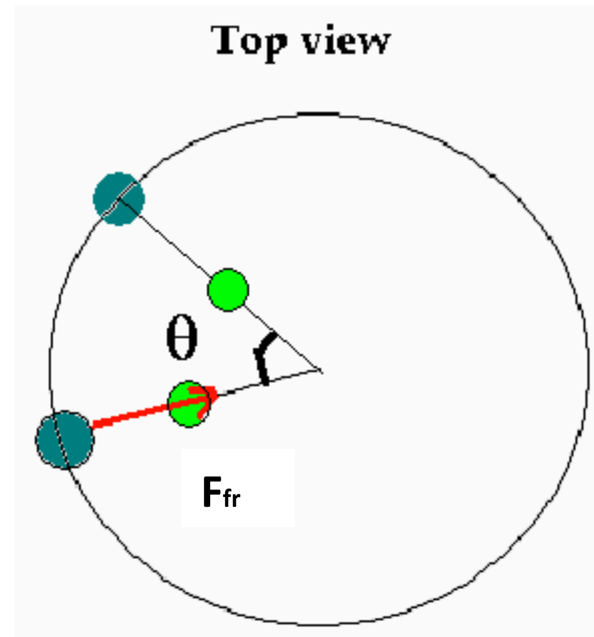
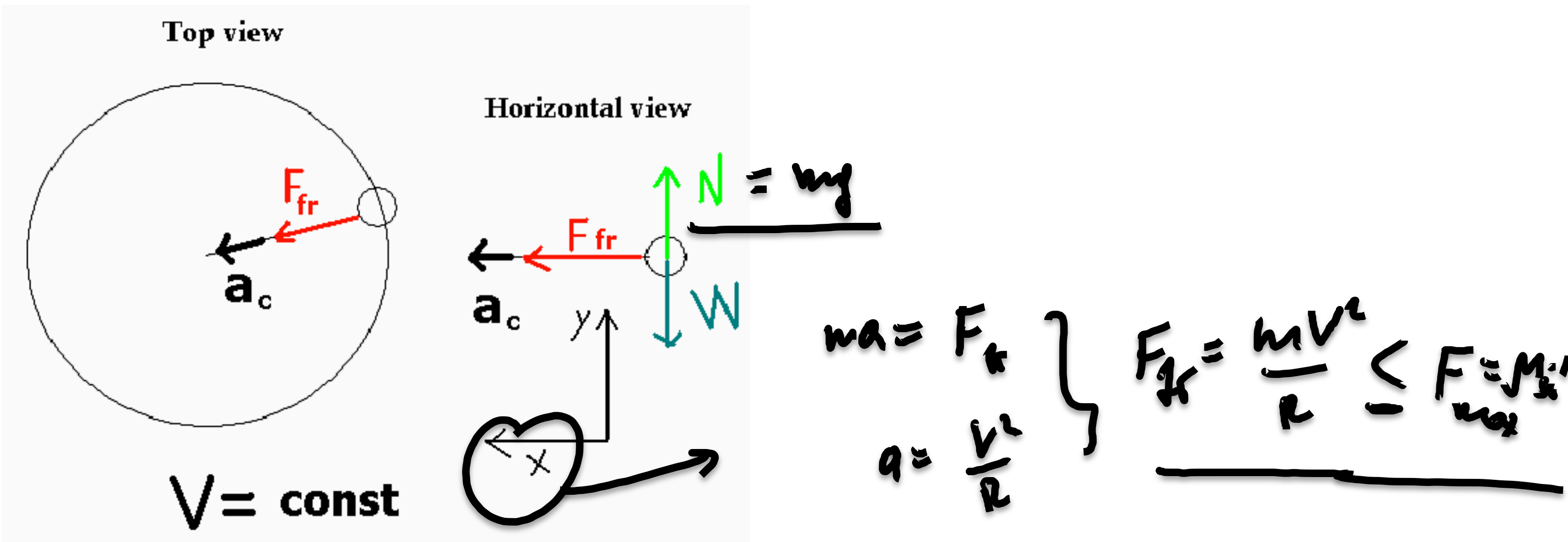
Problem

Masses on a turntable

Two identical masses on a horizontal turntable are located at different distances from the center of the turntable. As the turntable's rotation rate increases, which mass starts to slide off the turntable first?



1. closer to the center
2. another one
3. simultaneously



When an object is moving circularly it MUST have acceleration, hence the net force MUST be not equal to zero.

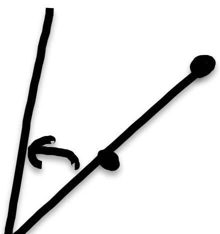
Only **force of static friction can (in this case) be the reason for acceleration. When the system cannot produce the force of friction strong enough to give the object the needed acceleration (V^2/R), the objects start sliding.**

$v \rightarrow v$

$\omega \rightarrow$

θ

$$\frac{m V^2}{R} \le \mu_{sf} \cdot m g$$

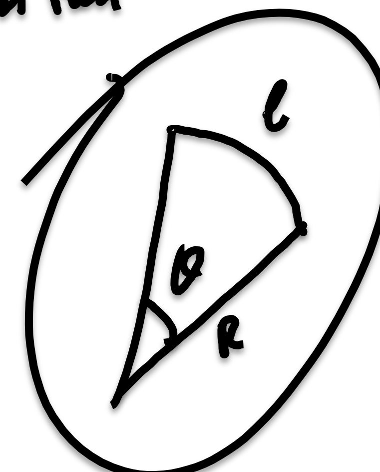


$$\omega^2 \cdot R \le \mu_s \cdot g$$

$$V = \omega \cdot R$$

$180° = \pi$ rad

$360° = 2\pi$ rad



$$\omega = \frac{v}{R}$$

$$\theta = \frac{l}{R} \,/\, t$$

$$\frac{\theta}{t}$$

rad

Unbanked Curves

On an unbanked curve, the static frictional force provides the centripetal force.

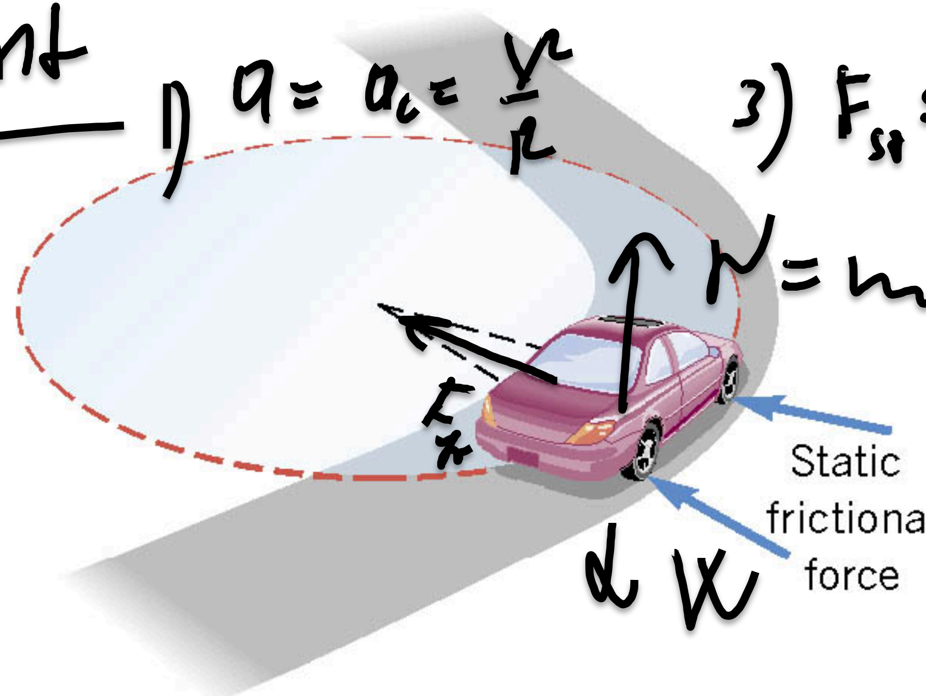
$|v| = $ const

1) $a = a_c = \frac{v^2}{R}$

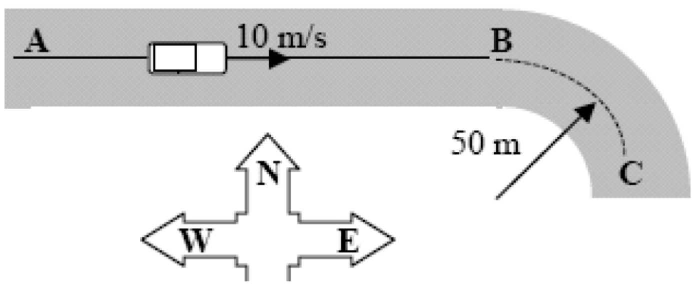
2) $\Sigma F = m \cdot a$

3) $F_{st} \leq F_{max\,st} = \mu_s \cdot N$

$N = mg$ 4)



Problem



A 1000-kg Jeep travels along a straight 500-m portion of highway (from **A** to **B**) at a constant speed of 10 m/s. At **B**, the Jeep encounters an unbanked curve of radius 50 m. The Jeep follows the road from **B** to **C** traveling at a constant speed of 10 m/s while the direction of the Jeep changes from east to south.

A) What is the magnitude of the acceleration of the Jeep as it travels from A to B?

Speed is constant, acceleration is 0.

B) What is the magnitude of the acceleration of the Jeep as it travels from B to C?

Now it has a centripetal acceleration $a_c = V^2/R = 10^2/50 = 2\ m/s^2$

C) What is the magnitude of the frictional force between the tires and the road as the Jeep negotiates the curve from B to C?

The force of friction (static!) is the reason for the acceleration. According the Newton's II Law (in the projection on the direction of the acceleration)

$F_{fr} = F_{net} = ma = ma_c = 1000*2 = 2000\ N$ $= \mu_s N = \mu_s 10000$

Problem

A racecar is traveling at constant speed around a circular track. What happens to the centripetal acceleration of the car if the speed is doubled?

(1) The centripetal acceleration remains the same.
(2) The centripetal acceleration increases by a factor of 2.
(3) The centripetal acceleration increases by a factor of 4.
(4) The centripetal acceleration is decreased by a factor of one-half.
(5) The centripetal acceleration is decreased by a factor of one-fourth

$$a_c = \frac{V^2}{R} \rightarrow a_c^* = \frac{(2V)^2}{R}$$

$$a_c^* = \frac{(2v)^2}{R} = \frac{2^2 v^2}{R} = 4\,\frac{v^2}{R} = \underline{4a}$$

Problem

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(B) The centripetal acceleration increases by a factor of 2.

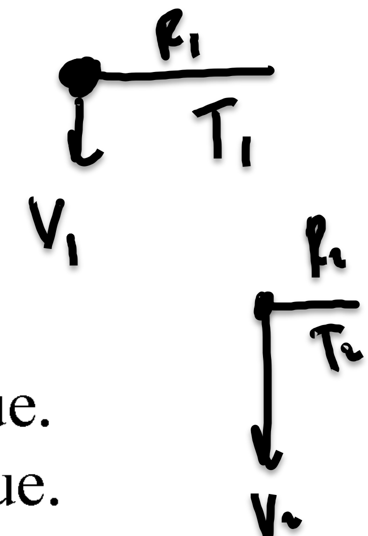
(C) The centripetal acceleration increases by a factor of 4.

(D) The centripetal acceleration is decreased by a factor of one-half.

(E) The centripetal acceleration is decreased by a factor of one-fourth

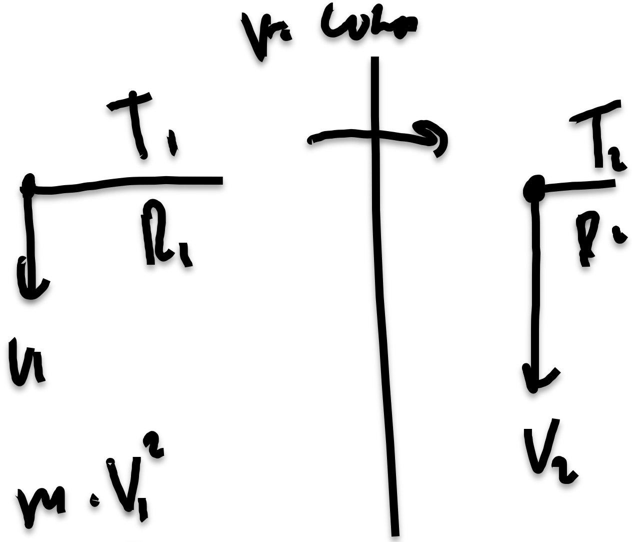
Problem

A boy is whirling a stone around his head by means of a string. The tension in the string is T1.
The boy then speeds up the stone, making the radius of the circle twice shorter, and the speed V of the ball is doubled.



What happens to the tension in the string?

(1) None of the answers below.
(2) The tension reduces to half of its original value.
(3) The tension increases to twice its original value.
(4) The tension increases to four times its original value.
(5) The tension reduces to one-fourth of its original value.



0 1 2 3 4 5 6 7 8 9

$$T_1 = \frac{m \cdot V_1^2}{R_1}$$

$$T_2 = \frac{m V_2^2}{R_2} = \frac{m (2V_1)^2}{\frac{1}{2} R_1} = \frac{m \cdot V_1^2}{R_1} \cdot \boxed{?}$$

~~$V_1 = 2 \cdot V_2$~~

$R_2 = \frac{1}{2} R_1$

$V_2 = 2 \cdot V_1$

1. Yes
2. a/b
3. ☺

$$T_2 = \frac{m(2v_1)^2}{\frac{1}{2}\cdot R_1} = \frac{m\cdot 4\cdot v_1^2}{R_1}\cdot 2 = 8\cdot T_1$$

Example

A boy is whirling a stone around his head by means of a string. The tension in the string is T_1. The boy then speeds up the stone, making the radius of the circle twice shorter, and the speed V of the ball is doubled. What happens to the tension in the string?

1. Draw FBD for case 1, show T_1, V_1, R_1, and a_1.

Write N 2nd L for the ball, use definition for a_c.

2. Draw FBD for case 2, show T_2, V_2, R_2, and a_2.

Write N 2nd L for the ball, use definition for a_c.

3. Write relations for V_1 and V_2, R_1 and R_2, use in N 2nd L and conclude on relation for T_2 and T_1.

Problem

A boy is whirling a stone around his head by means of a string. The tension in the string is T1.
The boy then speeds up the stone, making the radius of the circle twice shorter, and the speed V of the ball is doubled.

What happens to the tension in the string?

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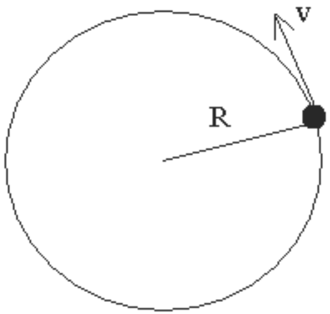
(E) The tension reduces to one-fourth of its original value.

The tension increases to eight times its original value

Example

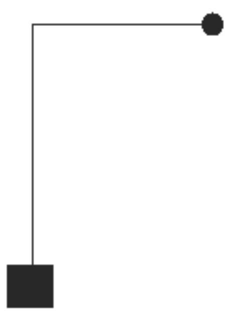
The top view



A ball which can freely roll on a table (see the pictures) makes a circle of the radius R = 20 cm. Use $g = 10\ m/s^2$.

If the ball has the mass of a half of the mass of the weight, try to find

its centripetal acceleration



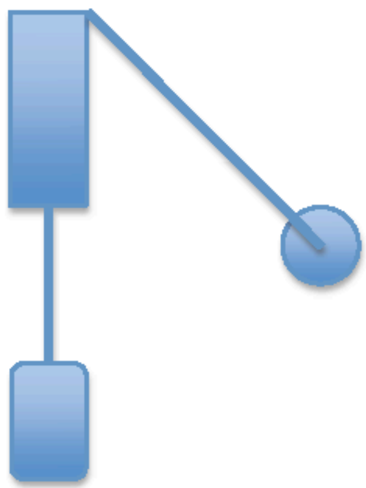
The side view

its linear speed

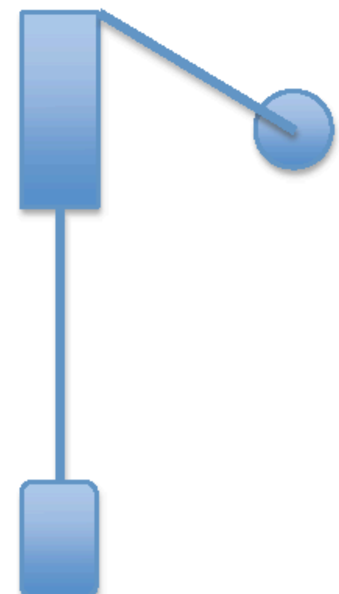
A question

The same conic pendulum is used twice (see the pictures).

Case 1



Case 2



In which case is the tension in the string larger?

1. Case 1 2. Case 2 3. The tension is the same

A question

The same conic pendulum is used twice (see the pictures).

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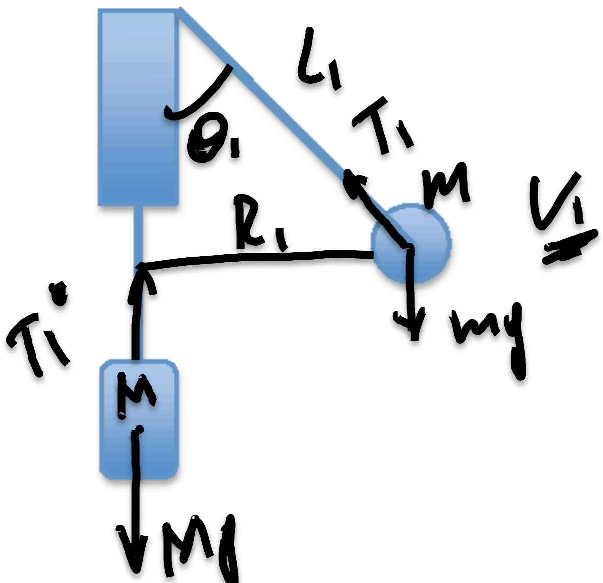
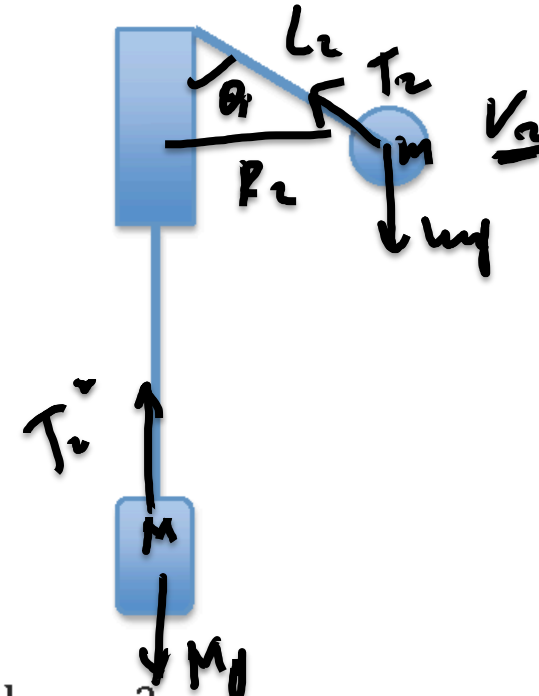


Case 2



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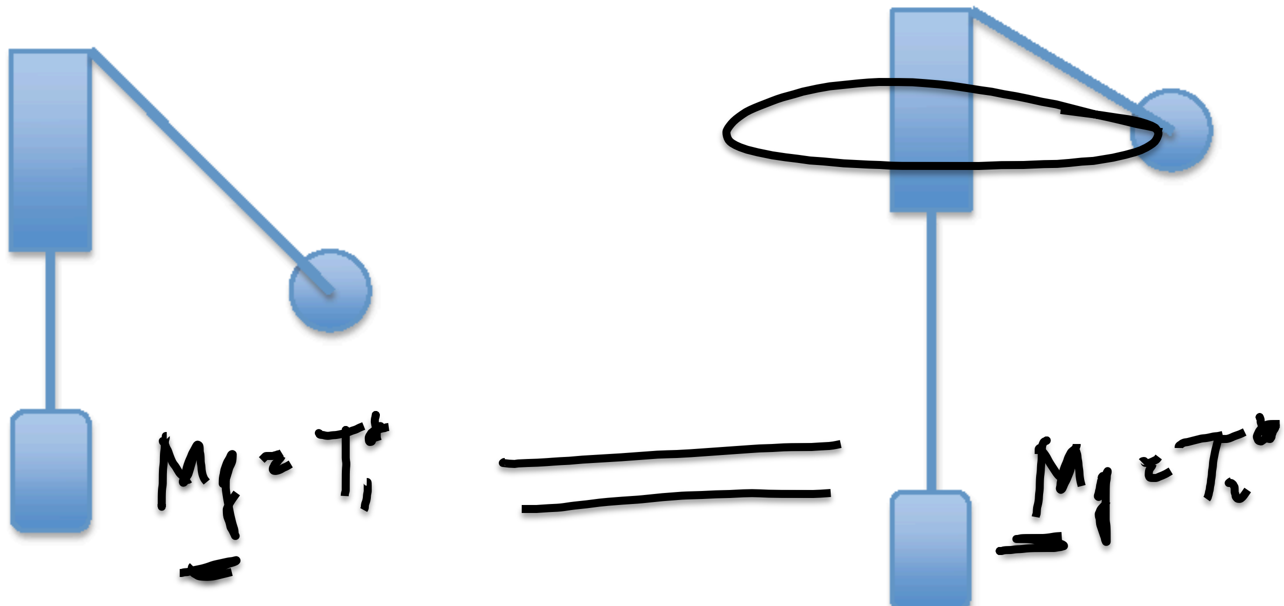
A question

The same conic pendulum is used twice (see the pictures).

Case 1 Case 2

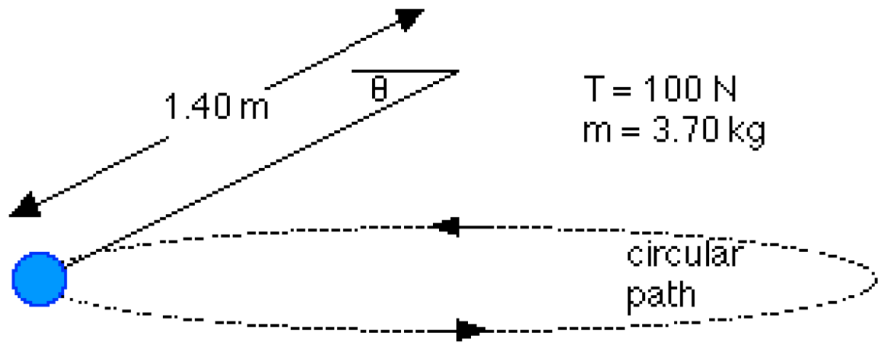


In which case is the tension in the string larger?

3. The tension is the same

Question

A ball on string is being whirled in mid-air in a horizontal circle at a constant speed v.



How many more forces (*in addition to* tension *and* the force of gravity) also belong on the free-body diagram?

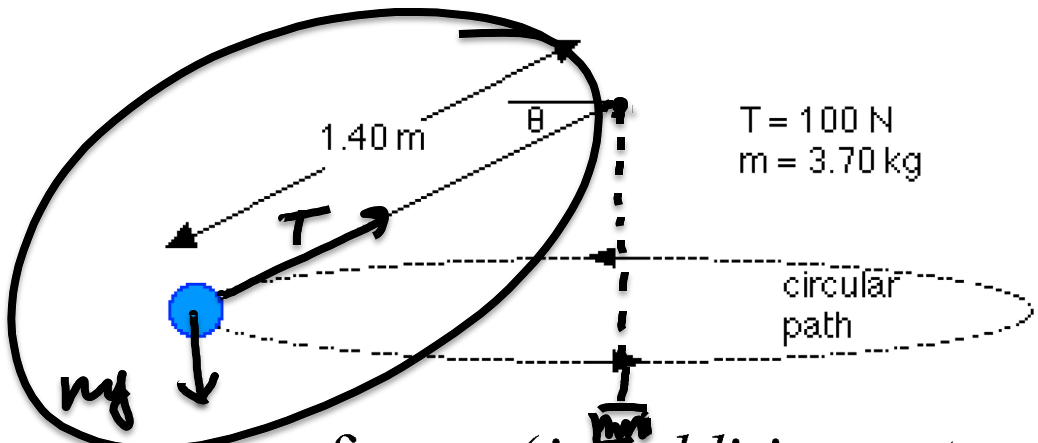
None !

1. 0
2. 1
3. 2
4. 4

A conic pendulum

Question

A ball on string is being whirled in mid-air in a horizontal circle at a constant speed v.



How many more forces (*in addition to* <u>tension</u> *and* the <u>force of gravity</u>) also belong on the free-body diagram?

1. 0
2. 1
3. 2
4. 4

A conic pendulum

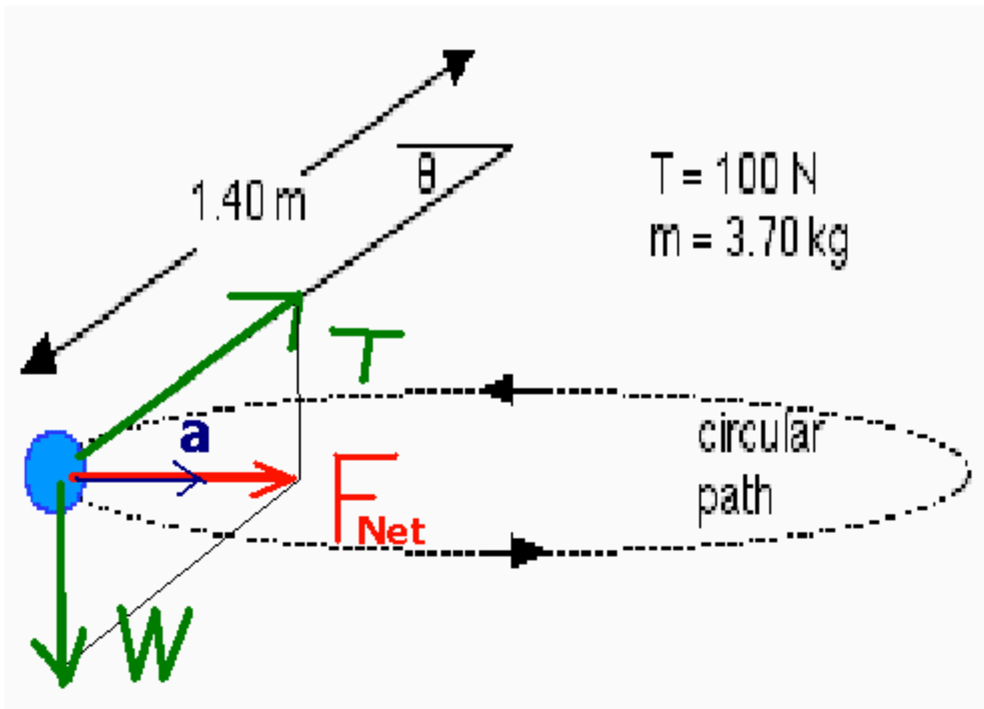
Problem

A ball on a 1.4-meter long string is being whirled in mid-air in a horizontal circle at a constant speed v.

The tension in the string is 100 N. The mass of the ball is 3.70 kg.

What is v?

Just T and W ! No more!



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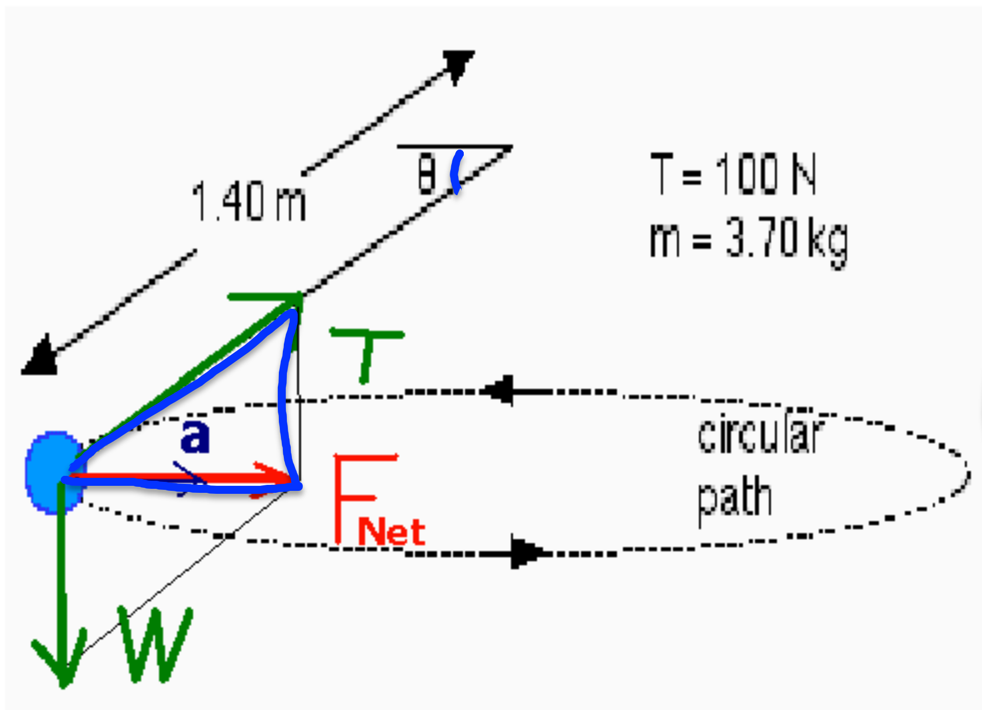
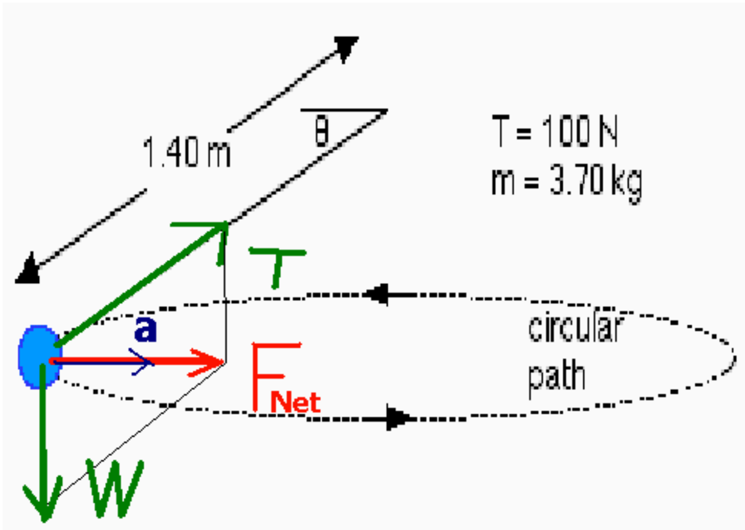




From the Newton's second law and the right triangle (involving forces) formulae we have:

$$m\ a = T\cos\theta$$

$$0 = T\sin\theta - W$$

From definitions of a weight and centripetal acceleration we have two more formulae:

$$W = m\ g \qquad a = V^2/R$$

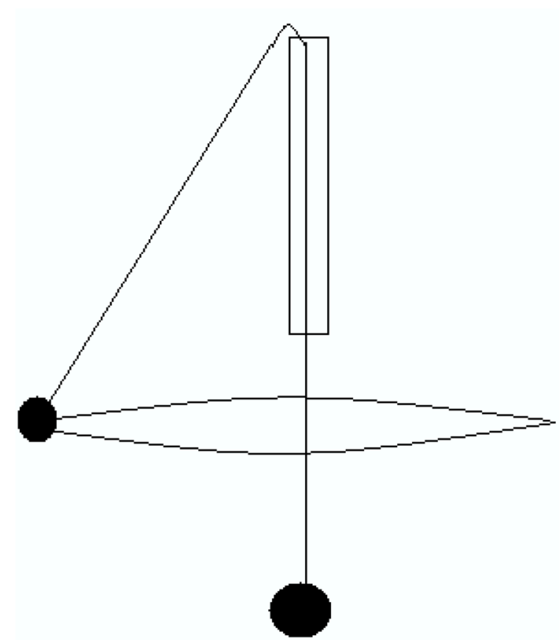
Finally, from a right involving the string and the radius: $R = L\cos\theta$

Combining all the equations and using $\sin^2\theta + \cos\theta^2 = 1$

The answer is $V^2 = \frac{TL}{m}\left(1-\left(\frac{mg}{T}\right)^2\right)$

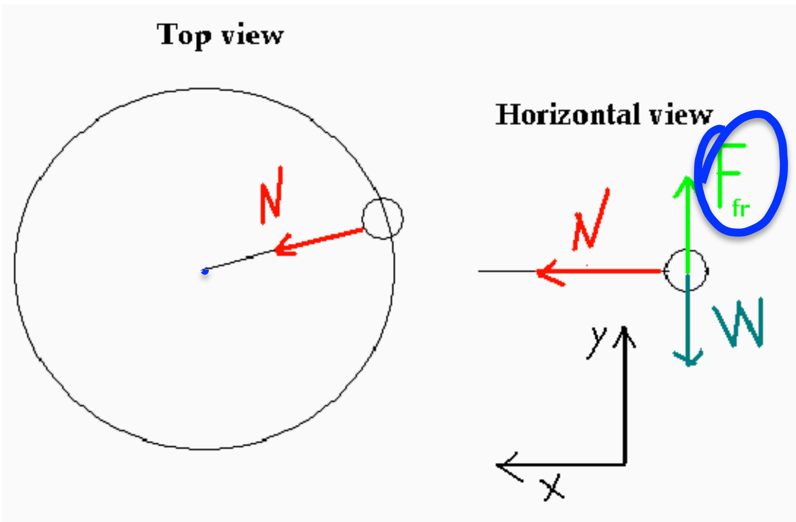
Problem

A 100 gram conical pendulum (a ball on a string) is rotating at a constant speed in a circle. To the other end of the string another ball is attached, which has the mass of 300 gram and it is stationary. If the speed of the rotation ball is 2 rev/s, find the radius of the circle.



The Gravitation VS. Friction

Carnival Ride



3) $F_{fr_{max}} = \mu N$

$N = m\,a$

$a = V^2/R$

1) $m\frac{V^2}{R} = N$

2) $F_{fr} = W \leq F_{max}$

When the speed is decreasing, the centripetal acceleration is decreasing, hence the normal force is decreasing, hence, the force of static friction eventually becomes not strong enough to support the sponge.

Actual value of Ffr has to be = mg, but if Fmax < mg, actual value also < mg, now mg is not canceled and the sponge starts sliding down.

1) $m\frac{v^2}{R} = N$

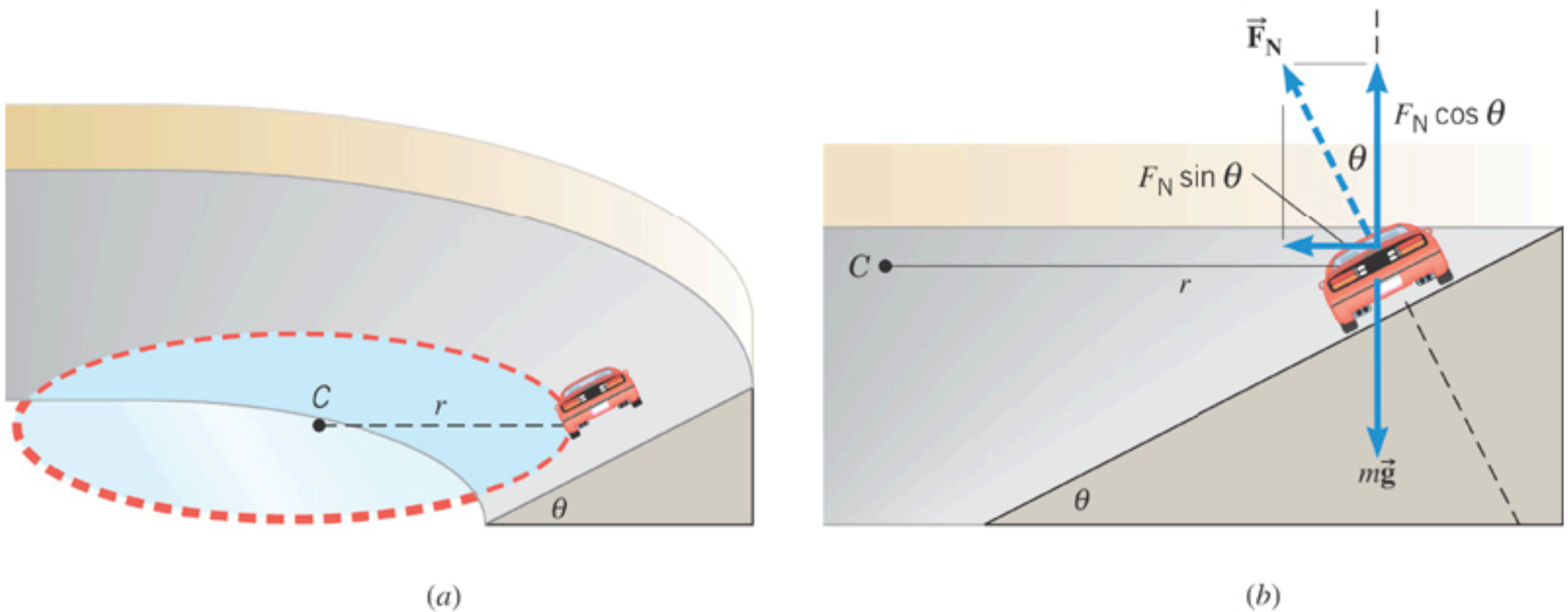
2) $mg = F_{fr} \le \mu \cdot N$

$$mg = F_f \le \mu \cdot \frac{m v^2}{R}$$

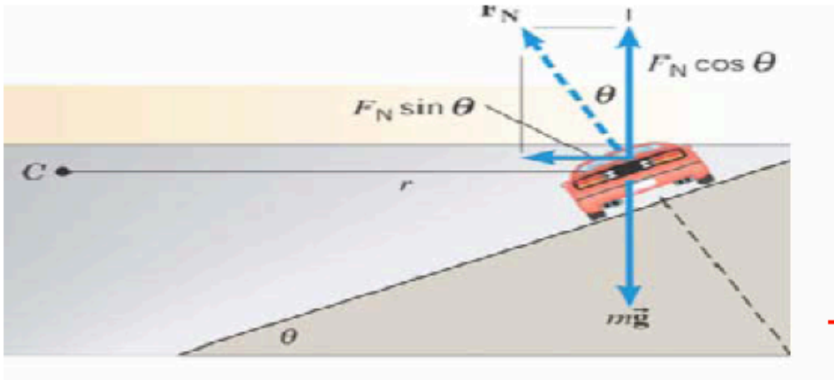
$$g \le \mu \frac{v^2}{R} \rightarrow \sqrt{\frac{g \cdot R}{\mu}} \le v$$

Banked Curves

On a frictionless banked curve, the centripetal force is the horizontal component of the normal force. The vertical component of the normal force balances the car's weight



Problem



The turns at the Daytona International Speedway have a maximum radius of 316 m and are steely banked at 31°.

Suppose these turns were frictionless.

As what speed would the cars have to travel around them?

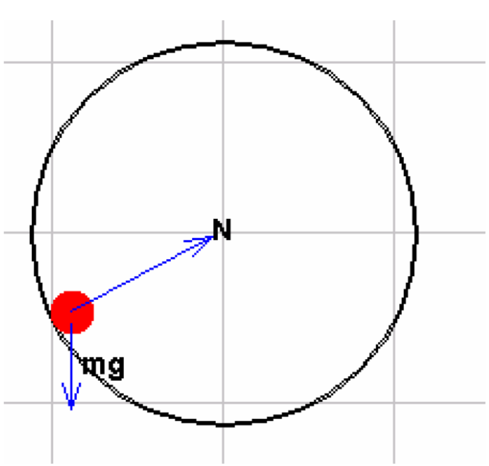
Let's write the Newton's II law in the projection on the direction of the centripetal acceleration: $ma_c = F_N \sin\theta$, hence $mV^2/R = F_N \sin\theta$

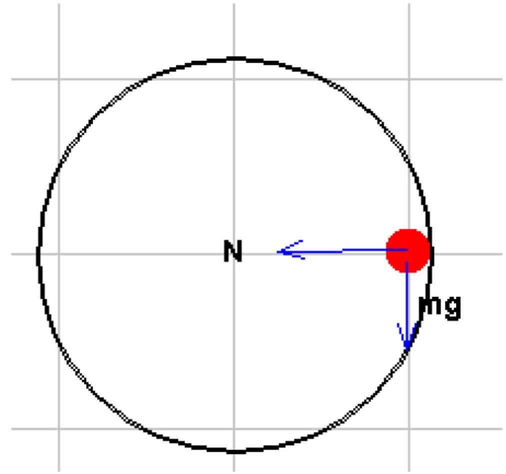
Now let's write the Newton's II law in the projection on the direction of the force of gravity: $m*0 = mg - F_N \cos\theta$

Now we can solve this system for the variable V.

$$V = \sqrt{\frac{RF_N \sin\theta}{m}} = \sqrt{\frac{gRF_N \sin\theta}{mg}} = \sqrt{\frac{gRF_N \sin\theta}{F_N \cos\theta}} = \sqrt{\frac{gR \sin\theta}{\cos\theta}} = \sqrt{Rg \tan\theta}$$

Vertical circular motion





The situation of *vertical* circular motion is fairly common. Examples include:

- roller coasters
- water buckets
- cars traveling on hilly roads
- a ball on a string

Vertical Circular Motion

Resistive and propulsion forces are NOT shown!

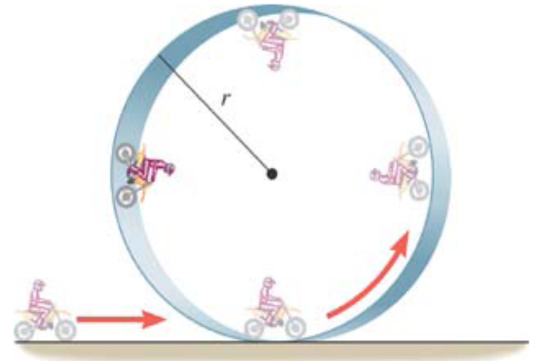
$$F_{Net\ radial} = m\,a_c$$

$$F_{N1} - mg = m\frac{v_1^2}{r}$$

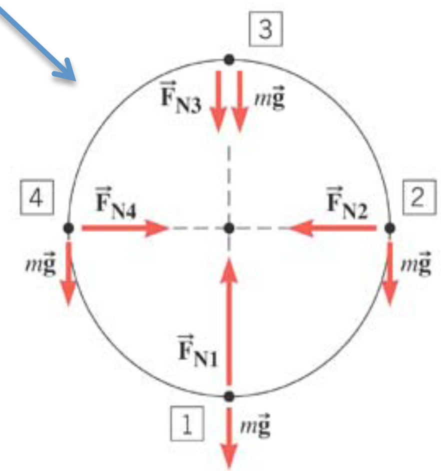
$$F_{N2} = m\frac{v_2^2}{r}$$

$$F_{N4} = m\frac{v_4^2}{r}$$

$$F_{N3} + mg = m\frac{v_3^2}{r}$$



(a)



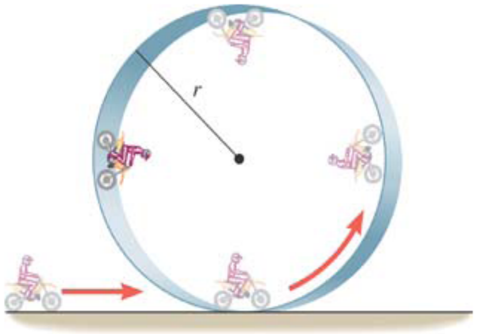
(b)

Vertical Circular Motion

$$v1 = v2 = v3 = v4 = v$$

Resistive F $\neq 0$ => v *can* be constant (!)

$$F_{Net\ radial} = m\,a_c$$



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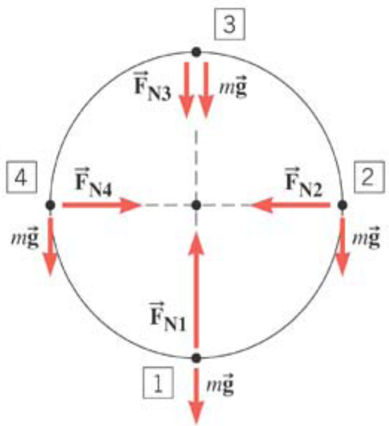
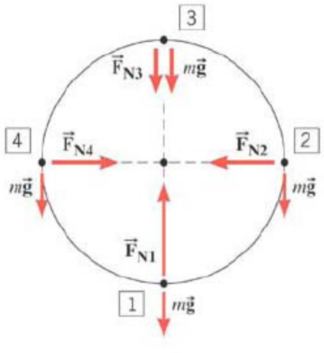


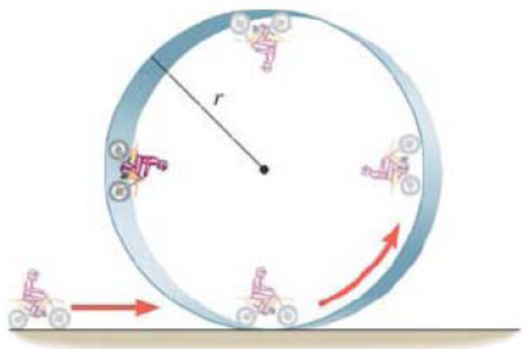
(b)

Resistive and propulsion forces are NOT shown!

PRS Question



(b)



A motorcyclist is moving at constant speed.

At which point does the motorcyclist experience the highest apparent weight?

1) 1

2) 2

3) 3

4) 4

5) Impossible to tell

$$F_{N1} - mg = m\frac{v_1^2}{r}$$

$$F_{N2} = m\frac{v_2^2}{r}$$

$$F_{N4} = m\frac{v_4^2}{r}$$

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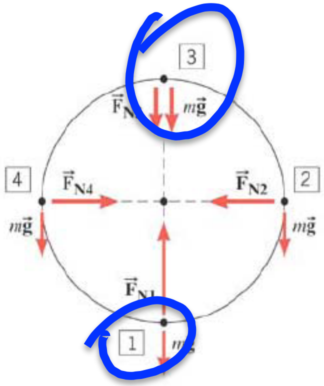
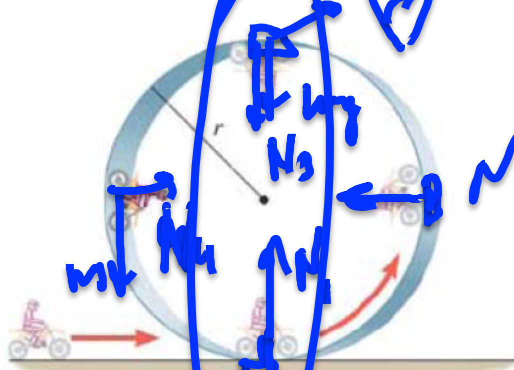
PRS Question



(b)



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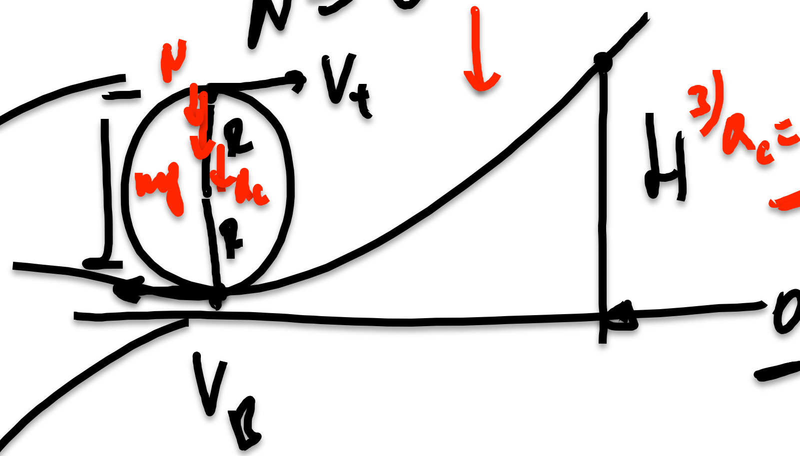
$$F_{N4} = m\frac{v_4^2}{r}$$

$$F_{N3} + mg = m\frac{v_3^2}{r}$$

$N = 0$



$m a_c = mg + N$, $N \to 0$

3) $a_c = g$

$W_{nc} = 0$ / no f_r

$mgH + 0 = \frac{mv^2}{2} + 0$

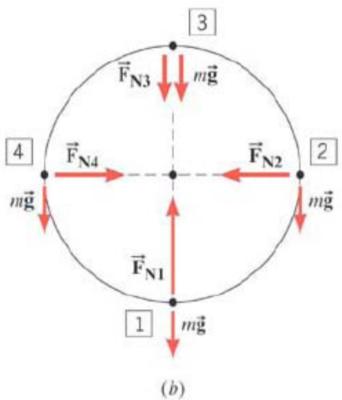
1) $V_B = \sqrt{2gH}$

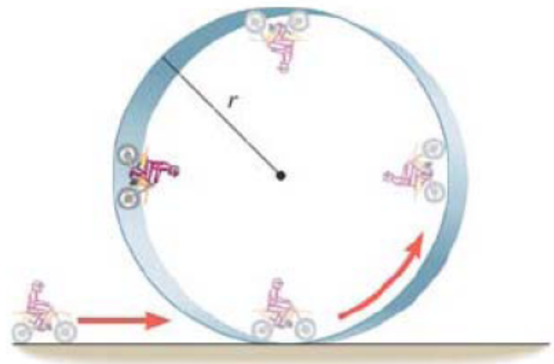
$V_B \overset{?}{\neq} V_t$

$a_c = \frac{V_t^2}{R}$ 4)

2) $\frac{mV_B^2}{2} + 0 = \frac{mV_t^2}{2} + mg \cdot 2R$

PRS Question





$$F_{N1} - mg = m\frac{v_1^2}{r}$$

$$F_{N2} = m\frac{v_2^2}{r}$$

$$F_{N4} = m\frac{v_4^2}{r}$$

$$F_{N3} + mg = m\frac{v_3^2}{r}$$

A motorcyclist is moving at constant speed.

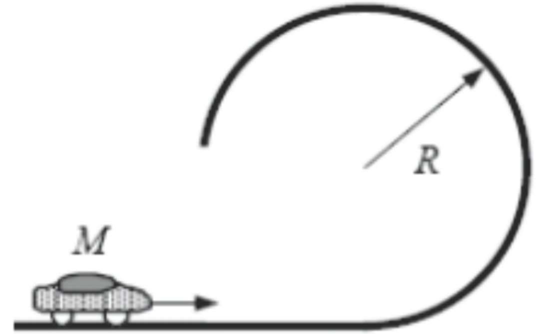
At which point does the motorcyclist experience the highest apparent weight?

Point 1

Example

A small car of mass M travels along a straight, horizontal track. The track then bends into a vertical circle of radius R.

What is the minimum acceleration that the car must have at the top of the track if it is to remain in contact with the track?



A

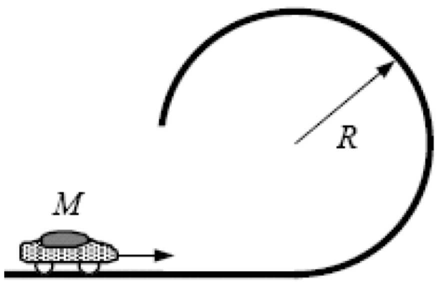
Find the minimum speed that the car must have at the top of the track if it is to remain in contact with the track?

If the speed of the car at the top point v is grater than the minimum speed, what is the normal force on the car at that point?

Find the apparent weight of the car at the bottom of the loop.

(Consider two cases: a) The car maintains its speed constant.
b) The track is frictionless after point A.)

Problem



A small car of mass M travels along a straight, horizontal track. The track then bends into a vertical circle of radius R.

A) What is the minimum acceleration that the car must have at the top of the track if it is to remain in contact with the track?

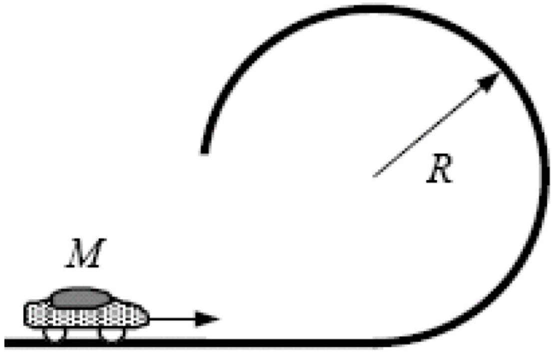
The smallest apparent weight (normal force acting on the car) is reached at the highest point of the track (the point 3, see the PRS question) and the minimum value of it is zero.

For the point 3 from $F_{N3} + mg = ma$ by setting $F_{N3} = 0$ we have the result

$$a = g$$

The minimum acceleration of the car at the top point has to be 9.8 m/s^2, otherwise the car will take off the track at hat point.

Problem



A small car of mass M travels along a straight, horizontal track. The track then bends into a vertical circle of radius R.

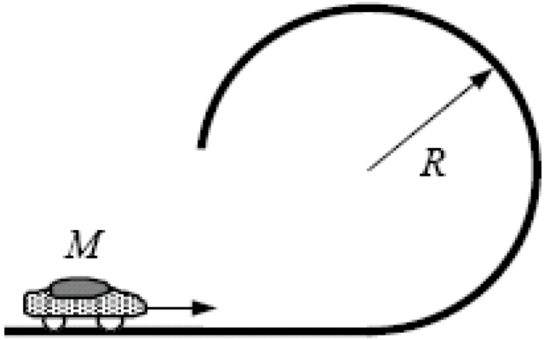
B) Find (algebraically) the minimum speed that the car must have at the top of the track if it is to remain in contact with the track?

The minimum speed of the car is directly related to the minimum acceleration.

At the top point the acceleration of the car $a = g$ also is equal to the normal acceleration (there is no tangential acceleration at the point 3).

Hence: $v^2/R = a_c = a = g$, and $v = \sqrt{Rg}$

Problem



A small car of mass M travels along a straight, horizontal track. The track then bends into a vertical circle of radius R.

C) If V is the speed of the car, what is the normal force on the car at the top?

If the speed of the car V > $\sqrt{Rg}$, the car exerts a force on the track (the apparent weight W) and the track exerts a force on the car (the normal force F_{N3}).

At the point 3 the normal force acting on the car can be found from

$$F_{N3} + mg = m\frac{V^2}{R}$$

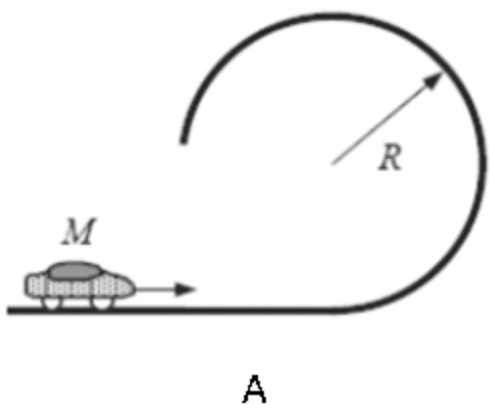
The answer is $F_{N3} = m\frac{V^2}{R} - mg$

If the speed of the car at the top point v

Find the apparent weight of the car at the bottom of the loop.

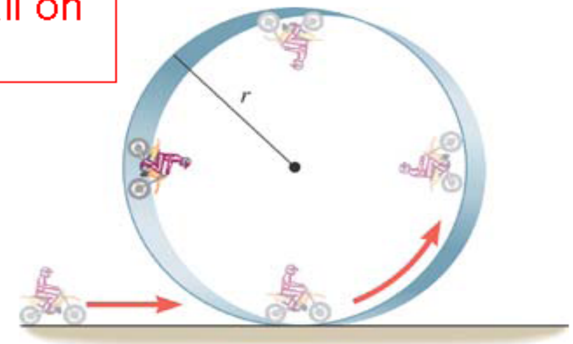
(Consider two cases: a) The car maintains its speed constant.

~~b) The track is frictionless after point A.)~~



Vertical Circular Motion

Friction = 0 => v ≠ constant (!) (like a ball on a string)



(a)

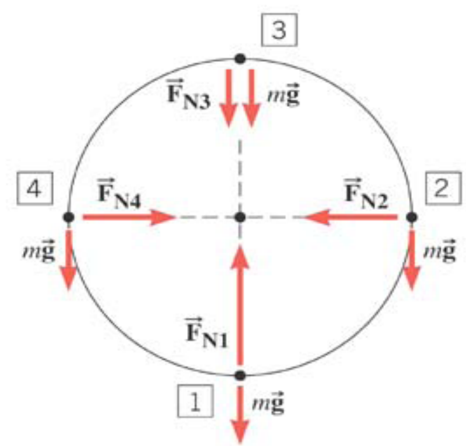
$$F_{Net\ radial} = m\,a_c$$

$$F_{N1} - mg = m\frac{v_1^2}{r}$$

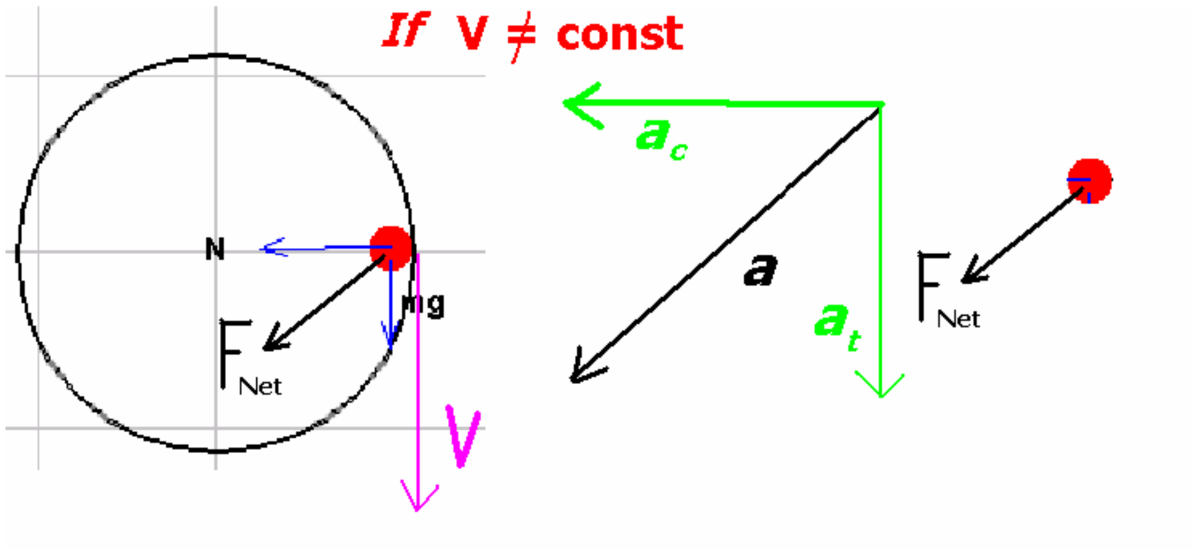
$$F_{N2} = m\frac{v_2^2}{r}$$

$$F_{N4} = m\frac{v_4^2}{r}$$

$$F_{N3} + mg = m\frac{v_3^2}{r}$$



(b)



When the motion *is* circular, but the speed is *not* constant,

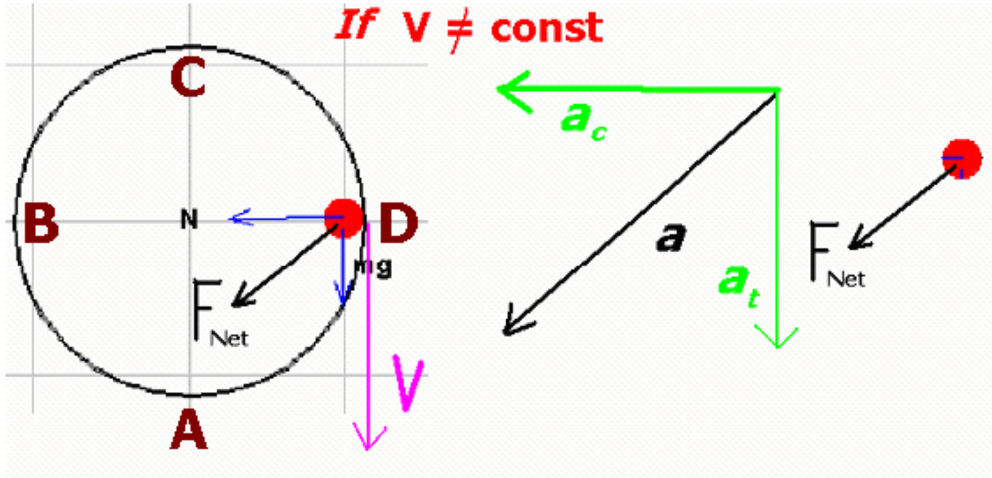
the acceleration **a** (the total acceleration) is NOT directed

to the center (!), but is has TWO components:

$\mathbf{a_c}$ is directed to the center of the circle = centripetal acceleration

$\mathbf{a_t}$ is directed along the velocity = tangential acceleration

(a ball on a string)

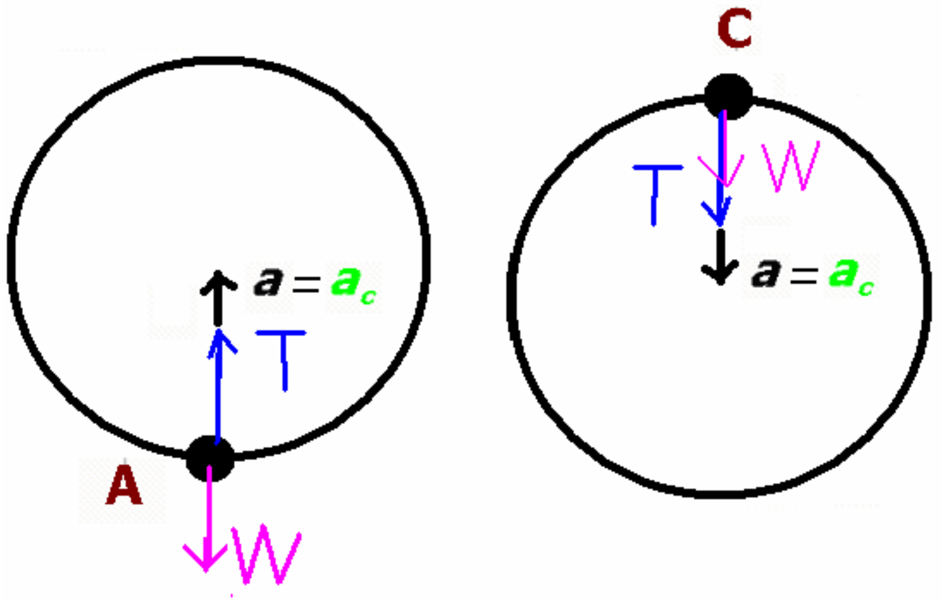


However!!!

There are TWO special locations of the object where

$$\mathbf{a} = \mathbf{a_c}$$

A and C



At the points A and C (only! When the speed is *not* constant)

we can use the relation

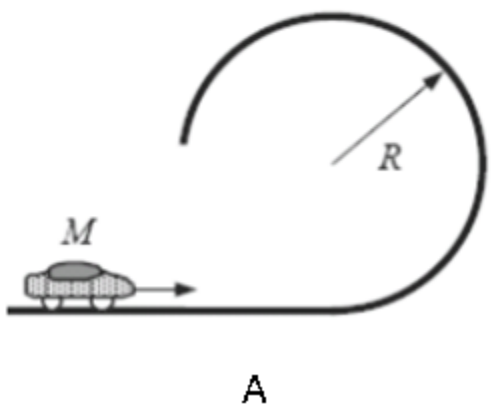
$$\mathbf{a = a_c = V^2/R}$$

(remember: R is the radius of the circle the object is making)

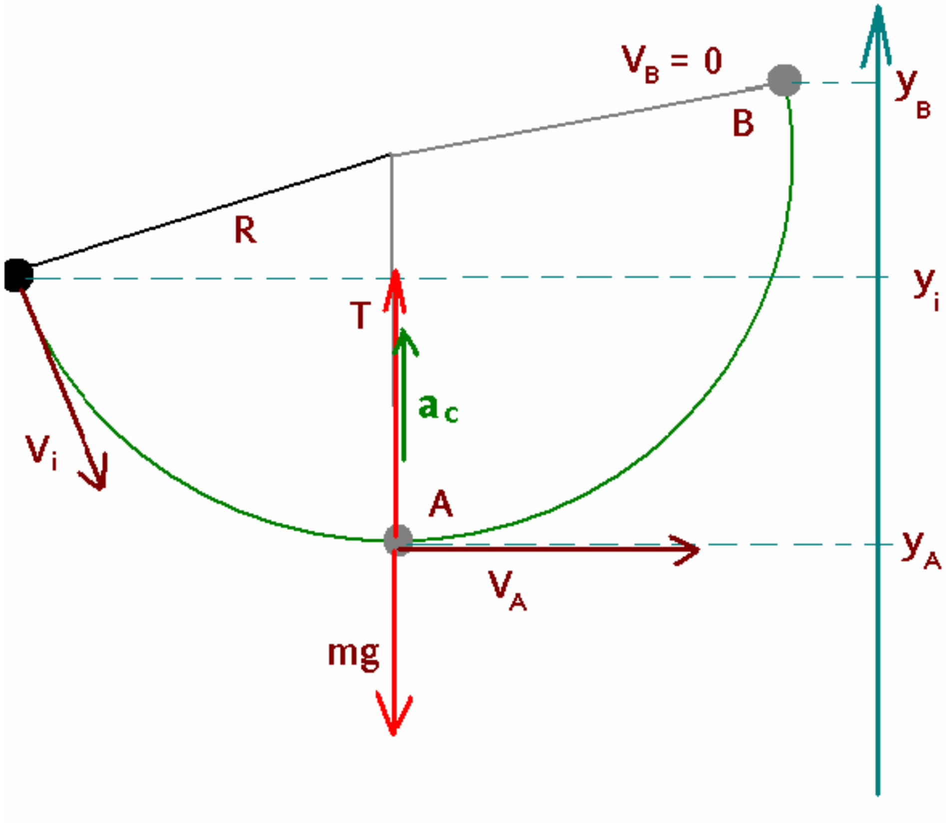
If the speed of the car at the top point v

Find the apparent weight of the car at the bottom of the loop.

(Consider two cases: ~~a) The car maintains its speed constant.~~
b) The track is frictionless after point A.)



Problem



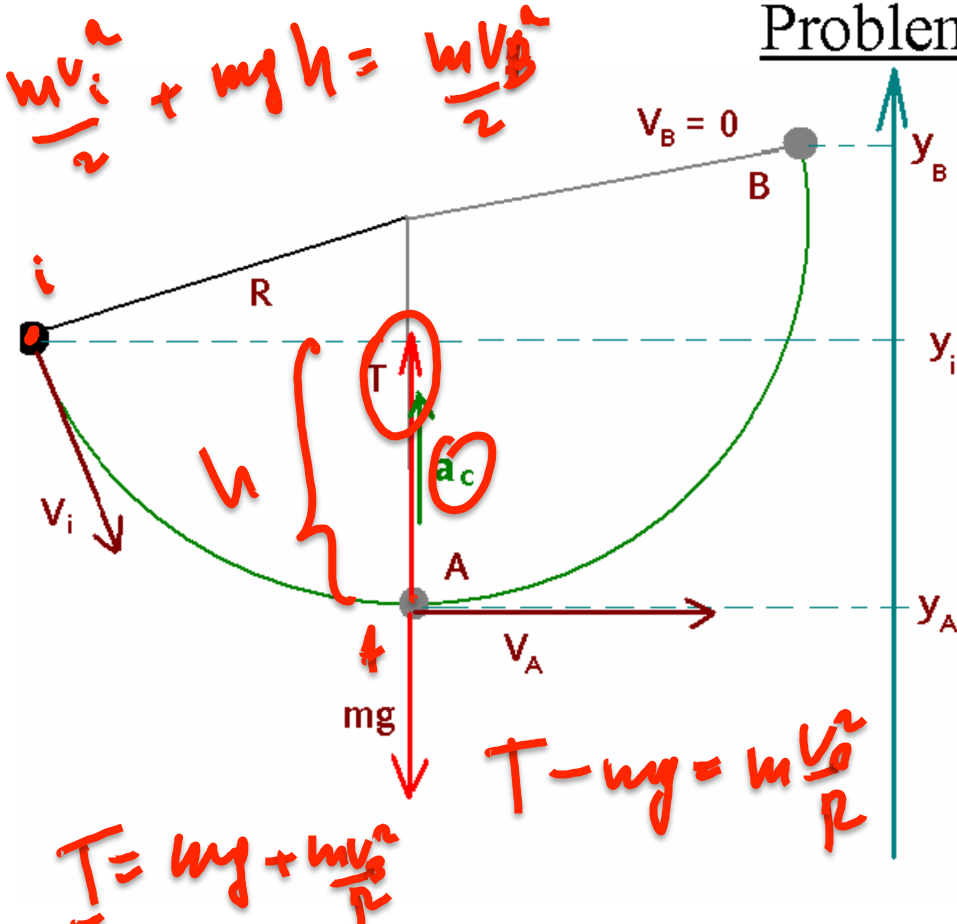
A pendulum is released with some initial velocity from initial height.

What is happening to its velocity?

What is the maximum speed it can have if the maximum tension in the string is T?

From which maximum height it can be released from rest?

Problem



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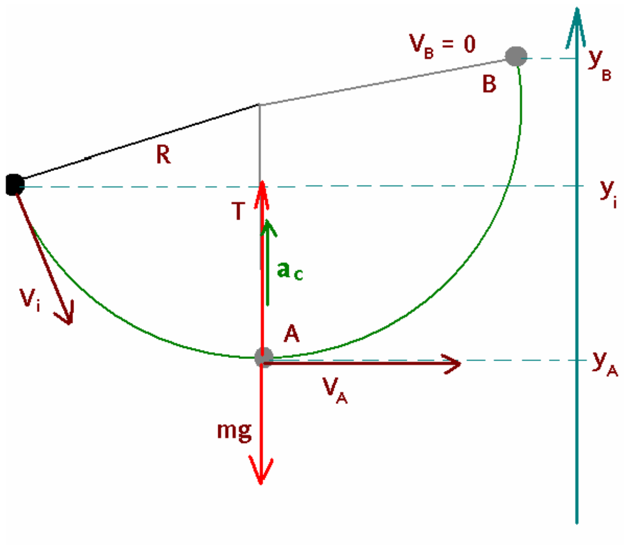


Problem

Two forces are acting: T and mg

$$K_2 - K_1 = W_{total} = W_T + W_{mg}$$

$W_T = 0$

$W_{mg} = mg(y_i - y)$

And we can set $y_A = 0$

Hence,

$$\tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_i^2 = mg(y_i - y)$$

Point A: $\tfrac{1}{2}mv_A^2 - \tfrac{1}{2}mv_i^2 = mgy_i$ and we can solve it for v_A

Point B: $\tfrac{1}{2}mv_B^2 - \tfrac{1}{2}mv_i^2 = mg(y_i - y_B)$ or $-\tfrac{1}{2}mv_i^2 = mg(y_i - y_B)$

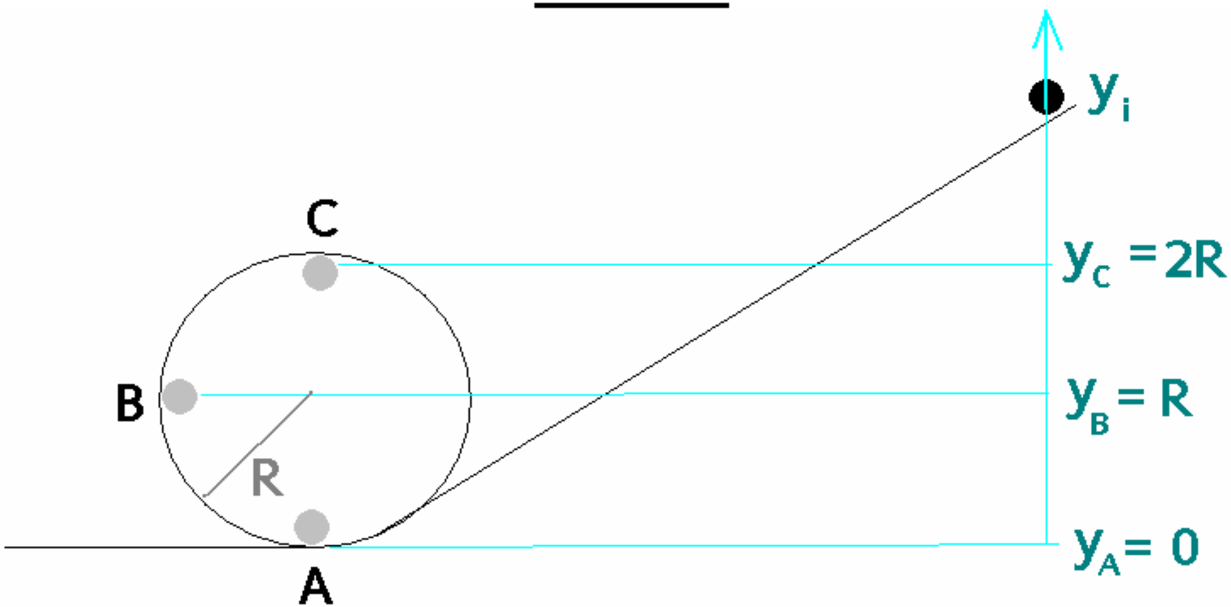
And we can solve it for y_B

Moving from rest from B to A: $\tfrac{1}{2}mv_A^2 = mgy_B$ (and solve for v_A)

Maximum speed is reached at the point A: $ma_c = T_{max} - mg$

$$a_c = v_A^2/R$$

Problem

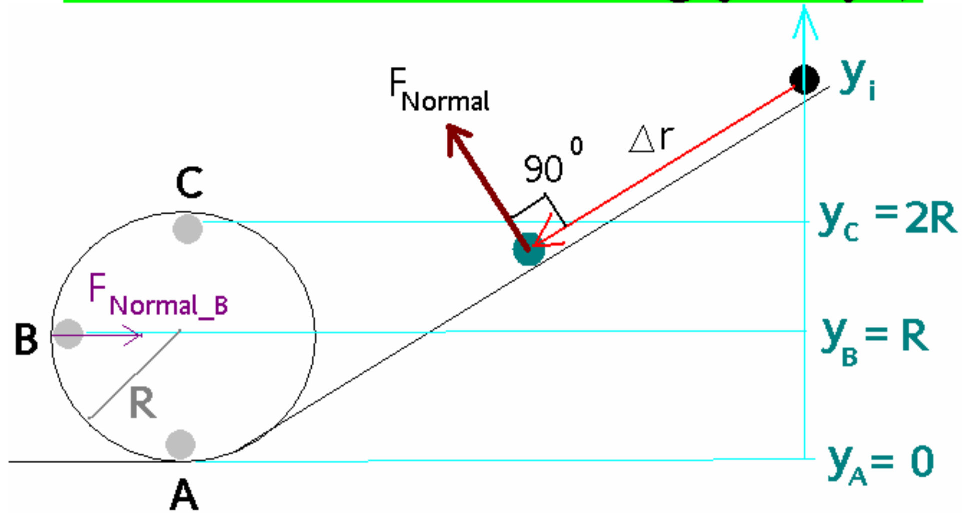


The ball is *sliding* down in the loop-the-loop demo.

(We are assuming there is no friction, so, the ball is *not* rolling; the rolling ball will be a *different* problem)

Compare the normal force acting on the ball at points A, B, and C.
What is the minimum height to get through the loop?

$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = mg(y_1 - y_2)$$



Point B (from y_i): $y_B = R \quad v_i = 0$

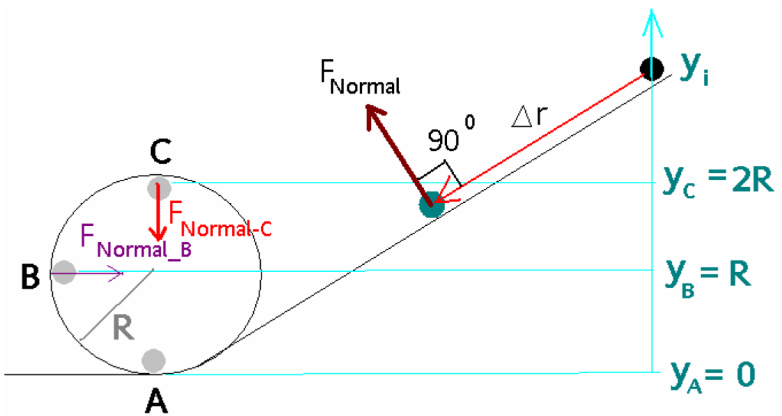
$$\frac{1}{2}mv_B^2 = mg(y_i - R)$$

We can solve it for v_B

From the N II L: $F_{Normal\text{-}B} = ma_{c\text{-}B}$ and $a_{c\text{-}B} = v_B^2/R$

That gives us the values of the $F_{Normal\text{-}B}$ at the pint B.

$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = mg(y_1 - y_2)$$



Point C (from y_i):

$y_C = 2R \quad v_i = 0$

$$\frac{1}{2}mv_C^2 = mg(y_i - 2R)$$

From the N II L:

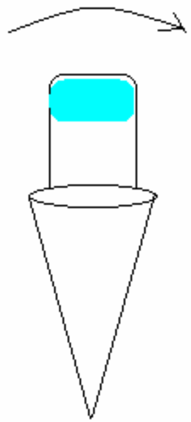
$F_{Normal\text{-}C} + mg = ma_{c\text{-}C}$ and $a_{c\text{-}C} = v_C^2/R$

To get through the loop-the-loop, the ball has to get through the pint C, hence the minimum height y_i can be found from the condition $F_{Normal\text{-}C} = 0$ (the minimum possible values of the normal force). This gives us the known result, $a_{c\text{-}C} = g$.

That leads to the condition on the velocity $v_C^2/R = g$.

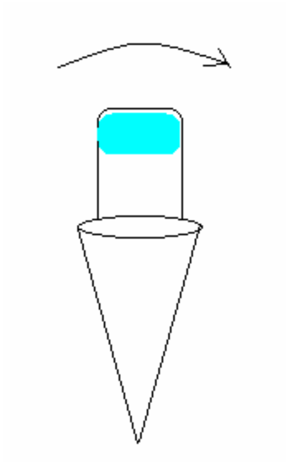
An finally, to the equation on the y_i $\frac{1}{2}mgR = mg(y_i - 2R)$

The solution is $y_i = 5R/2$ (for the sliding ball!)



A bucket with water in it is rotating in a vertical plane.

What is the minimum speed so the water stays in the bucket?



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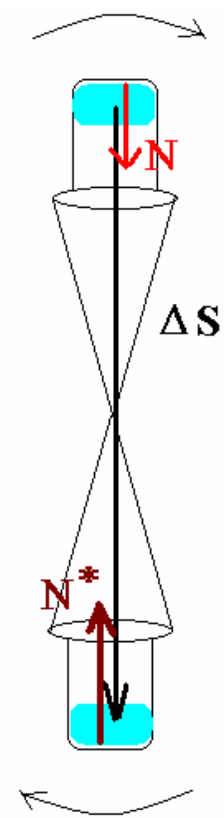

What work is done by the normal force acting on the water from the bucket over a half of the period?

Two more practice problems

(Use the discussion sections and office hours if you need a help)

Problem 1

An object of mass m = 0.50 kg is tied to a string and whirled in a vertical circle of radius r = 1.0 m. The string breaks at the tension of 13 N. What is the minimum speed of the ball required to break the string?

Problem 2

A water bucket of mass m is being whirled in a vertical circle of radius r. What is the minimum speed required so that the water remains in the bucket?

Conceptual question

When you are on a roller coaster or traveling fast on a hilly road, when you at the bottom of the loop-the-loop (or the valley) you fell pressed into your seat. But when you are at the top of the loop-the-loop (or the hill), you feel almost weightless. How can you explain this (Use a free body diagram and Newton's Laws)?