This week's topics

A solid object, rotational motion, axis of rotation, an arc, angular displacement, angular velocity, angular acceleration, degrees vs. radians, connections RM to LM, torque, lever arm, calculating torque, rotational inertia (RI), Newton's 1st law for RM, Newton's 2nd law for RM, static equilibrium, conditions for static equilibrium, solving problems on static equilibrium, Table of RI, parallel axis theorem, applications of Newton's laws for RM, angular momentum, rotational kinetic energy, rotational impulse rotational work, work-kinetic energy theorem, rolling, rolling without slipping, special cases of rolling (a spool, racing objects, Atwood's machine), law of conservation of energy, law of conservation of angular momentum (***the last topic of test 2***)

If we place a rod on a tabletop and apply two forces to it like shown in the picture; forces have the same magnitude but opposite direction, what is happening to the rod?

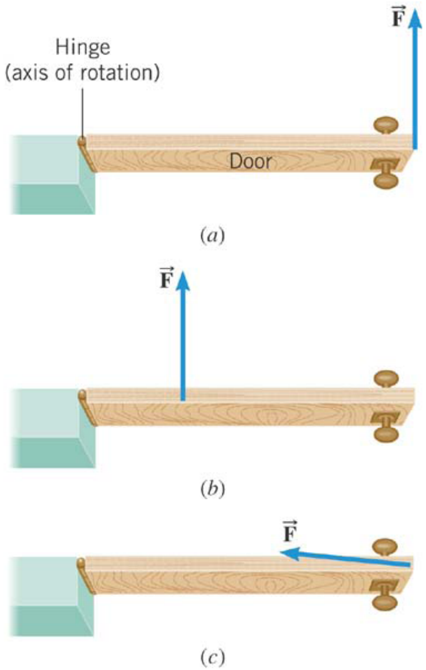
For each force $|F| = 1$ N

What is the value of the net force acting on the rod?

<u>0 N</u>   1 N   2 N

What does make the rod rotate?

Torque



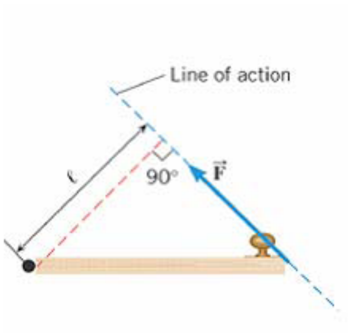
According to Newton's second law, a net force causes an object to have an acceleration.

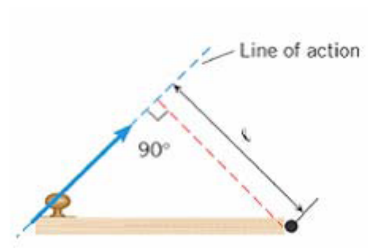
What causes an object to rotate?

The force alone does not provide the answer.

The rotational effect of the force depends on the orientation of the force!

Direction: The torque is positive when the force tends to produce a counterclockwise rotation about the axis.



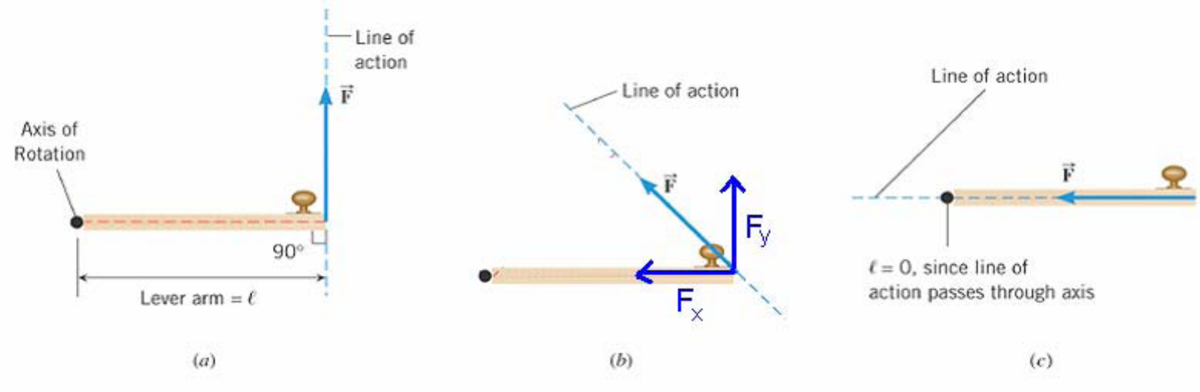


Positive torque
(to us from
the screen)

Negative torque
(from us into
the screen)

(use the right hand rule!)

Torque



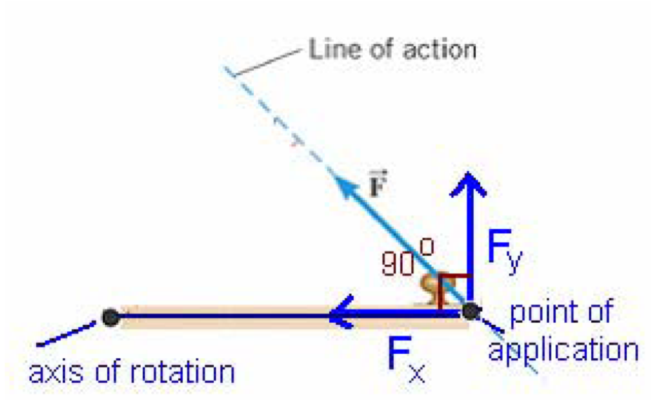
In the picture (a) the force F "uses" all its "force" to rotate the door.

In the picture (b) the force F partly rotates the door, and partly squeezes it. Only F_y component of the force makes the rotational effect. F_x component squeezes the door.

In the picture (c) the force F does not make any rotational effect, it tries just to squeeze the door.

Torque

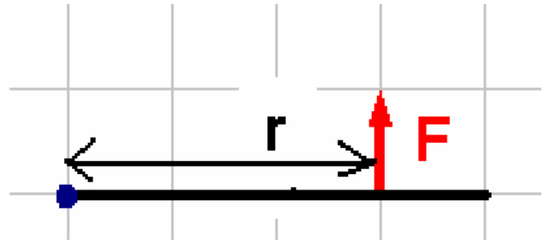
Fact #1



The rotational effect of the force depends on the *component of the force* which is *perpendicular* to the line connecting the axis of rotation and the point of application.

Torque

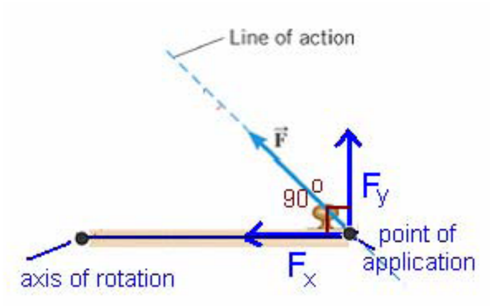
Fact # 2



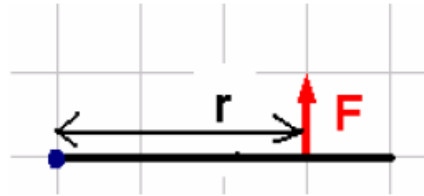
The rotational effect of the force depends on the *distance* between the axis of rotation and the point of application.

Torque

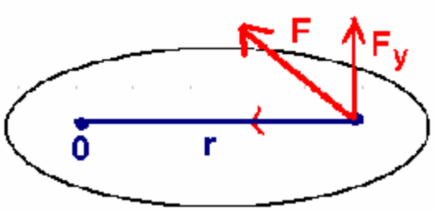
Fact #1



Fact # 2



Now we can combine the facts together and define a new physics quantity TORQUE.



The torque τ of the force F is a vector with the magnitude of

$$\tau = r F_y$$

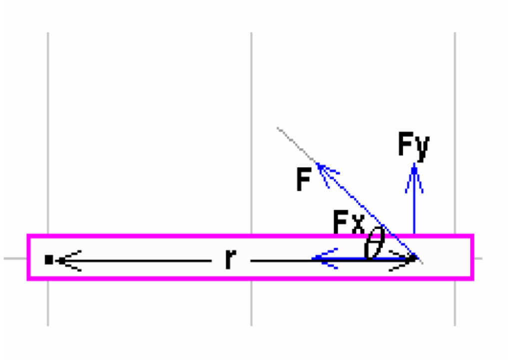
r is the distance from the point of application of the force to the axis of rotation

$\mathbf{F_y}$ is the component of the force perpendicular to the line connecting the point of application and the axis of rotation.

Calculating Torque

There are two equivalent ways to determine the magnitude of the torque about a rotation axis.

Method 1



Resolve the force into two components:

F_x is directed to the axis

F_y is perpendicular to F_x

Only the F_y component is responsible for a rotational effect

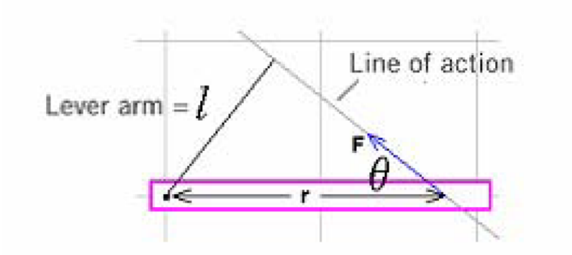
$$\tau = r\,F_y$$

from the triangle for F, F_x and F_y we have

$$\tau = r\,F\,\sin(\theta)$$

Calculating Torque

There are two equivalent ways to determine the magnitude of the torque about a rotation axis.



Method 2

From the method 1

$$\tau = r\,F\,\sin(\theta) = F\,r\,\sin(\theta)$$

From the right triangle $l = r\sin(\theta)$ hence

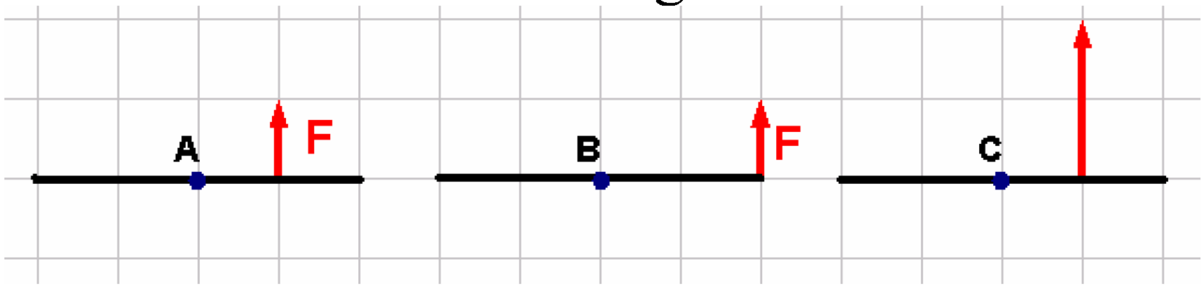
$$\tau = F\ell$$

F is the magnitude of the force (not the component!)

l is the distance form the axis of rotation to the line of action of the force (*the lever arm*)

Torque Questions

Consider the following three situations.



Rank these situations based on the magnitude of the torque, from largest to smallest.

A) C >A >B

B) C > B >A

C) C >A = B

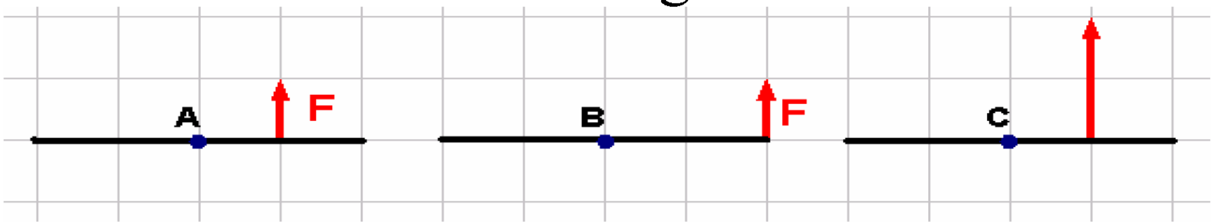
D) B = C >A

E) B >A > C

F) B >A = C

Torque Questions

Consider the following three situations.



Rank these situations based on the magnitude of the torque, from largest to smallest.

This is the situation when reading all the possible answers is useless until you figure out the right one.

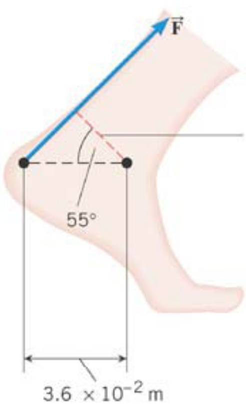
Clearly, the door A experiences the smallest torque. There are only to answers with A at the end: ether B) C > B >A or D) B = C >A.

Forces B and C create the same torque, hence the answer is

D) B = C >A

The Achilles Tendon

(a)



(b)

The tendon exerts a force of magnitude 790 N. Determine the torque (magnitude and direction) of this force about the ankle joint.

Solution

$$\tau = F\ell \qquad \cos 55^\circ = \frac{\ell}{3.6\times 10^{-2}\,\text{m}}$$

$$\tau = (720\text{ N})(3.6\times 10^{-2}\,\text{m})\cos 55^\circ$$

$$= 15\text{ N}\cdot\text{m}$$

Rigid Objects in Equilibrium

A rigid body is in equilibrium if it has zero translational acceleration and zero angular acceleration.

In equilibrium, the sum of the externally applied forces is zero, and the sum of the externally applied torques is zero.

In general, x and y components of the forces and z component of the torques have to be considered:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum \tau = 0$$

N II L
(balance of forces equation)
Net Force = 0

a new condition
(balance of torques equation)
Net Torque = 0

Equilibrium

Equilibrium is the absence of ANY motion:
translational as well as rotational.

Conditions for equilibrium:

1) an object is initially at rest

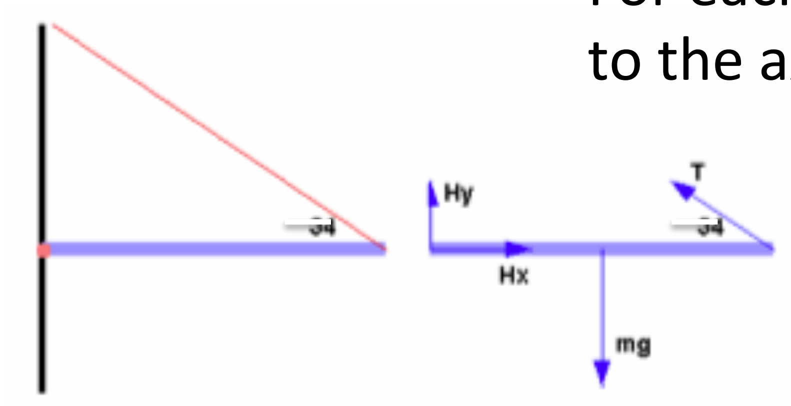
2) $F_{Net} = 0$ (no translational motion!)

3) $\tau_{Net} = 0$ (no rotational motion!)

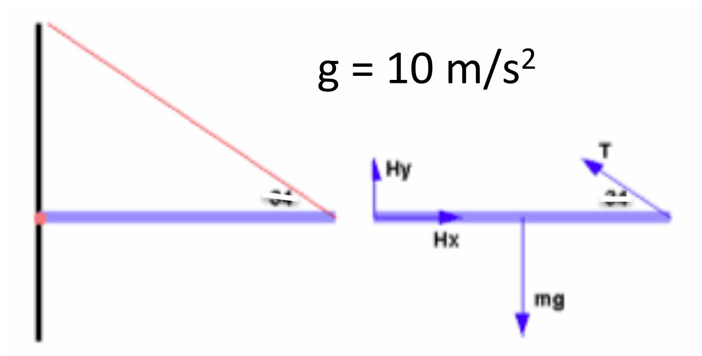
Reasoning Strategy

- Select the object to which the equations for equilibrium are to be applied.
- Draw a free-body diagram that shows all of the external forces acting on the object.
- Choose a convenient set of x, y axes and resolve all forces into components that lie along these axes.
- Apply the equations that specify the balance of forces at equilibrium. (Set the net force in the x and y directions equal to zero.)
- Select a convenient axis of rotation. Set the sum of the torques about this axis equal to zero.
- Solve the equations for the desired unknown quantities.

For each force write torque relative to the axis of your choice.



Example



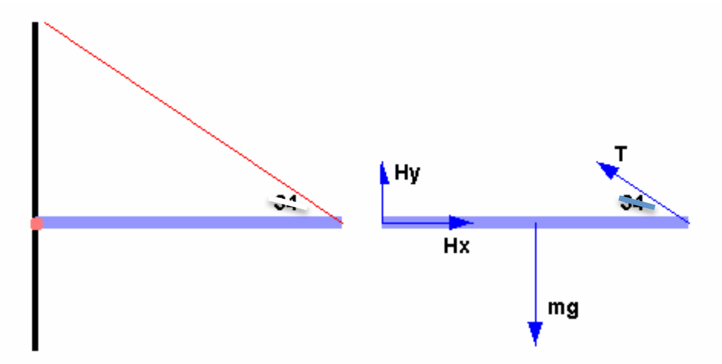
A uniform horizontal hinged rod is supported by a rope at one end. The rod has a mass of 1.4 kg, and there is an angle of 30° between the rope and the rod.

(a) What is the tension in the rope?

(b) What are the two components of the support force exerted on the rod by the hinge?

T = … 1. 10   2. 11   3. 12   4. 13   5. 14   6. 15   7. 16   (N)

Example



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(a) What is the tension in the rope?

(b) What are the two components of the support force exerted on the rod by the hinge?

$\Sigma \mathbf{F_y} = 0$ Hence, $H_y + T\sin(30°) - mg = 0$

$\Sigma \mathbf{F_x} = 0$ Hence, $H_x - T\cos(30°) = 0$

$\Sigma \boldsymbol{\tau} = 0$ Hence (for the axis at the hinge)

$$0.5Lmg - LT\sin(30°) = 0$$

$$H_y + T\sin(30°) - mg = 0$$

$$H_x - T\cos(30°) = 0$$

$$0.5Lmg - LT\sin(30°) = 0$$

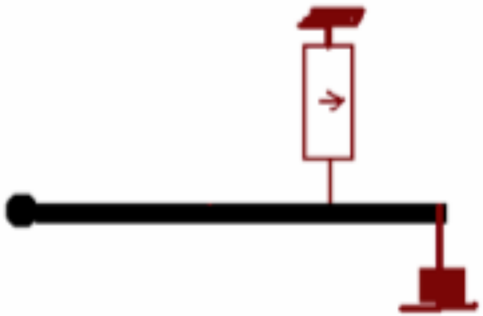
Now we can solve the system of equations:

$$T = \frac{mg}{2\sin(30°)} = 14 \text{ N}$$

$$H_x = T\cos(30°) = 12 \text{ N}$$

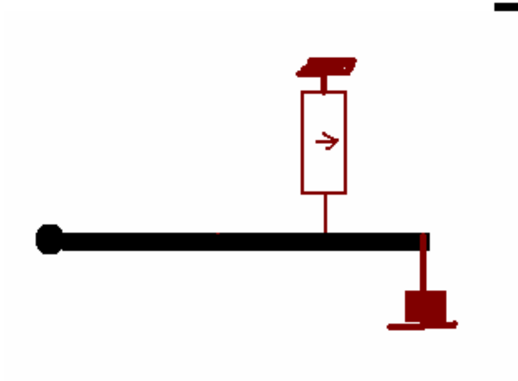
$$H_y = mg - T\sin(30°) = 7 \text{ N}$$

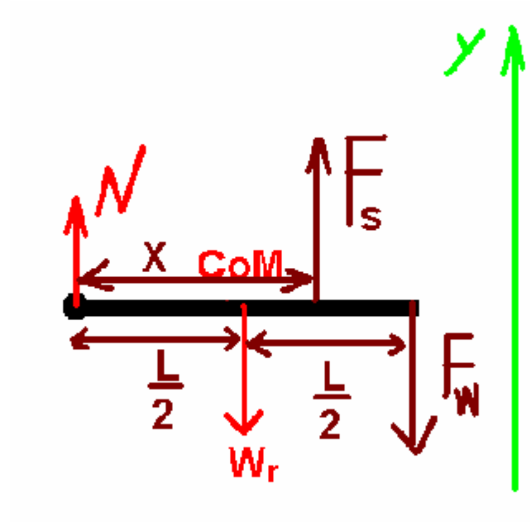
The rod on a hinge



For each force write torque relative to the axis of your choice.

The rod on a hinge





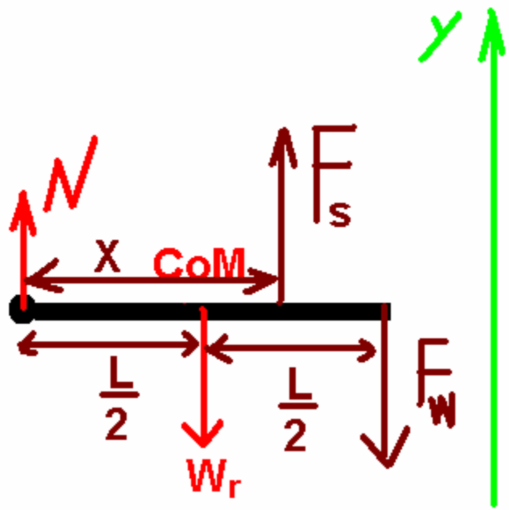
Four forces are acting on the rod.

$$\sum F_y = 0 \quad \Rightarrow \quad N + F_s - W_r - F_W = 0$$

$\sum \tau = 0$ (let's set the rotation axis at the hinge)

$$\Rightarrow \quad N*0 + F_s*X - W_r*L/2 - F_W*L = 0$$

The rod on a hinge



$$N + F_s - W_r - F_W = 0$$

(the rotation axis is at the hinge)

$$N*0 + F_s*X - W_r*(L/2) - F_W*L = 0$$

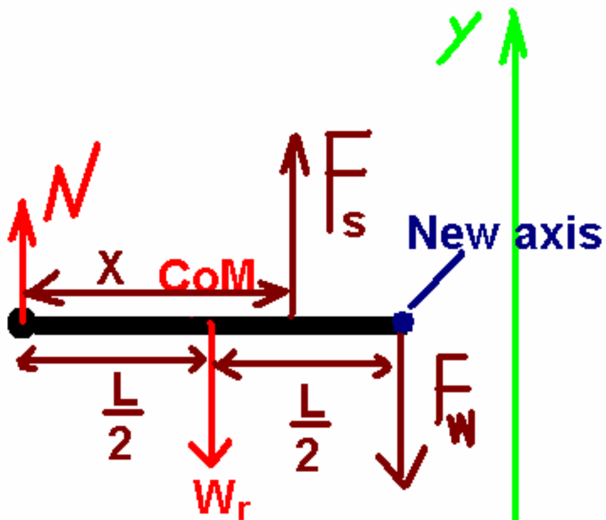
$$W_r = M_{rod}*g \qquad F_W = m_{weight}*g$$

Now we have the system of equations we can solve for the needed variables.

Note: we can choose the axis of rotation at ANY point!

The rod on a hinge

Let's choose the axis of rotation at the right end of the rod.

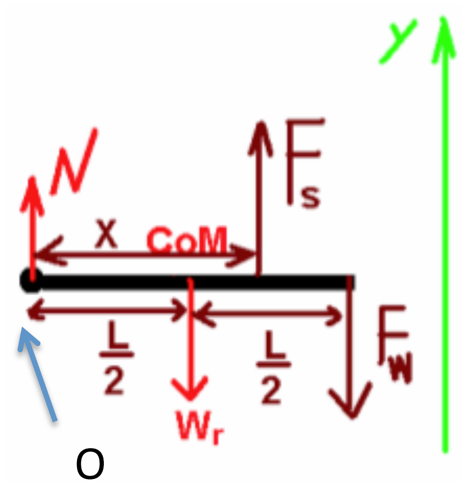


New axis means forces have new lever arms!

Now $\sum \tau = 0$ leads to

$$-N*L + W_r*(L/2) - F_s*(L - X) + F_W*0 = 0$$

$$N + F_s - W_r - F_W = 0$$



$$W_r = M_{rod} * g \qquad F_W = m_{weight} * g$$

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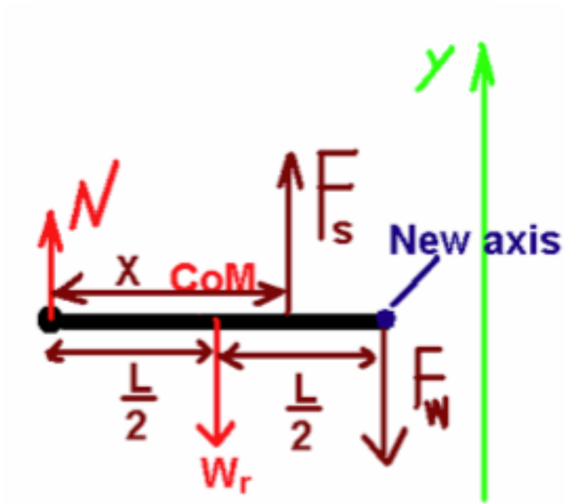

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