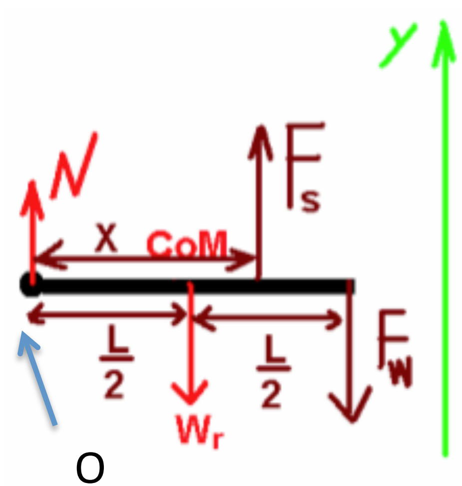
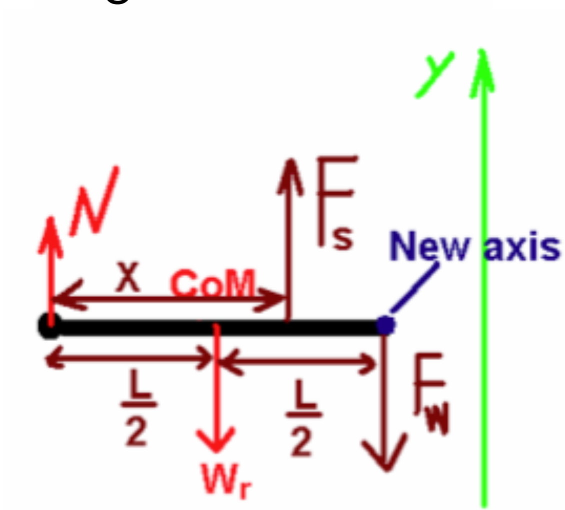
$$N + F_s - W_r - F_W = 0$$



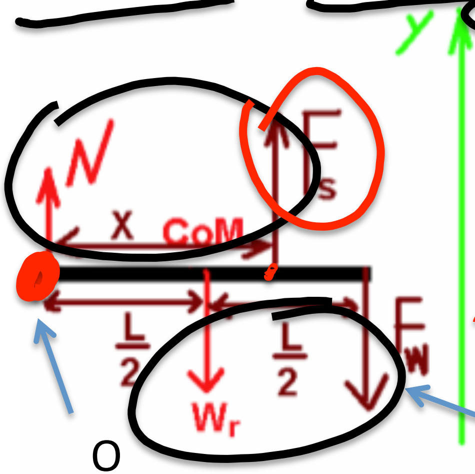
$$W_r = M_{rod} * g \qquad F_W = m_{weight} * g$$

$$N*0 + F_s*X - W_r*(L/2) - F_W*L = 0$$



$$-N*L + W_r*(L/2) - F_s*(L - X) + F_W*0 = 0$$

$$N + F_s - W_r - F_W = 0$$

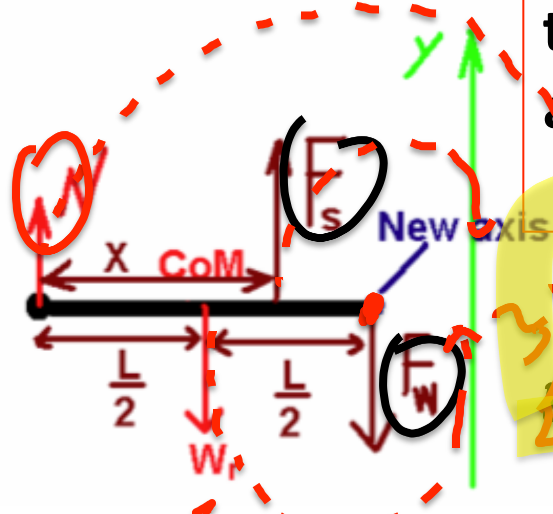


$$W_r = M_{rod} * g \qquad F_W = m_{weight} * g$$

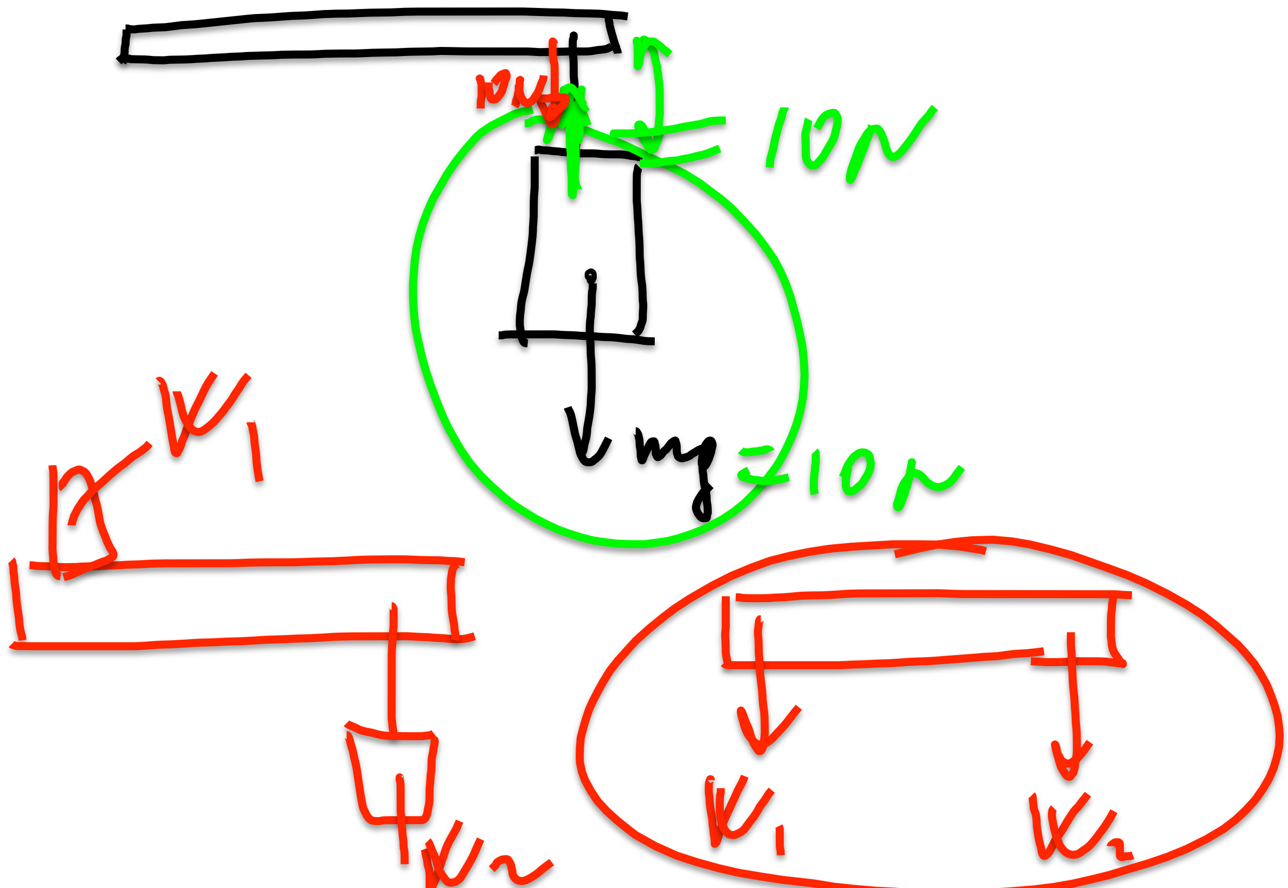
$$N*0 + F_s*(X) - W_r*(L/2) - F_W*L = 0$$

F_w is acting on the weight. Which law says that the force with exactly same magnitude is acting on the rode?

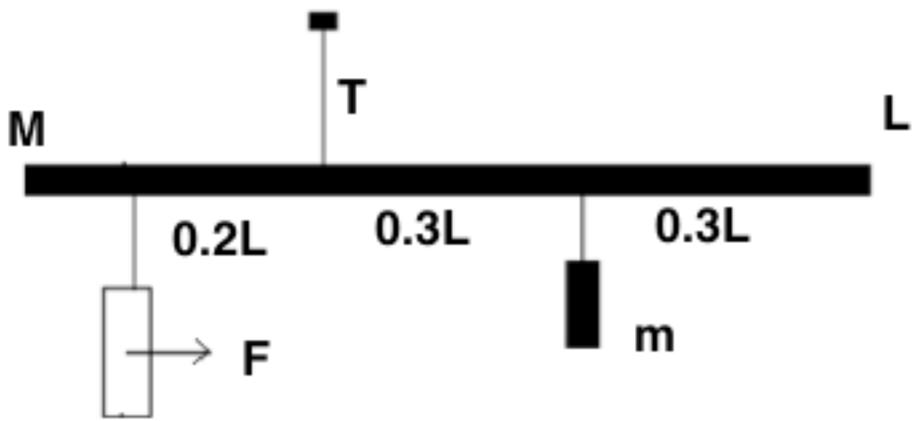
1. 1 st N L 2. 2nd N L 3. 3d N L



$$-N*L + W_r*(L/2) - F_s*(L - X) + F_W*0 = 0$$


10N
10N
mg=10N
K_1
K_2
K_1
K_2

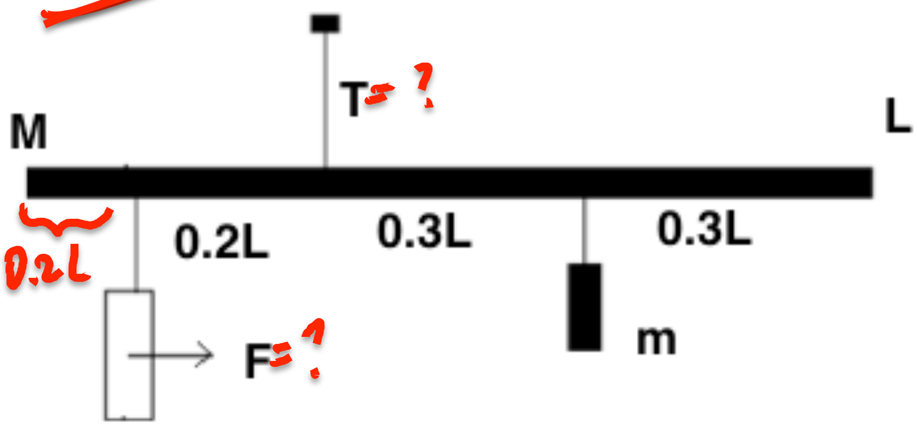
A rod on a pivot



1. Draw a FBD
2. Choose X and Y axes
3. Write x: FBE
4. Write y: FBE
5. Draw lines of action
6. Choose O
7. Show l's
8. Write torques
9. Write O: TBE
10. Repeat from 6 (if needed)

M, m, L

A rod on a pivot



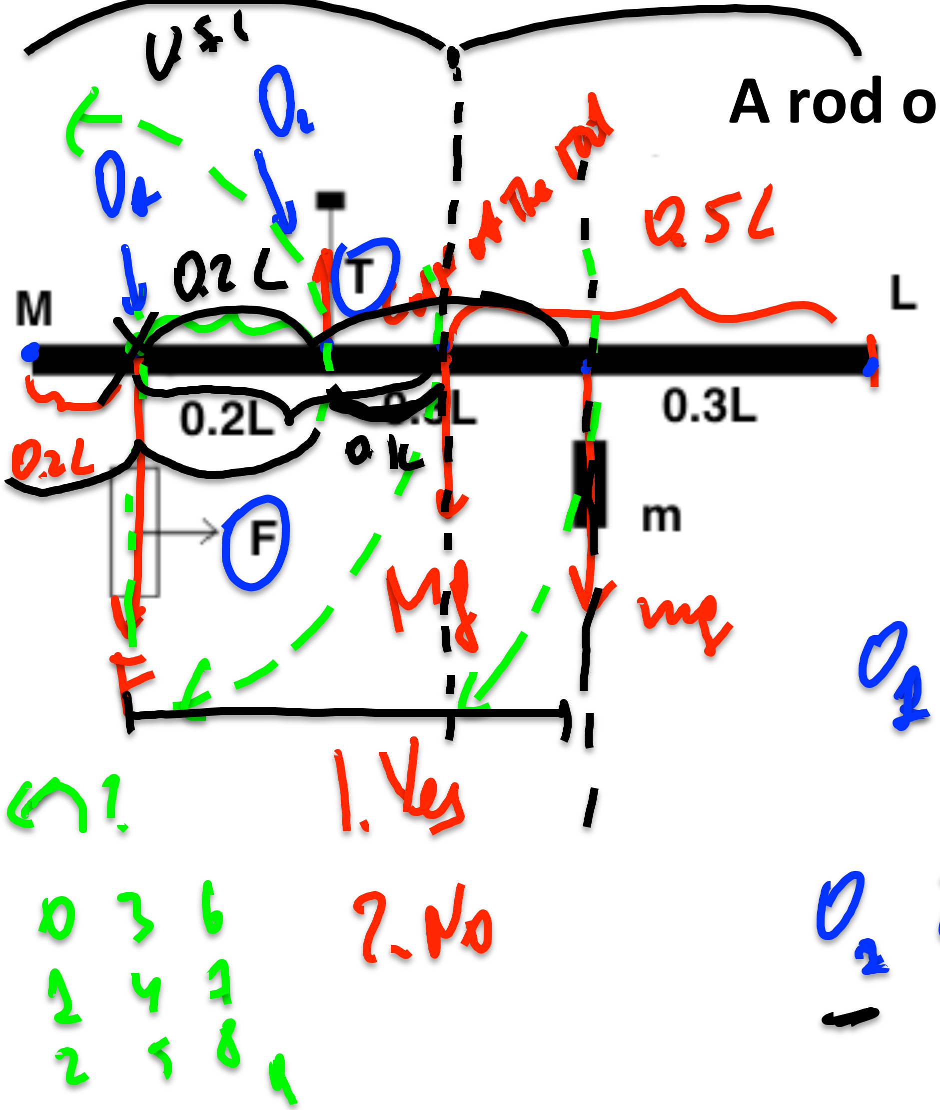
? $T = mg + F$

? $t_F = 0.9 * F * L$

1. Yes
2. No
3. Not enough informtion

1. Draw a FBD
2. Choose X and Y axes
3. Write x: FBE
4. Write y: FBE
5. Draw lines of action
6. Choose O
7. Show ℓ's
8. Write torques
9. Write O: TBE
10. Repeat from 6 (if needed)

A rod on a pivot



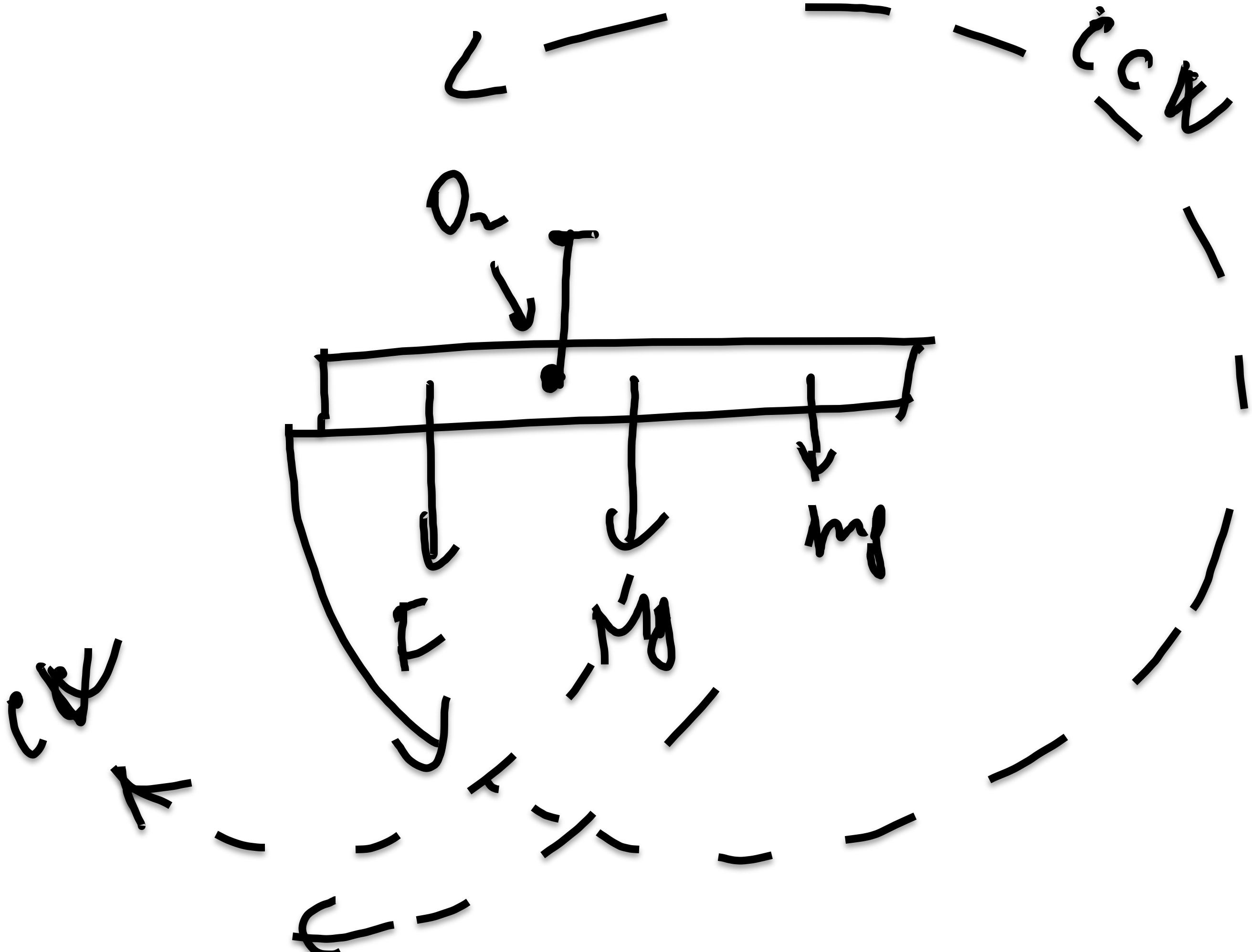
1) $T = F + Mg + mg$

$T = ?$ $F = ?$

2) $T \cdot 0.2L = Mg \cdot 0.3L + mg \cdot 0.5L$

$T = Mg\frac{3}{2} + mg\frac{5}{2}$

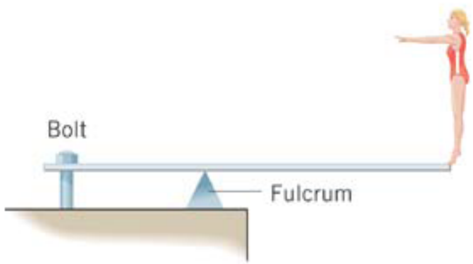
2) $F \cdot 0.2L = Mg \cdot 0.1L + mg \cdot 0.3L$



CCW
O₂
mg
F
Mg
CW

A Diving Board

A woman whose weight is 530 N is poised at the right end of a diving board with length 3.90 m.



(a)

The board has the mass of 12 kg and is supported by a fulcrum 1.40 m away from the left end.

Find the forces that the bolt and the fulcrum exert on the board.

A woman whose weight is 530 N is poised at the right end of a diving board with length 3.90 m.

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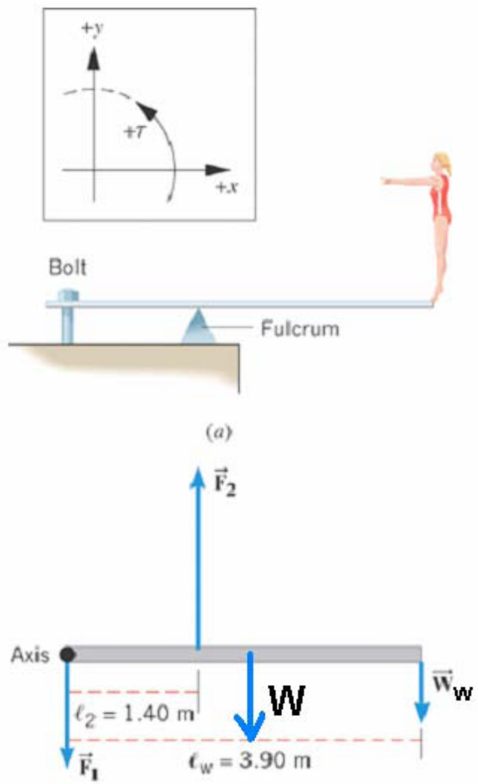
A Diving Board



(b) Free-body diagram of the diving board

$W_W = 530$ N $\quad l_W = 3.9$ m

$M = 12$ kg $\quad (W = 12*9.8 = 117.6$ N$)$,

$l_2 = 1.4$ m

$F_1 = ? \quad F_2 = ?$

From the balance of forces condition:

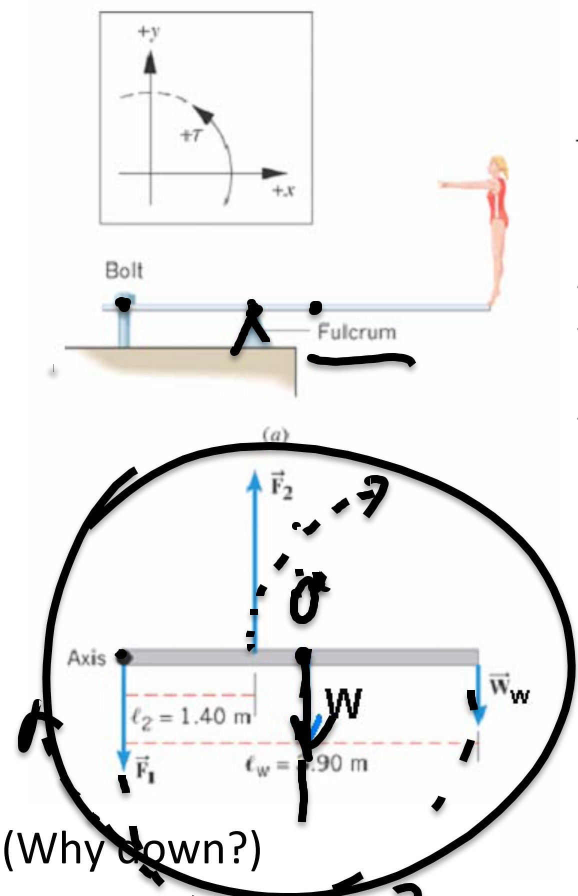
$$F_2 + W = F_1 + W_W$$

From the balance of torques condition (written relative to the bolt):

~~F_1*0~~ $+ F_2*l_2 - W*(l_W/2) - W*l_W = 0$

The second equation gives as F_2 and then the first equation gives F_1.

A Diving Board



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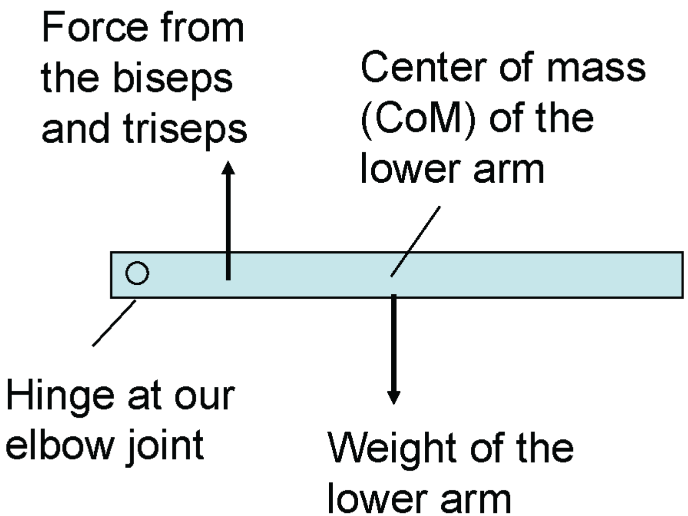
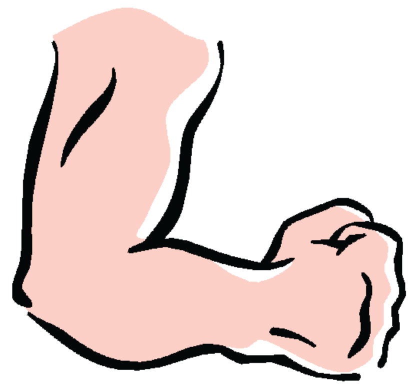
~~F_1*0~~ $+ F_2*l_2 - W*(l_W/2) - W*l_W = 0$

The second equation gives as F_2 and then the first equation gives F_1.

Example : Model of our lower arm

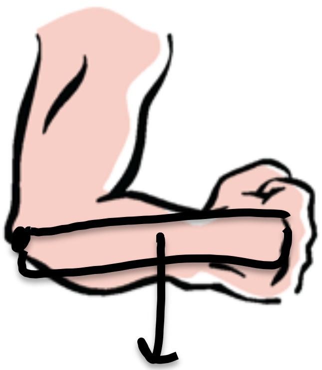
This is a model of our lower arm, with the elbow being the hinge.

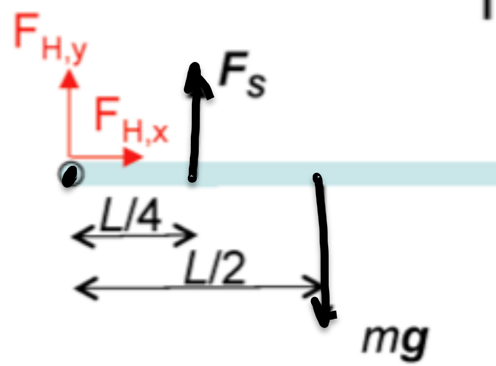




An equilibrium example

The rod is held horizontal by an upward force applied by a spring scale ¼ of the way along the rod.





$F_{Hx} =$ $F_{Hy} =$

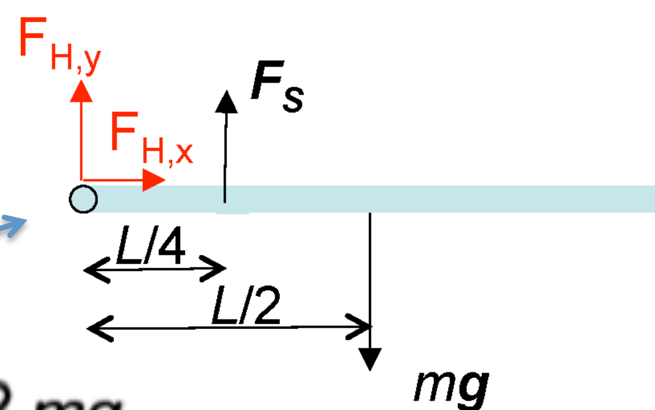
1. -2 mg
2. -mg
3. 0
4. Mg
5. 2mg

Example Model of our arm (cont'd)

Draw a free-body diagram for a horizontal rod that is hinged at one end. The rod is held horizontal by an upward force applied by a spring scale ¼ of the way along the rod.

How does the hinge force (F_H) and reading on the spring scale (F_S) compare to the weight of the rod if the rod is at equilibrium?

Let F_H be the hinge force, and we decompose it into $F_{H,x}$ and $F_{H,y}$ along the x and y direction, respectively.



$$\frac{L}{4} \times F_S = \frac{L}{2} \times mg$$

$$F_S = 2\,mg$$

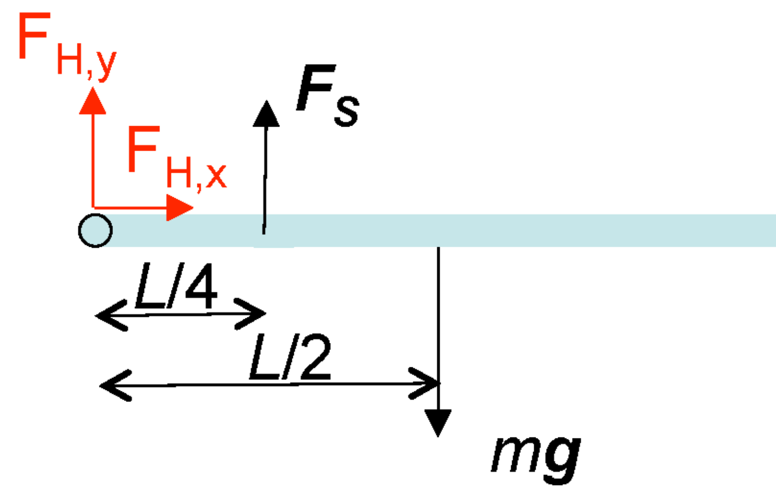
To find the hinge force, we can applied $\Sigma F_x = 0$ and $\Sigma F_y = 0$ to the system.

$$\Sigma F_x = 0 \Rightarrow F_{Hx} = 0$$

$$\Sigma F_y = 0 \Rightarrow F_{Hy} + F_S = mg$$

$$\Rightarrow F_{Hy} = mg - F_S$$

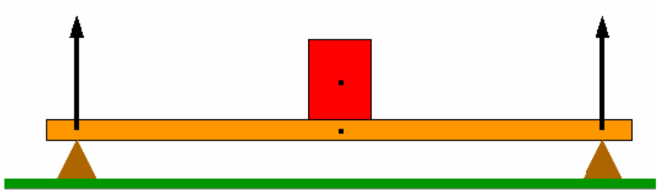
$$\Rightarrow F_{Hy} = -mg$$



The negative sign means that the hinge force is actually pushing down, i.e., directed opposite to what is drawn for F_{Hy} in the above picture.

Equilibrium

Example - a board and mass



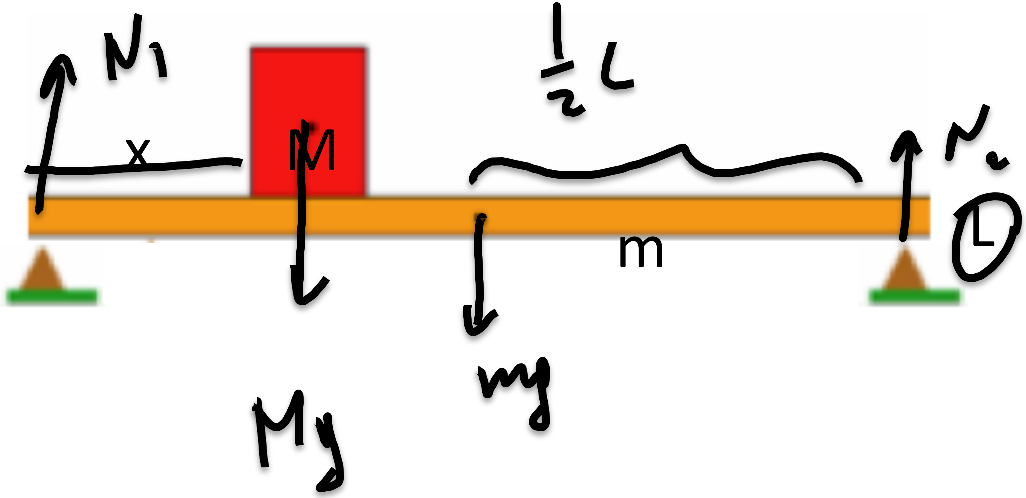
A board has a box in the middle. The board is supported by two supports, one on the right and one on the left. If the box is moved to the left what will happen to the two support forces?

A) The force from the left support increases and the force from the right support decreases

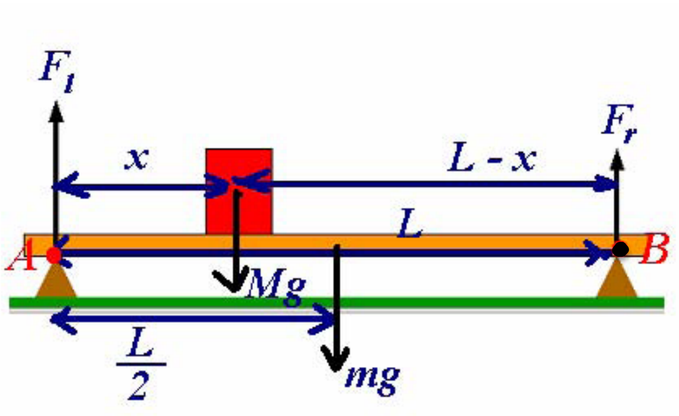
B) The force from the left support increases and the force from the right support stays the same

C) The force from the left support decreases and the force from the right support increases

D) The force from the left support decreases and the force from the right support stays the same


N1
x
M
1/2 L
N2
m
L
Mg
mg

Example - a board and mass



The force from the left support increases and the force from the right support decreases

Let's prove it mathematically.

Set the axis at the point A: $\sum\tau = 0$

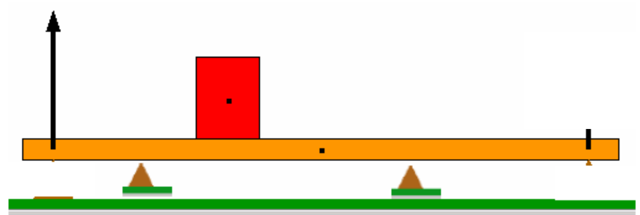
$Mgx + mgL/2 = F_r L$ or $F_r = \{Mgx + \frac{1}{2}mgL\}/L$ $x\downarrow \Rightarrow F_r\downarrow$

Set the axis at the point B: $\sum\tau = 0$

$mgL/2 + Mg(L - x) = F_l L$ or $F_l = \{\frac{1}{2}mgL + Mg(L - x)\}/L$

$x\downarrow \Rightarrow F_l\uparrow$

Example · a board and mass

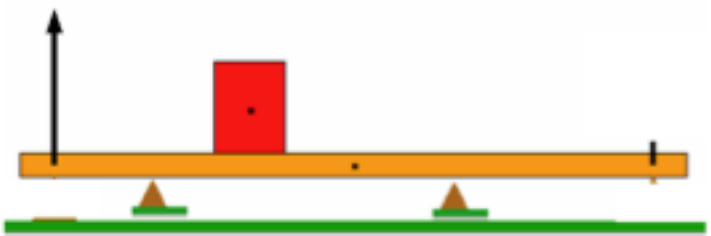


A board has a box   The board is supported by two supports

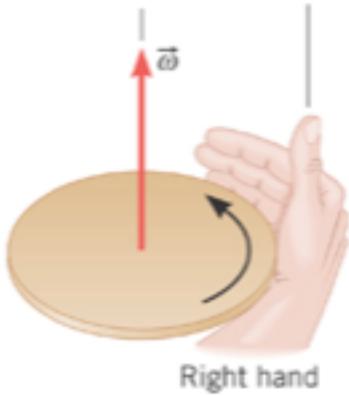
Find the force we should apply to the right (left) end of the board to lift it off the right (left) fulcrum.

If we attach two cables to the ends of board, what forces should be applied in order to lift the board up keeping it horizontal?

If we had the cables attached to the same crane hook, which cable would be longer?



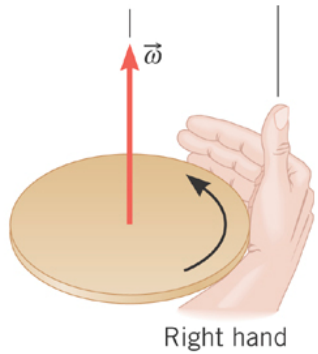
Find the force we should apply to the right (left) end of the board to lift it off the right (left) fulcrum.



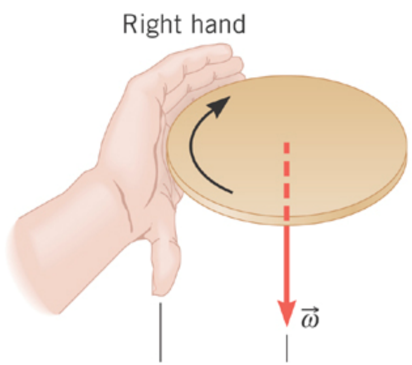
In the picture on the left a guy is trying to ***stop*** a wheel. What is the direction of the angular acceleration?

1. The same as the direction of the angular velocity
2. Opposite to the direction of the angular velocity

Right-Hand Rule:

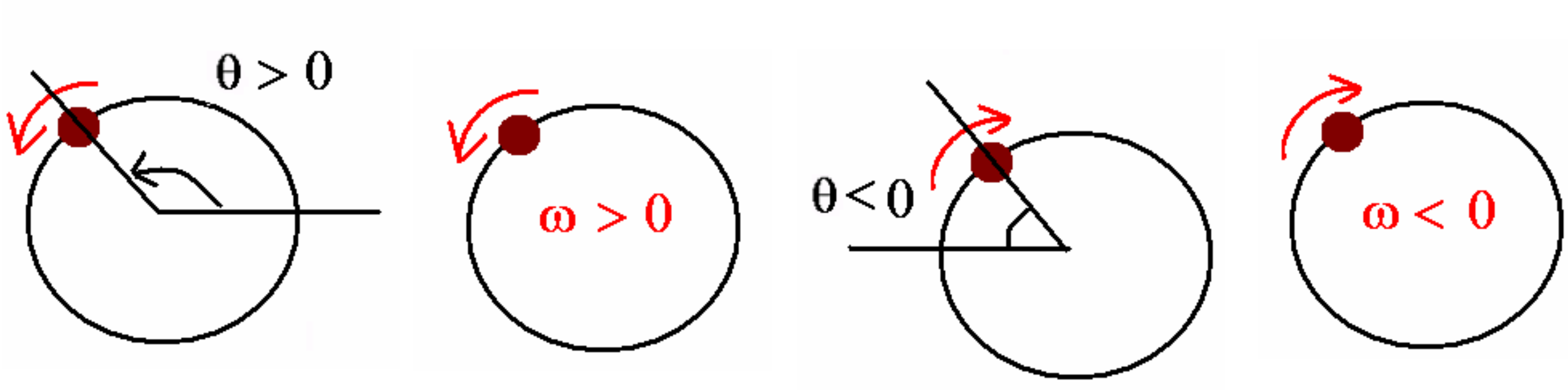


Grasp the axis of rotation with your right hand, so that your fingers circle the axis in the same sense as the rotation.



Your extended thumb points along the axis in the direction of the angular velocity.

Directions for angular variables
Standard choice



To us (from the screen) **From us (in the screen)**

When $|\omega| \uparrow$ (speeding up), $\alpha \uparrow\uparrow \omega$

When $|\omega| \downarrow$ (slowing down), $\alpha \uparrow\downarrow \omega$

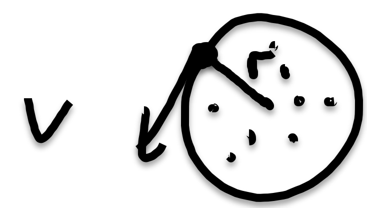
Period of Motion

When the angular velocity ω = constant; $\alpha = 0$ and $a_t = 0$.

T is the period, the time to go around once.

$\omega = \frac{\theta}{t}$

$$v = \frac{2\pi r}{T}$$



The angular speed is: $\omega = \frac{v}{r} = \frac{2\pi}{T}$

Angular vs. Linear variables

Angular displacement (angle): $\theta = \dfrac{s}{r}$ Angular speed: $\omega = \dfrac{v}{r}$

Angular acceleration is a rate of the change of ω

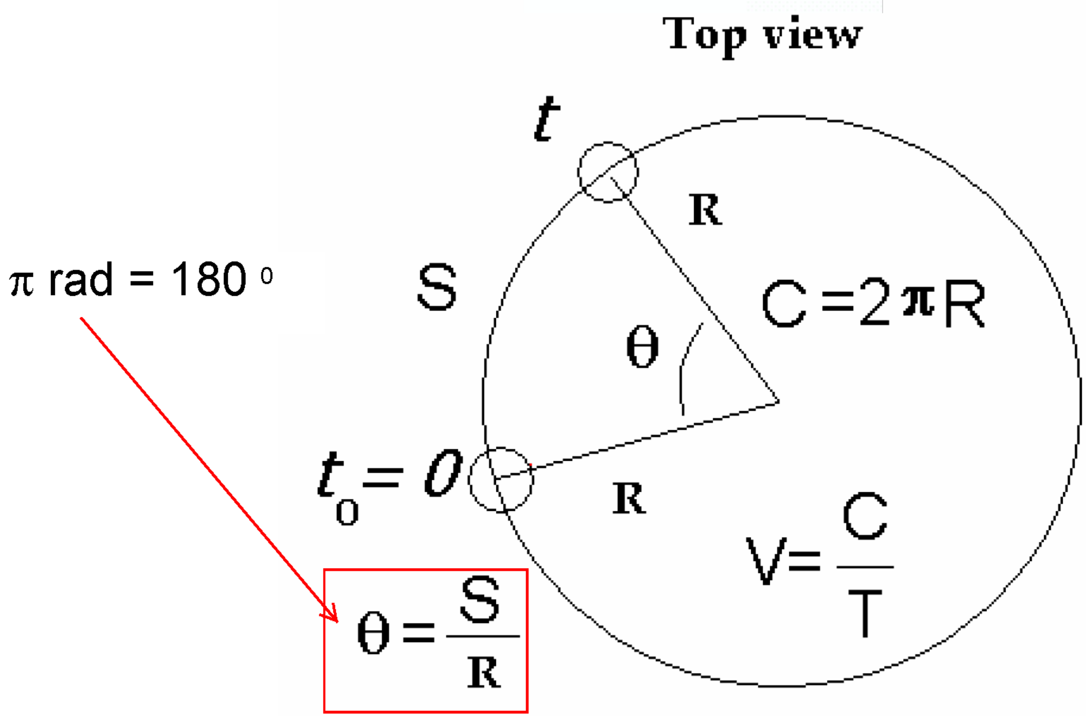
Angular acceleration: $\alpha = \dfrac{a_t}{r}$

The angular acceleration α is connected with the tangential acceleration a_t,
***not* with the centripetal acceleration a_c!**

For a circular motion $a_c = v^2/R \neq 0$,
but a_t and α might be = or $\neq 0$.

When angular velocity (as well as linear one) is constant;
$\alpha = 0$ and $a_t = 0$.

Rotational Motion



t/N = T period

N/t = n frequency

n = 1/T

V = 2πr/T

$S = Vt$ $\quad \theta = \frac{S}{R} = \frac{Vt}{R} = \frac{V}{R}t = \omega t$ where $\omega = \frac{V}{R}$ is angular velocity

angular velocity is a rate of the change of angular displacement

Analogies

Variable	Straight-line motion	Rotational motion	Connection
Displacement	$s = x - x_0$	θ	$\theta = \frac{s}{r}$
Velocity	v	ω	$\omega = \frac{v}{r}$
Acceleration	a	α	$\alpha = \frac{a_t}{r}$

Tangential acceleration is NOT centripetal acceleration!!

Analogies for constant angular acceleration situations

Straight-line motion	Rotational motion
$v = v_o + at$	$\omega = \omega_o + \alpha t$
$s = v_o t + \frac{1}{2} a t^2$	$\theta - \theta_o = \omega_o t + \frac{1}{2} \alpha t^2$ or $(\theta_o = 0)$ $\theta = \omega_o t + \frac{1}{2} \alpha t^2$
$v^2 = v_o^2 + 2 a s$	$\omega^2 = \omega_o^2 + 2 \alpha (\theta - \theta_o)$ or $(\theta_o = 0)$ $\omega^2 = \omega_o^2 + 2 \alpha \theta$

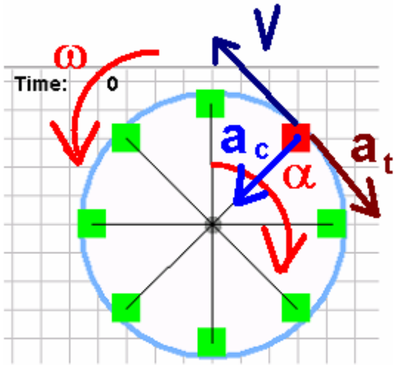
$$\omega = \frac{V}{R}$$

$$\theta = \frac{S}{R}$$

$$\alpha = \frac{a}{R}$$

(we can set $\theta_o = 0$ for almost all problems!)

Angular acceleration is NOT centripetal acceleration!!

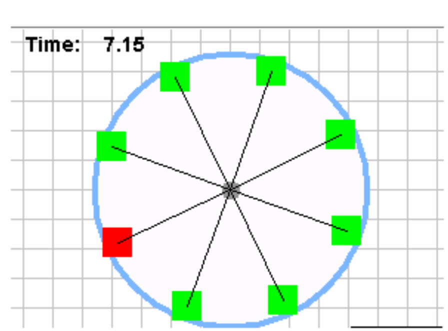


What if the turntable is gradually slowing down (sim. 2)?

(a_t is tangential acceleration)

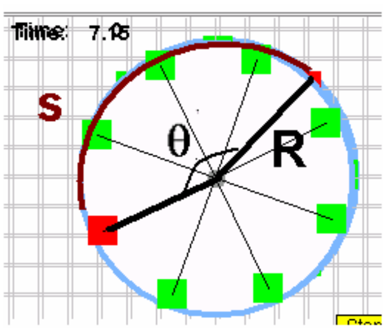
$$s = v_0\, t + \tfrac{1}{2}\, a_t\, t^2$$

(in this particular case a < 0)



Let's make a transition to angular variables:

$$\frac{s}{R} = \frac{v_0}{R} t + \frac{1}{2} \frac{a_t}{R} t^2$$



$\frac{S}{R} = \theta$ *and* $\frac{V}{R} = \omega$ If we define $\frac{a_t}{R} = \alpha$

we have $$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

Constant angular acceleration situations

1. Draw a picture.
2. Choose an origin.
3. Choose a positive direction (generally counter-clockwise).
4. Make a table summarizing everything you know.
5. Write the equations
6. Solve the system of equations

Problem

Masses on a turntable

Two identical masses on a horizontal turntable are located at different distances from the center of the turntable. Which of the masses has grater angular velocity?

$$\omega = \theta / t$$

A) The mass closest to the center.

B) The mass furthest from the center.

C) Both have the same angular velocity.

Sample problem

You are on a ferris wheel that is rotating at the rate of 1 revolution every 8 seconds. The operator of the ferris wheel decides to bring it to a stop, and puts on the brake. The brake produces a constant acceleration of -0.11 radians/s^2.

(a) If your seat on the ferris wheel is 4.2 m from the center of the wheel, what is your speed when the wheel is turning at a constant rate, before the brake is applied?

(b) How long does it take before the ferris wheel comes to a stop?

(c) How many revolutions does the wheel make while it is coming to a stop?

(d) How far do you travel while the wheel is slowing down?

Solution

Variable	Given Value	Value	Value
θ (N)			
v_0			3.30 m/s
ω_0	1rev/8s	0.125rev/s	0.125rev/s
α	-0.11 rad/s^2		
r	4.2 m		
t			
s			

a) $\omega = 1$ rev/8s = 0.125 rev/s =

0.125 rev/s *2π rad/rev = 0.785 rad/s

$v = r\,\omega = 4.2 * 0.785 = 3.30$ m/s

Solution

Variable	Given Value	Value	Value
θ (N)			
v_0			3.30 m/s
ω_0	1rev/8s	0.125rev/s	0.125rev/s
α	-0.11 rad/s^2		
r	4.2 m		
t			7.14 s
s			

? Why?

b)

$$\omega = \omega_o + \alpha\, t$$

$$t = \frac{\omega - \omega_o}{\alpha} = \frac{0 - 0.785}{-0.11} = 7.14 \text{ s}$$

Solution

Variable	Given Value	Value	Value
θ (N)			2.80 rad
v_0			3.30 m/s
ω_0	1rev/8s	0.125rev/s	0.125rev/s
α	-0.11 rad/s^2		
r	4.2 m		
t			7.14 s
s			

(c) $(\theta_o = 0)$: $\theta = \omega_o\, t + \frac{1}{2}\, \alpha\, t^2$

$$\theta = (0.785 * 7.14) + \tfrac{1}{2}\,(-0.11) * (7.14)^2 = 2.80 \text{ radians}$$

Solution

Variable	Given Value	Value	Value
θ (N)		0.446 rev	2.80 rad
v_0			3.30 m/s
ω_0	1rev/8s	0.125rev/s	0.125rev/s
α	-0.11 rad/s^2		
r	4.2 m		
t			7.14 s
s			

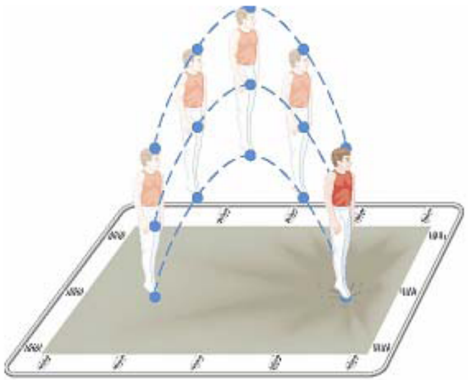
This can be converted to revolutions:

$$\frac{2.80 \text{ rad}}{2\pi \text{ rad/rev}} = 0.446 \text{ revolutions}$$

Solution

Variable	Given Value	Value	Value
θ (N)		0.446 rev	2.80 rad
v_0			3.30 m/s
ω_0	1rev/8s	0.125rev/s	0.125rev/s
α	-0.11 rad/s^2		
r	4.2 m		
t			7.14 s
s			11.8 m

(d) $s = r\,\theta = 4.2 * 2.80 = 11.8$ m

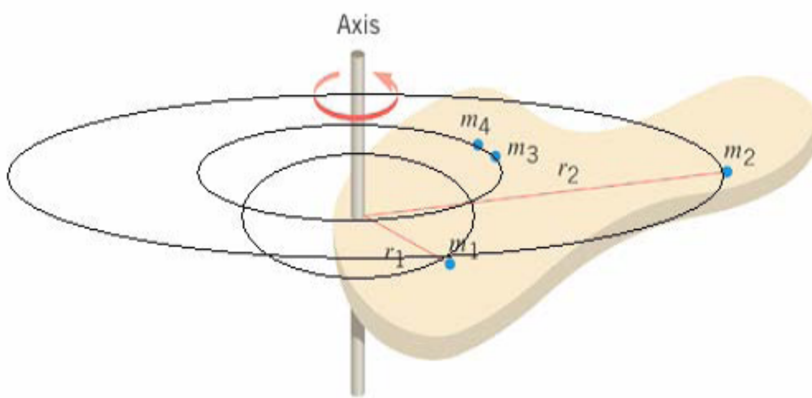


Translation

Translational Motion

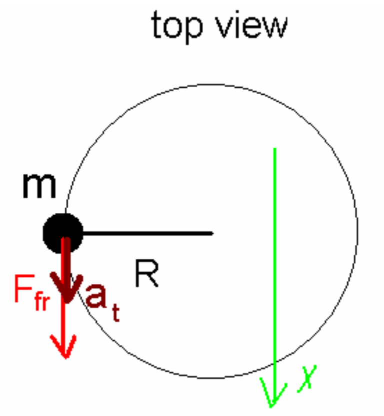
In translational motion, all points on an object travel on parallel paths (the trajectories are alike).

Rotational motion



In rotational motion, all points on an object travel on circular paths, and all the circle are parallel to each other and have the same axis of rotation.

Rotational Dynamics



$$ma_t = F_{fr}$$

Let's make a transition to angular variables, i.e.

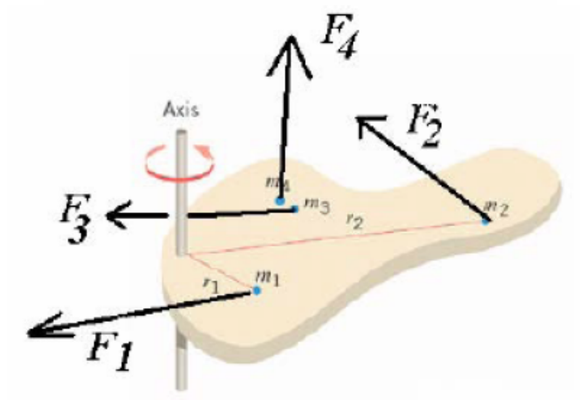
angular acceleration $\alpha = a_t/R$:

and torque $\tau = RF_{fr}$

$m\alpha R = \tau/R$ or $mR^2\alpha = \tau$

Newton's second Law for linear motion $ma = F$	Newton's second law for rotational motion $I\alpha = \tau \quad (I = mR^2)$

Newton's Second Law for Rotational Motion About a Fixed Axis



$$\sum \tau = \sum \left(mr^2\right)\alpha$$

Net external torque — Total moment of inertia

Standard expression:

Newton's Second Law for Rotation!!! =>

$$\tau_{Net} = I\alpha$$

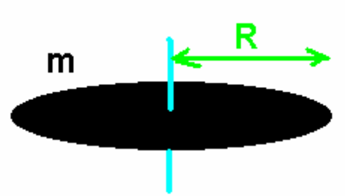
Moment of inertia about the axis passing through the CoM of the object for some objects

Moment of inertia $I = \sum \left(mr^2 \right)$ depends on:

1) distribution of the mass of the object (i.e. its shape!)

2) the location and orientation of the axis of rotation.

For example:



a uniform solid disk of mass m and radius R about the axis perpendicular to the disk and passing through its center:

$$I = \frac{1}{2} mR^2$$

Torque

Torque is the rotational equivalent of force.

Force is the source for linear acceleration (when an object is in a linear motion)	Torque is the source for angular acceleration (when an object is in a circular motion)

$\mathbf{F_{Net} \neq 0 \Rightarrow a \neq 0}$ $\boldsymbol{\tau_{Net} \neq 0 \Rightarrow \alpha \neq 0}$

$$\tau_{Net} = I\alpha$$

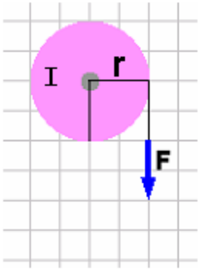
Force is a vector Torque is a vector!

Direction: The torque is positive when the force tends to produce a counterclockwise rotation about the axis.

Newton's First Law for Rotation

Newton's first law: an object at rest tends to remain at rest, and an object that is spinning tends to spin with a constant angular velocity, *unless* it is acted on by a **nonzero net torque** or **there is a change in the way the object's mass is distributed**.

Pulley and a force



We have a pulley in the shape of a solid disk of mass M = 2.0 kg and radius R = 0.50 m. If we apply a constant force F = 8 N to a string wrapped around the outside of the pulley what is the pulley's angular acceleration (Neglect the friction)?

According to the Newton's Second Law for Rotation $\tau_{Net} = I\,\boldsymbol{\alpha}$

There is one torque only: $\tau_{Net} = \tau = r\,F\,\sin 90^\circ$

And, for the solid disk we have $I = \frac{1}{2} Mr^2$

So, $\boldsymbol{\alpha} = \tau / I = rF/(\frac{1}{2}Mr^2)$

$$\alpha = \boxed{\frac{2\,F}{M\,R}} = \frac{2*8}{2*0.5} = 16.0 \text{ rad/s}^2$$