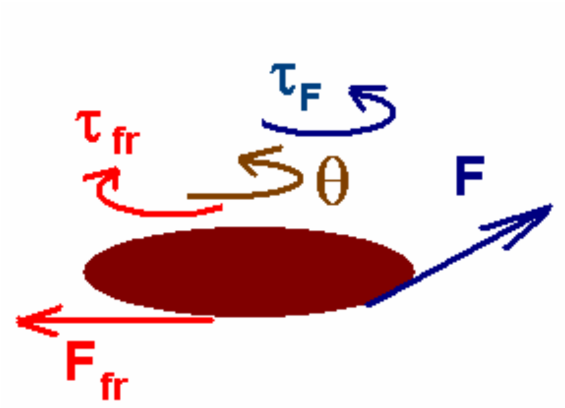
SI Unit of Rotational Work: joule (J)

Work

In a rotational situation, for a constant torque,

$$W = \tau \, \Delta\theta \cos\theta$$

In our problems $\cos\theta$ can be equl to 1 or -1 only!



When the torque is in the same direction as the displacement the work of the torque is positive (accelerating force create positive torque).

When the torque is opposite to the displacement the work of the torque is negative (friction creates negative torque).

Power

Power is the rate at which work is done.

$$P = \frac{W}{\Delta t} = \frac{\tau \Delta\theta \cos\theta}{\Delta t} = \tau\omega\cos\theta$$

For a rotation object: $P = \tau\ \omega\ \cos\theta$ (when τ is const)

$P = \tau_{ave}\ \omega_{ave} \cos\theta$ (when τ is not const)

(here τ and ω are absolute values)

θ is the angle between the torque and the angular velocity.

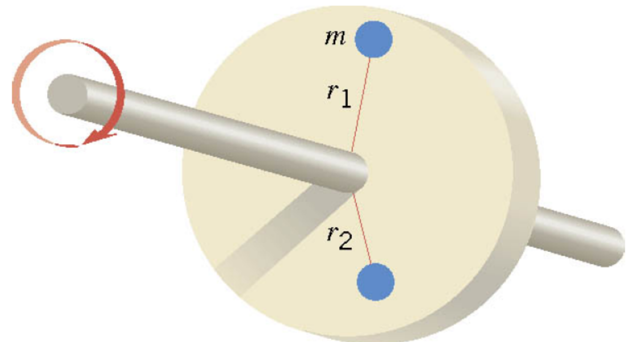
Normally: $\cos\theta = 1$ or -1

Rotational Energy

For a point-like object

$$KE = \tfrac{1}{2} m v_T^2 = \tfrac{1}{2} m r_1^{\,2} \omega^2$$

$$v_T = r_1 \omega$$



For a solid object

$$KE = \sum \left(\tfrac{1}{2} m r^2 \omega^2\right) = \tfrac{1}{2}\left(\sum m r^2\right)\omega^2 = \tfrac{1}{2} I \omega^2$$

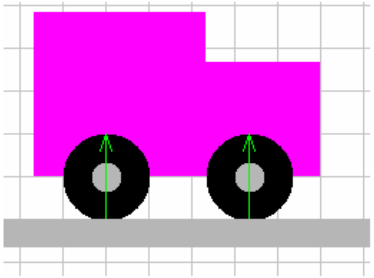
How would you write the Work – Kinetic Energy theorem for a rotating object?

Analogies

Variable	Straight-line motion	Rotational motion	Connection
Displacement	x	$\Delta\theta$	$\Delta\theta = \frac{x}{r}$
Velocity	v	ω	$\omega = \frac{v}{r}$
Acceleration	a	α	$\alpha = \frac{a_t}{r}$
Produces motion	F	τ	$\tau = r F \sin\theta$
Inertia	m	I	
Second Law	$\Sigma \mathbf{F} = ma$	$\Sigma \boldsymbol{\tau} = I \boldsymbol{\alpha}$	
Kinetic Energy	$K = \frac{1}{2} mv^2$	$K = \frac{1}{2} I\omega^2$	
Work	$W = F\, d \cos\theta$	$W = \tau\, \Delta\theta \cos\theta$	
Power	$P = F\, v \cos\theta$	$P = \tau\, \omega \cos\theta$	

An accelerating car

(have we seen this before?)



A front-wheel drive car accelerates from rest.

The force of friction plays a crucial role in the process of the motion.

If there is no slipping between your car tires and the road, in what direction is the force of friction from the road acting on the front tires?

A) The force of friction acting in the direction of the traveling.

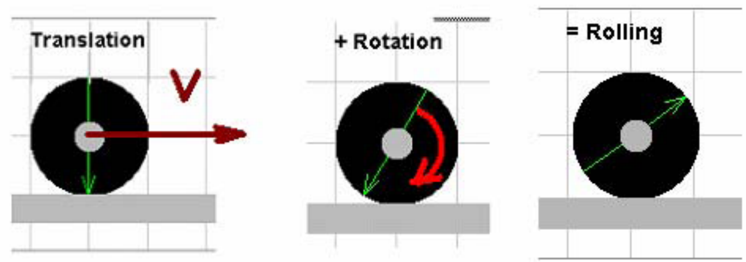
B) The force of friction acting opposite to the direction of traveling.

1. St

2. K.

Circular motion + static friction = translational motion + rolling!

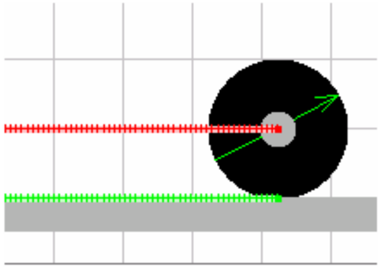
Rolling



Rolling can be viewed as a **combination** of two separate motions, a purely **translational** motion and a purely **rotational** motion.

Rolling involves both of these at the same time - rotation while the wheel is experiencing straight-line motion.

Rolling



When a wheel rolls **without slipping**, the straight-line distance traveled by the wheel's center-of-mass (in red on the simulation) is exactly equal to the rotational distance traveled by a point on the edge of the wheel (in green).

$d_{cm} = d_{rs}$

If the wheel has a constant angular velocity, the rotational speed is:

$$v_r = \frac{2\pi r}{T}$$

Because the distances and times are equal, the translational speed of the center of the wheel equals the rotational speed of a point on the edge of the wheel.

$$V_r = V_{CoM}$$

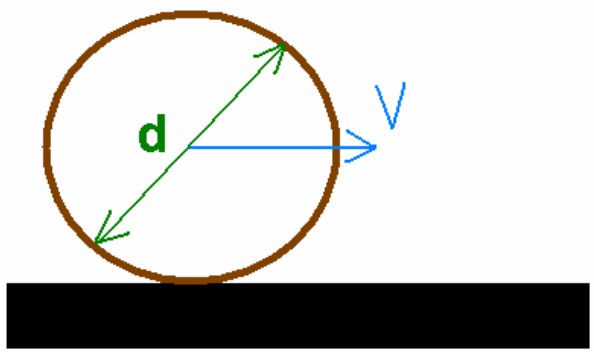
1. 1 m
2. < 1 m
3. > 1 m

We say "acceleration" we mean "the acceleration of the center of mass"!

Problem

A wheel of a bicycle has the diameter of 80 cm.

If the bicyclist rides the bicycle at the speed of 18 km/h, how many turns does the wheel make for 30 minutes?



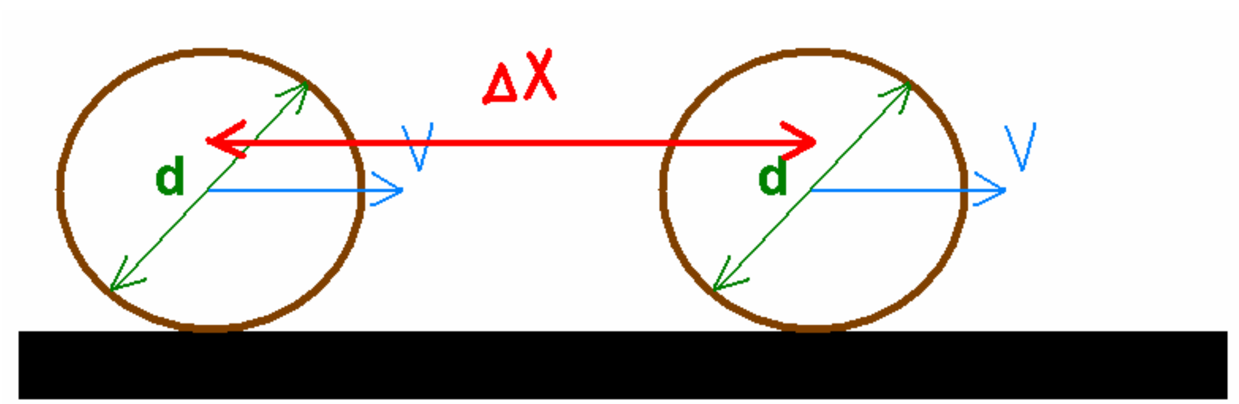
There is NO slipping between the wheels and the ground!

Solution

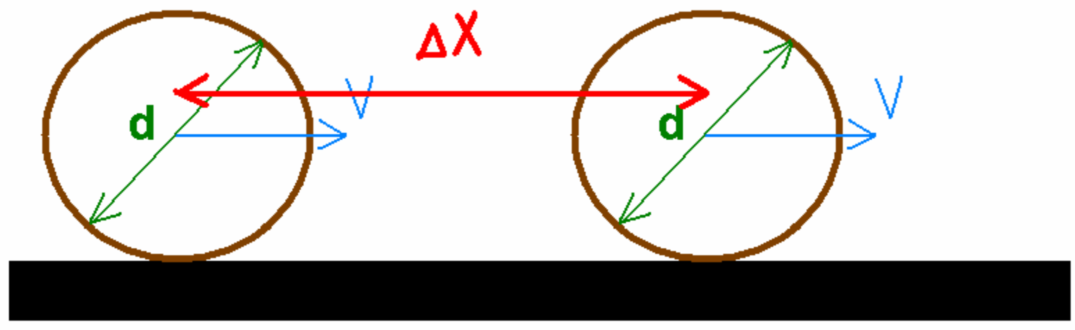
$R = \frac{1}{2}\, d = 0.4$ m. $\quad v_{center\text{-}wheel} = 18$ km/h $= 5$ m/s

$\Delta t = 30$ min $= 1800$ s

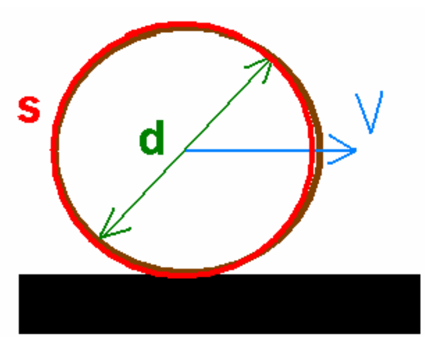
$\Delta x = v_{center\text{-}wheel} * \Delta t = 5 * 1800 = 9000$ m.



Solution



$$\Delta x = v_{\text{center-wheel}} * \Delta t = 5*1800 = 9000 \text{ m}$$



$$\mathbf{S = \Delta x}$$

The number of turns is

$$N = S/[2\pi r] = 9000/[2\pi 0.4] = 3580.9863$$

Example

If we pull the string, in which direction is the spool going to roll (no slipping)?

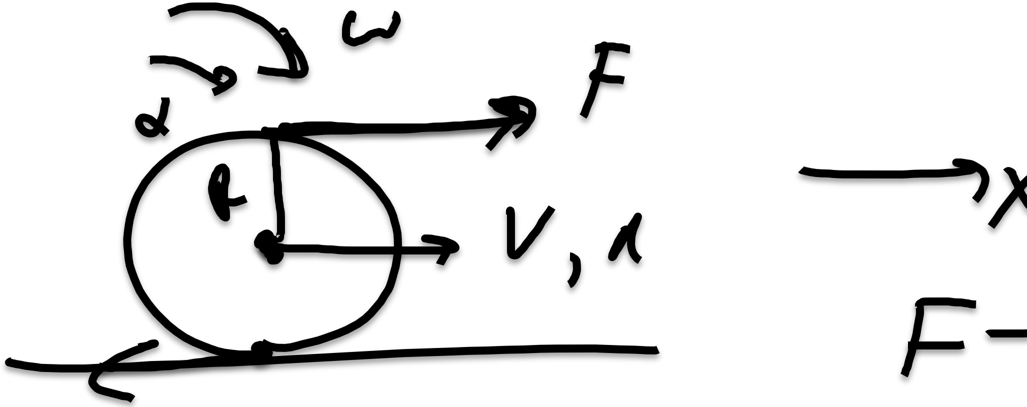
A) to the right

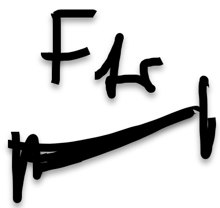
B) to the left

C) it won't move

If we pull the string, in which direction does the force of friction point (no slipping)?

If we pull the string, find the acceleration of the spool (no slipping)?





$$F - F_{fr} = M \cdot a$$

$$-F \cdot R - F_{f} \cdot R = -I \cdot \alpha$$

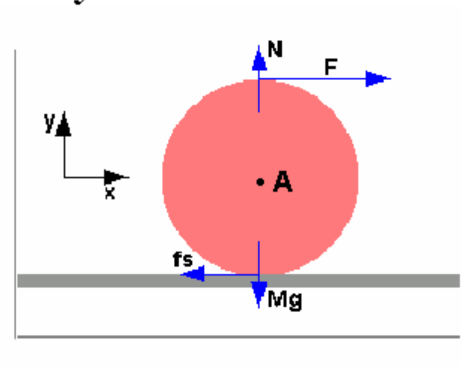
$$\alpha = \frac{a}{R}$$

$$I = \frac{1}{2} M R^2$$


F

Sample Problem

A cylinder of mass M and radius R has a string wrapped around it, with the string coming off the cylinder above the cylinder. If the string is pulled to the right with a force F, what is the acceleration of the cylinder if the cylinder rolls without slipping?



Let's draw FBD. Static friction is involved! We have to write the N II L for translation and rotational motion (relative to the center of the cilinder)!

$$F - f_s = Ma \quad | \quad - RF - Rf_s = -\tfrac{1}{2} MR^2\alpha$$

With no slipping $\quad a = a_t = \alpha \cdot R.$

Now we can solve the system for a.

$$F - f_s = Ma \quad | \quad - F - f_s = -\tfrac{1}{2} Ma$$

First we solve it for the force of friction f_s:

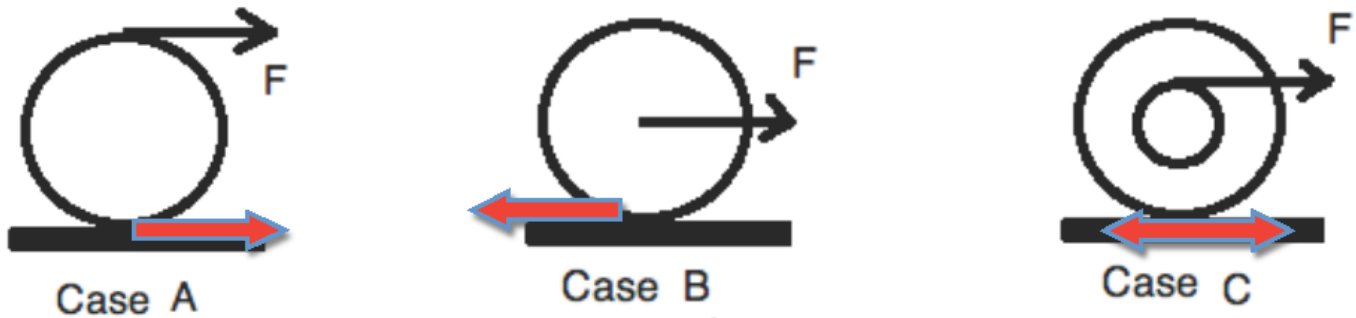
$$f_s = - \frac{Ma}{4}$$

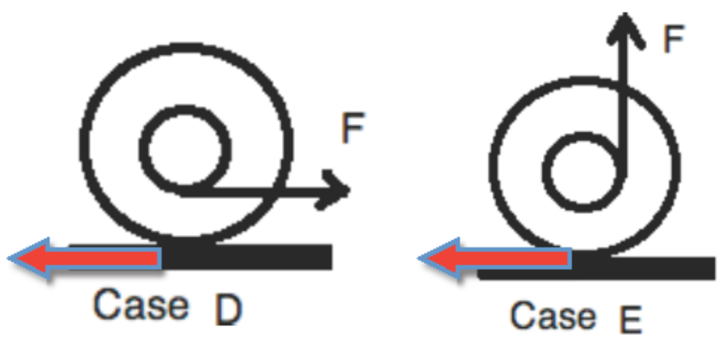
That means that the actual direction of the force of friction is to **the right**!

Now we can find the acceleration:

$$a = \frac{4\ F}{3\ M}$$

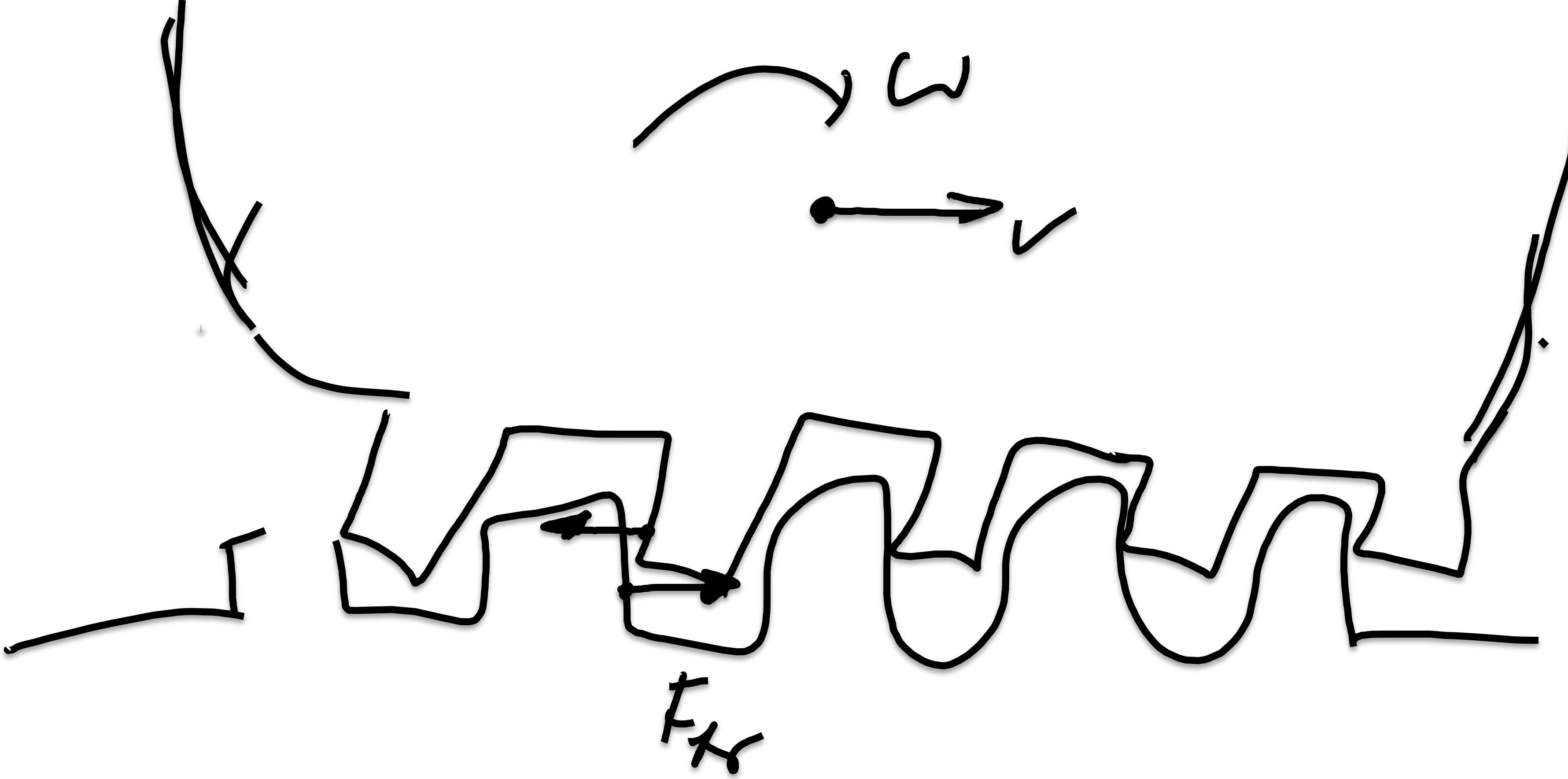
Pictures show a spool with a force F applied to it.
The spool rolls without slipping.





(the direction of friction depends on the ratio of the raddi)

The red arrow shows the direction of the force of friction.


ω
v
Fhr

A sphere is released form rest and is rolling down an incline with no slipping.

The direction of the force of friction acting on the sphere is …

1. Up the incline

2. Down the incline

3. not enough information