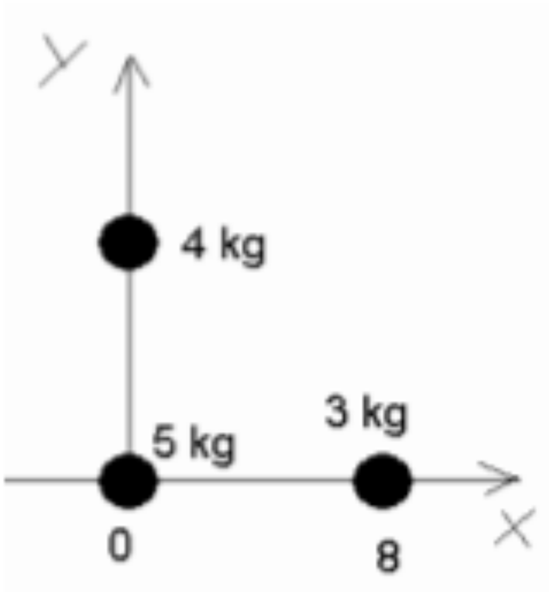
Problem



Three balls of a different mass are located as shown.

The fourth ball of what mass should be placed so the square made of the four balls would have $X_{center\text{-}of\text{-}mass} = 4$?

$m_4 = \ldots$ (kg)

1

2

3

4

5

Also find the $Y_{center\text{-}of\text{-}mass}$ in this situation.

$$X_{cm} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$$

This question is anonymous

I have taken physics course …

1. Never before (this is the first time I take physics)
2. Fairly recently (no more than 3 years ago)
3. A long time ago

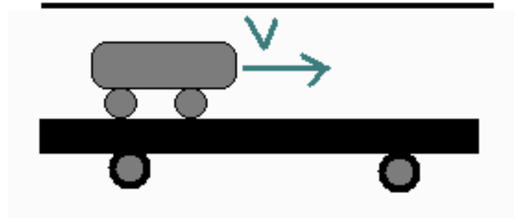
This is the *definition* of ***the center of mass*** of a system.

$$\sum_{\text{all EXTERNAL forces on the system}} F = M_{system} \bullet a_{CM}$$

The question is – what is happening to the CofM if all external forces are cancelling each other out?

1. It moves with a constant velocity
2. It is speeding up
3. It is slowing down
4. None of the above

A cart on a track



When a cart is moving on a track to the right, in what direction is ***the CM*** of the system moving?

A) To the right   B) To the left

C) Stays at the same location   D) Impossible to tell

$$\sum_{\text{all EXTERNAL forces on the system}} F = M_{system} \bullet a_{CM}$$

When the net external force is 0,
we call the system isolated

For an isolated system $$\sum_{\text{all EXTERNAL forces on the system}} F = 0$$

In this case

$$0 = M_{system} \bullet a_{CM}$$

or $a_{CM} = 0$ and $V_{CM} = \text{const}$

The CM of an isolated system has a constant velocity

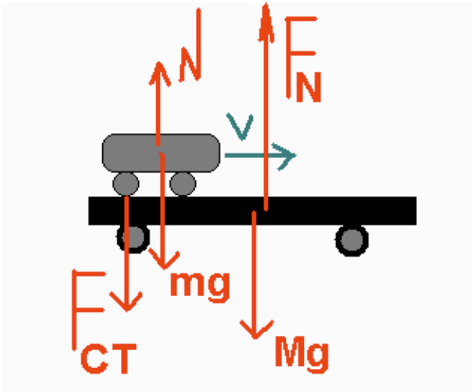
This is the *definition* of ***the center of mass*** of a system.

$$\sum_{\text{all EXTERNAL forces on the system}} F = M_{system} \bullet a_{CM}$$

When all external forces are cancelling each other out
CofM moves with a constant velocity!

=> For an isolated system: **If the initial velocity of the CofM is 0 => it remains being 0!**

The CofM of an isolated system does not move!



The cart is moving on a track to the right.

<u>The cart and the track make a system.</u>

All the external forces on ***all*** the objects of the system (the cart and the track) are directed ***vertically***, hence, cancel out each other, hence, the net external force is **B) 0 N**

$$\sum_{all\ EXTERNAL\ forces\ on\ the\ system} F = 0$$

From the Newton's II Law for the system

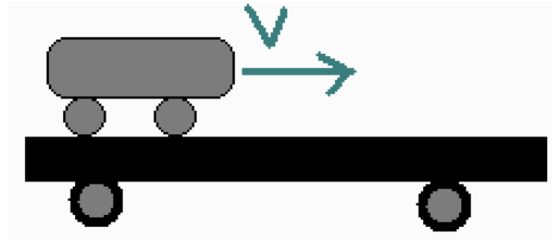
$$\sum_{all\ EXTERNAL\ forces\ on\ the\ system} F = M_{system} \bullet a_{CM}$$

Hence, $a_{CM} = 0$.

Because the initial velocity of the system is 0, the initial velocity of the CM is 0 and stays the same (no acceleration!).

Hence, the CM is NOT moving!!!

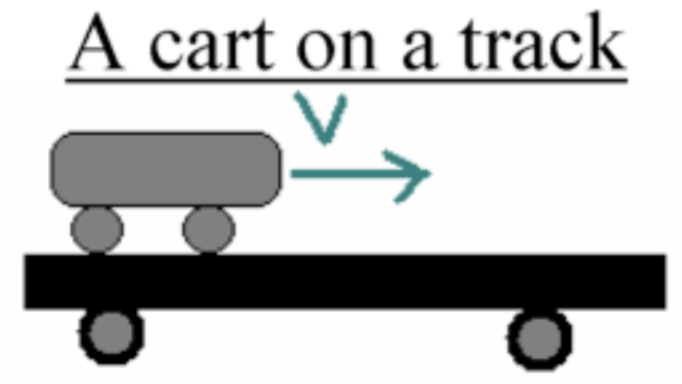
A cart on a track



When a cart is moving on a track to the right, in what direction is the CM of the system moving?

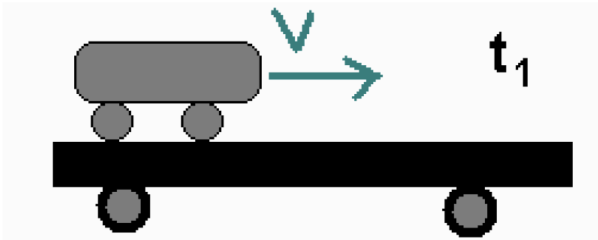
The cart is moving to the right, the track is moving to the left, but the CM of the system is NOT moving!

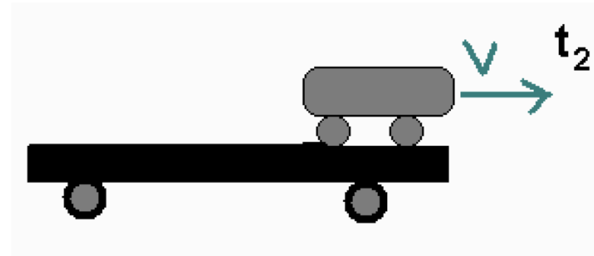
(Can we ignore friction in this situation?)



M_{cart} = 1.5 kg; M_{track} = 500 gram;
V_{track_table} = 16 cm per 4 s.
Find V_{cart_track}

A cart on a track





A cart is moving on a track to the right (the initial state).

If the cart moved 30 cm to the right, what would be the displacement of the track?

We solve a similar problem instead

A 90 kg man sits at one end of a 2.4 meter canoe that has a mass of 30 kg.

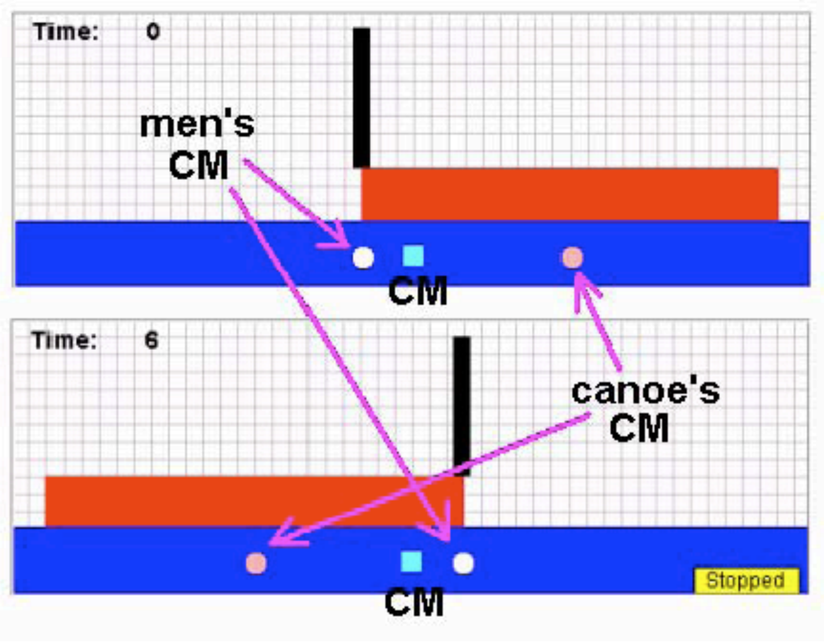
If the man moves to the opposite end of the canoe, how far does the canoe move?

Problem

SIM

A 90 kg man sits at one end of a 2.4 meter canoe that has a mass of 30 kg.

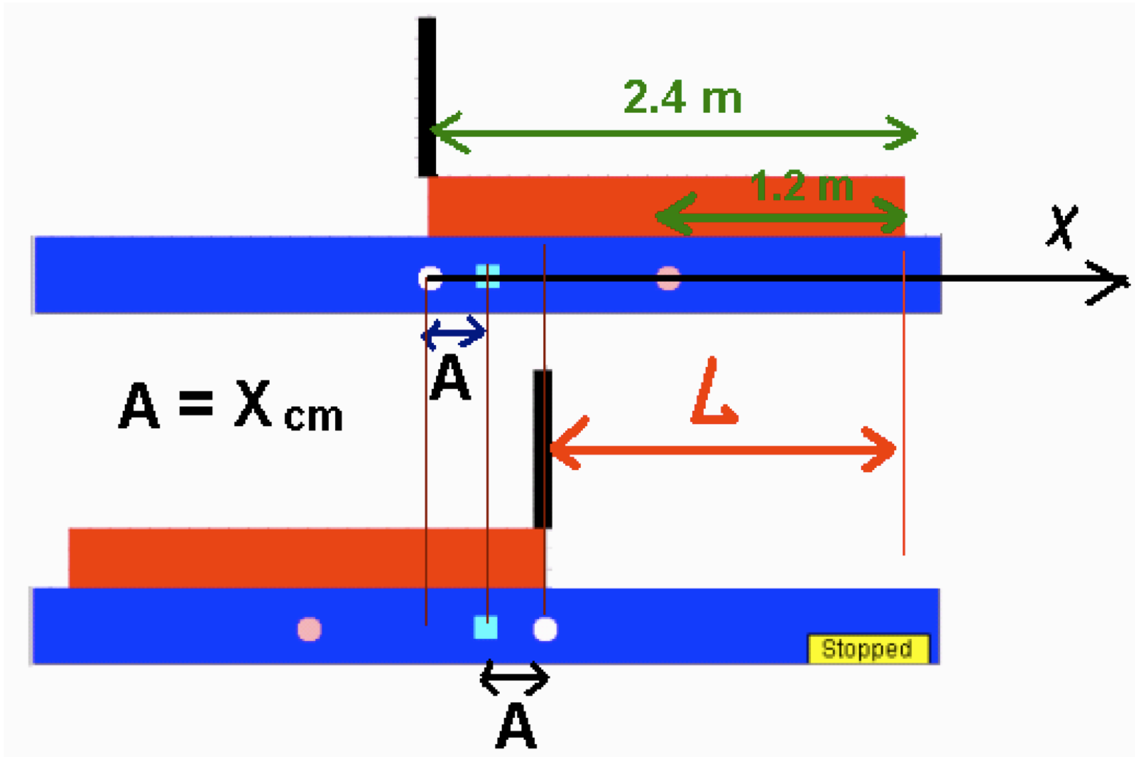
If the man moves to the opposite end of the canoe, how far does the canoe move?



We can find the CM first.

$$X_{CM} = \frac{90 \cdot 0 + 30 \cdot 1.2}{90 + 30} = 0.3\,m$$

Now this problem can be solved purely geometrically!



$$A = X_{CM} = 0.3 \text{ m}$$

$$L = 2.4 - A - A = 2.4 - 0.3 - 0.3 = 1.8 \text{ m}$$

PRINCIPLE OF CONSERVATION OF LINEAR MOMENTUM

The total linear momentum of an isolated system is constant (conserved). An isolated system is one for which the sum of the average external forces acting on the system is zero.

$$\vec{\mathbf{P}}_{\mathbf{f}} = \vec{\mathbf{P}}_{\mathbf{o}}$$

When $\sum_{all\ EXTERNAL\ forces\ on\ the\ system} F = 0$

the total linear momentum of the system is conserved!

Question: car crash

You are driving at high speed along a divided, multi-lane highway when all of sudden you see your evil twin, driving an identical car, going the wrong way and coming directly toward you. You both slam on your brakes, but it's too late to stop and there is about to be a collision. At the last instant you spot a large, very solid immovable object by the side of the road. Which is better for you, to hit your evil twin or to swerve and hit the immovable object instead?

Assume the speed when you collide is the same whether you hit your evil twin or the immovable object, and that your evil twin is going at the same speed you are. Either collision is a head-on collision.

It is better for you to:

1. Hit your evil twin
2. Hit the immovable object
3. Neither one - they're equivalent as far as you're concerned

The answer is that it doesn't matter. Either collision causes you to come to a complete stop in a distance equal to how far the front of your car collapses, which will be the same in both cases.

Ice Skaters

Starting from rest, two skaters push off against each other on ice where friction is negligible.

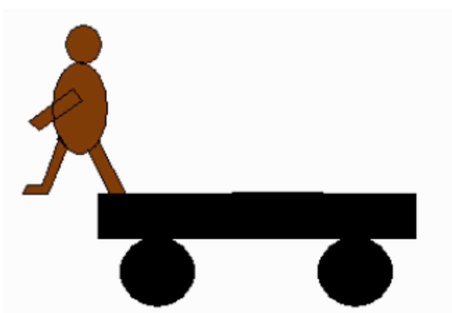
One is a 54-kg woman and one is a 88-kg man. The woman moves away with a speed of +2.5 m/s. Find the recoil velocity of the man.

The completely similar solution you would have to use for solving the problems like the following:

<u>The gun fires a bullet</u>

or

<u>A man (dog) jumps of the cart (or boat)</u>



or

<u>A grenade explodes into two parts</u>

Some special cases (inelastic collision or explosion)

An explosion

Two carts are placed together at rest. A spring-loaded piston attached to one cart is then released, causing the carts to move in opposite directions. What is conserved in this process?

1. Momentum but not kinetic energy
2. Kinetic energy but not momentum
3. Both momentum and kinetic energy
4. Neither momentum nor kinetic energy

An explosion

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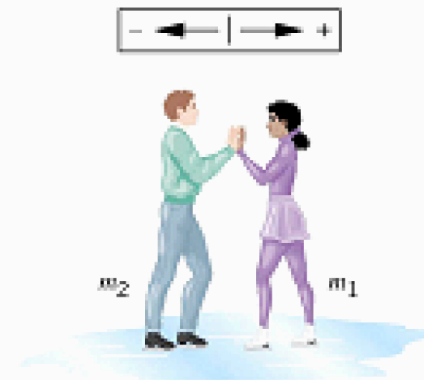

(a) Before

(b) After

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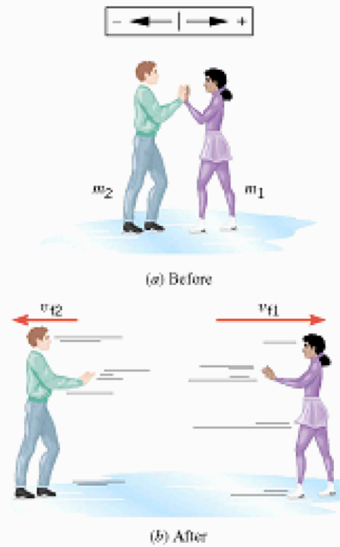

The total linear momentum of the system is conserved!

$$P_{system} = const$$

$$P_{i\text{-}system} = P_{f\text{-}system}$$

$$P_{iW} + P_{iM} = P_{fW} + P_{fM}$$

$$m_W \mathbf{V}_{iW} + m_M \mathbf{V}_{iM} = m_W \mathbf{V}_{fW} + m_M \mathbf{V}_{fM}$$

$$54*0 + 88*0 = 54*2.5 + 88*V_{fM}$$

$$0 = 54*2.5 + 88*V_{fM}$$

$$V_{fM} = -54*2.5/88 \text{ m/s}$$

Ice Skaters

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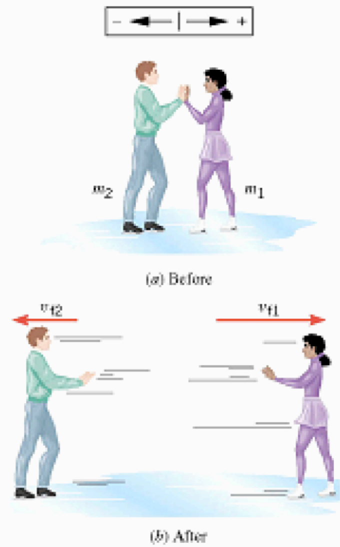


The total linear momentum of the system is conserved!

$$P_{system} = const$$

$$P_{i\text{-}system} = P_{f\text{-}system}$$

Is kinetic energy of the system conserved?

1. Yes
2. No
3. We need more information

Collisions and Elasticity

During a collision the objects involved generally apply equal-and-opposite forces on one another for a short time. There are usually no external forces or they are canceling each other, so the momentum of the system of objects is conserved.

Generally, **momentum is conserved** in all types of collisions.

There are *four* classes of collisions based on what happens during the collision and, in particular, what happens to the total kinetic energy of the system.

The elasticity of the collision is related to the ratio of the relative velocities of the two colliding objects after and before the collision:

$$k = \left| \frac{V_{2f} - V_{1f}}{V_{2i} - V_{1i}} \right|$$

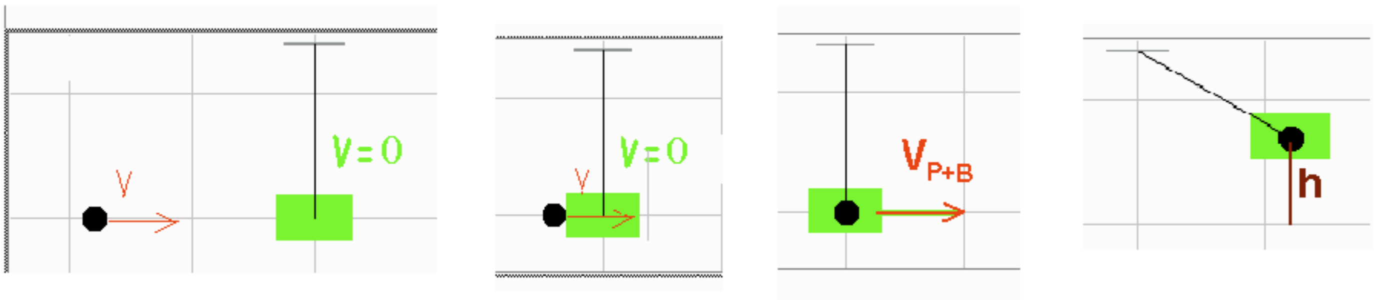
<u>The elasticity is related to the type of collision as follows (Sim. 1):</u>

TKEoS stands for Total Kinetic energy of the System

Type of Collision	**Description**	**Elasticity**
Super-elastic	TKEoS is larger after the collision (e.g., an explosion)	**k > 1**
Elastic	TKEoS is conserved (*no friction*)	**k = 1**
Inelastic	TKEoS is smaller after the collision (*some of kinetic energy lost because of friction*, *but the object are still separated*)	**k < 1**
Completely inelastic	TKEoS is smaller (lost because of friction), *and the objects stick together after the collision* (they make one new object)	**k = 0**

Ballistic Pendulum

Start	Before the collision	After the collision	End



Two pictures in the middle represent a collision

What kind of a collision is it?

A) Super-elastic

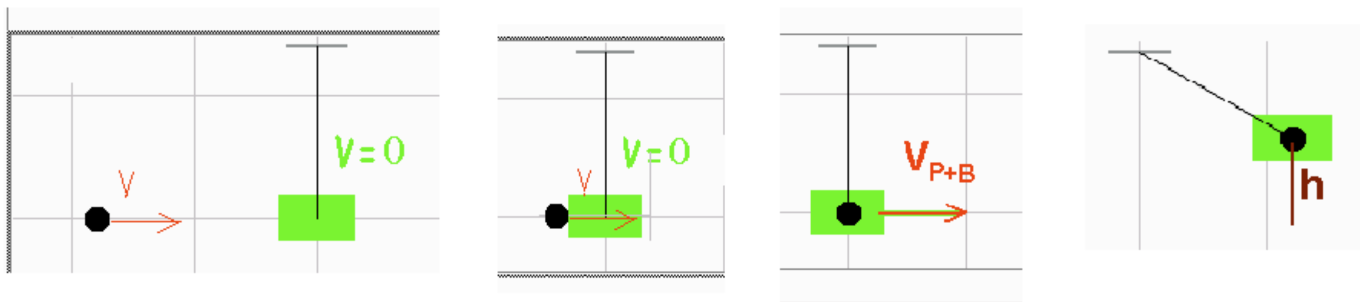
B) Elastic

C) Inelastic

D) Completely inelastic

Ballistic Pendulum

Start	Before the collision	After the collision	End



Two pictures in the middle represent
an *absolutely inelastic collision*

$m_B V_B = (m_B + M_P) V_{P+B}$ (Conservation of Momentum)

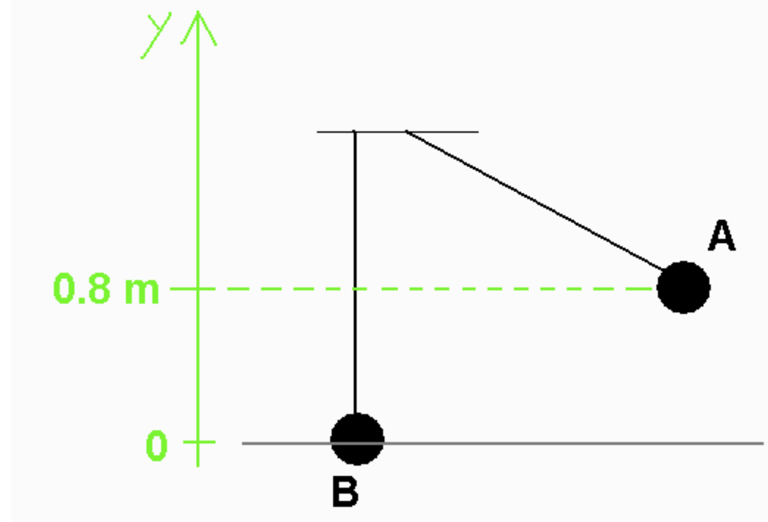
***During* the collision the mechanical energy of the system is *not* conserved!**

But *after* it, E is conserved!

$$\frac{(m_B + M_P) V_{P+B}^{\ 2}}{2} = (m_B + M_P) gh$$

Two balls

Two balls hang from strings of the same length.



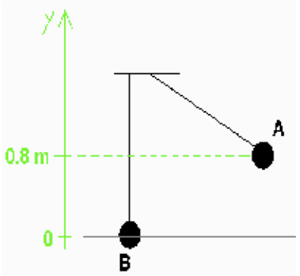
Ball A, with a mass of 4 kg, is swung back to a point 0.8 m above its equilibrium position an then released from rest and swings down and hits ball B.

After the collision ball A rebounds to a height of 0.2 m above its equilibrium position, and ball B swings up to a height of 0.05 m.

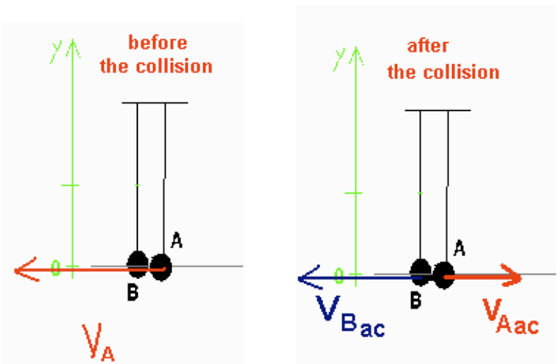
What is the mass of ball B? Is the collision elastic?

Two balls

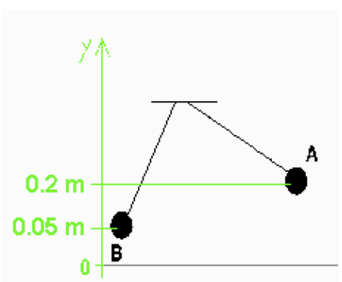
Let's analyze what is happening.



1) The ball A is moving down form rest to the point where it touches the ball B (Is the mechanical energy conserved?)



2) The ball A collides with the ball B (What kind of a collision is it? What is happening with the velocities of the balls?)

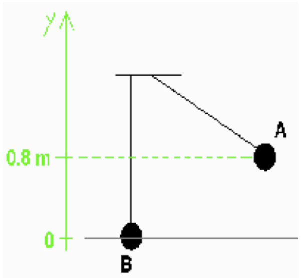


3a) The ball A rebounds to a height of 0.2 m above its equilibrium position (Is the mechanical energy conserved?)

3b) The ball B swings up to a height of 0.05 m (Is the mechanical energy conserved?)

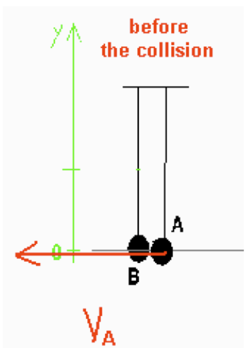
Two balls

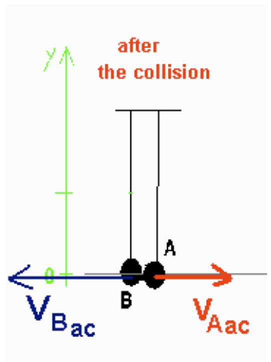
Let's analyze what is happening.



1) The ball A is moving down from rest to the point where it touches the ball B . No friction; Mechanical energy is conserved!

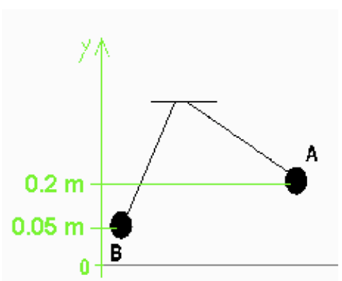
$$m_A g h_A = \frac{m_A V_A^2}{2}$$





2) The ball A collides with the ball B. We do not know if the collision is elastic (W_{nc} might be not 0).

$$m_A V_A = -m_A V_{Aac} + m_B V_{Bac} \qquad \frac{m_A V_A^2}{2} + W_{nc} = \frac{m_A V_{Aac}^2}{2} + \frac{m_B V_{Bac}^2}{2}$$

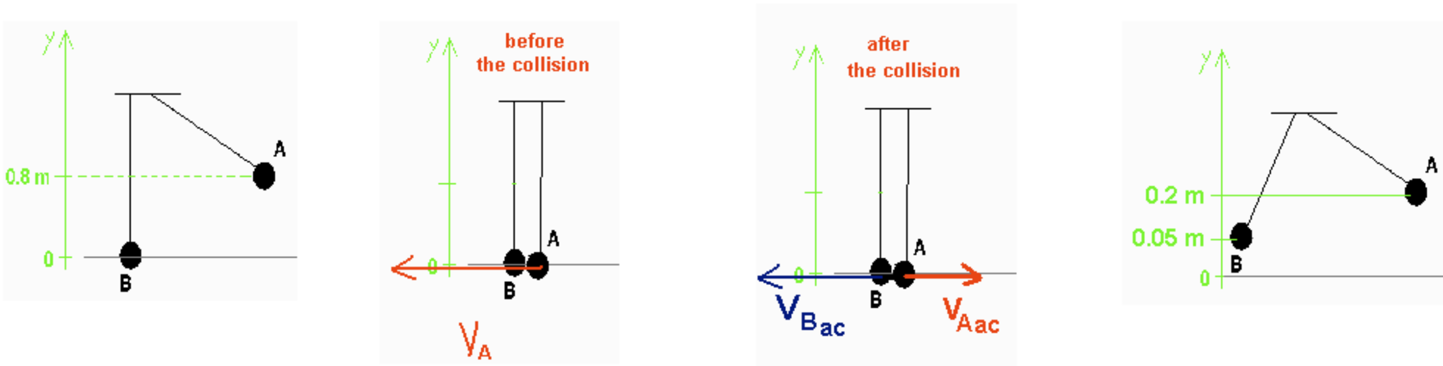


3a) The ball A rebounds to a height of 0.2 m above its equilibrium position
3b) The ball B swings up to a height of 0.05 m. Mechanical energy is conserved!

$$\frac{m_A V_{Aac}^2}{2} = m_A g h_{Aac} \qquad \frac{m_B V_{Bac}^2}{2} = m_B g h_{Bac}$$

Two balls

Let's analyze what is happening.



1) $m_A g h_A = \dfrac{m_A V_A^{\ 2}}{2}$ 2) $m_A V_A = -m_A V_{Aac} + m_B V_{Bac}$ 3) $\dfrac{m_A V_A^{\ 2}}{2} + W_{nc} = \dfrac{m_A V_{Aac}^{\ 2}}{2} + \dfrac{m_B V_{Bac}^{\ 2}}{2}$

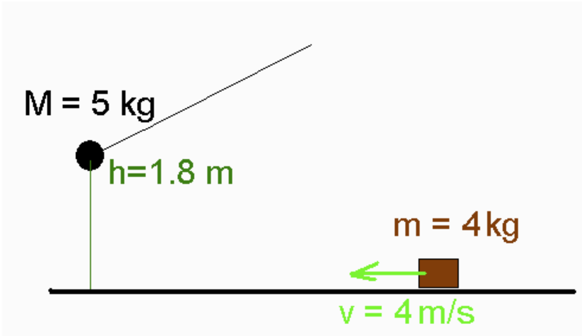
4) $\dfrac{m_A V_{Aac}^{\ 2}}{2} = m_A g h_{Aac}$ 5) $\dfrac{m_B V_{Bac}^{\ 2}}{2} = m_B g h_{Bac}$

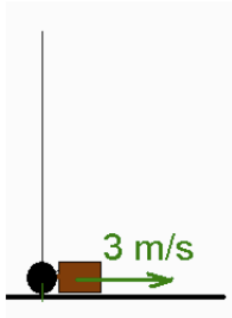
(1) gives $V_A = \sqrt{2g \cdot 0.8}$ | (4) gives $V_{Aac} = \sqrt{2g \cdot 0.2}$ | (5) gives $V_{Bac} = \sqrt{2g \cdot 0.05}$

Now, from (2) we can find m_B and from (3) we can find W_{nc} .

If $W_{nc} = 0$, the collision is elastic (or inelastic otherwise).

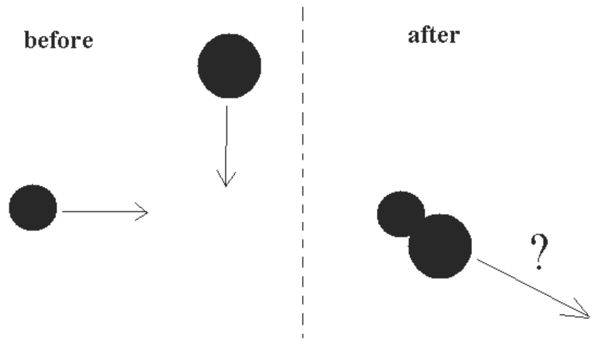
Problem





The 5 kg ball is released from the height of 1.8 m. When it reaches the lowest point it hits the block initially moving towards the ball at the speed of 4 m/s. After the ball collides with the block, the block reverses its velocity and moves at the speed of 3 m/s. Find the velocity of the ball immediately after the collision. What kind of a collision is it?

Example



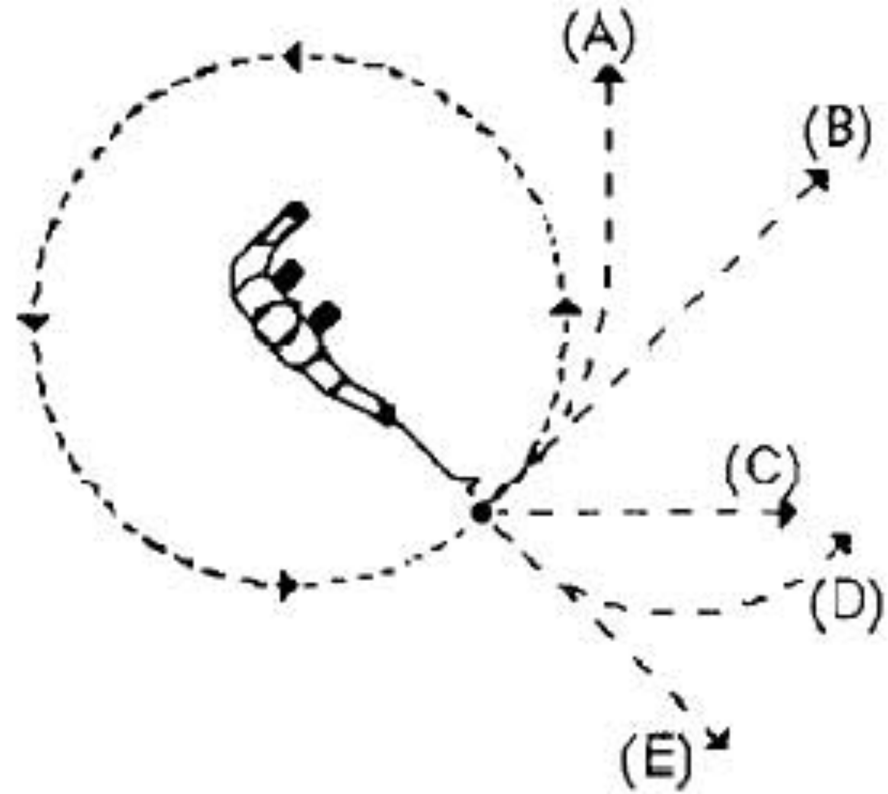
Two pucks collide and stick to each other.

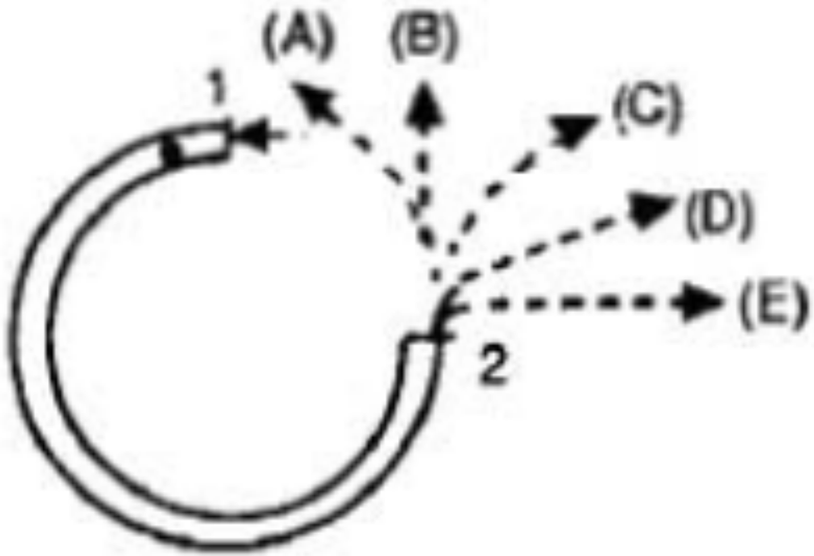
The big one is twice heavier of the small one.

Before the collisions their velocities are perpendicular.

Try to find (algebraically) the velocity (speed and directions) of the pucks after the collision.

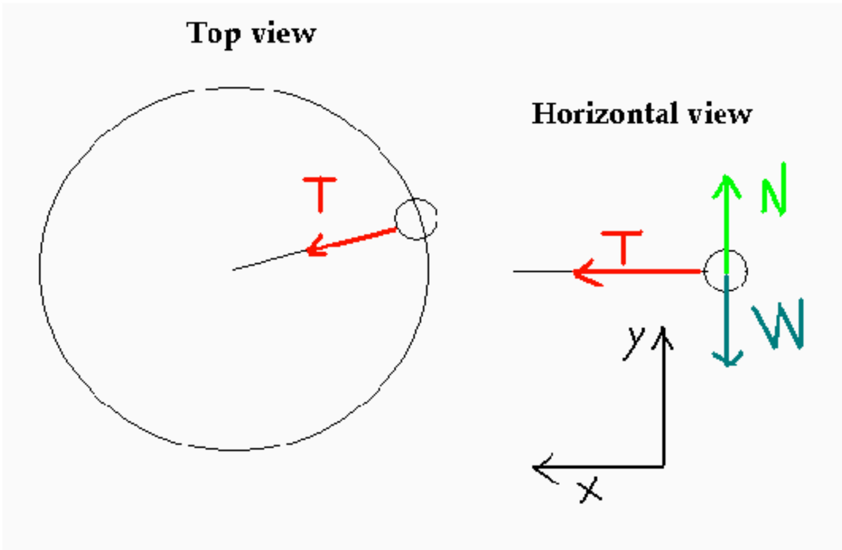
A heavy ball is attached to a string and swung in a circular path In a horizontal plane as illustrated in the picture. At the point indicated in the picture, the string suddenly breaks. If these events were observed from directly above, indicate the path of the ball after the string breaks.



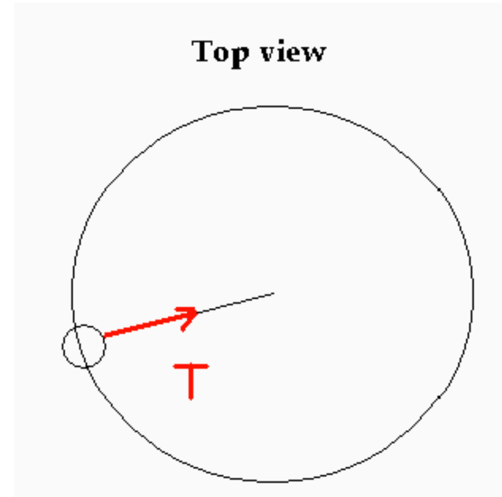


The picture shows a channel placed on a horizontal tabletop, and a small ball moving through it, entering the channel at point 1 and leaving it at point 2. Which of the path representations would most accurately correspond to the path of the ball as it exits the channel and rolls on a table top?

FBD at one moment of time



FBD at another moment of time



What is the direction of the acceleration?

1. up (like N)
2. to the center (like T)
3. down (like W)

(Circular motion belongs to Test II)

Uniform Circular Motion

Uniform circular motion is motion in a circular path at constant speed.

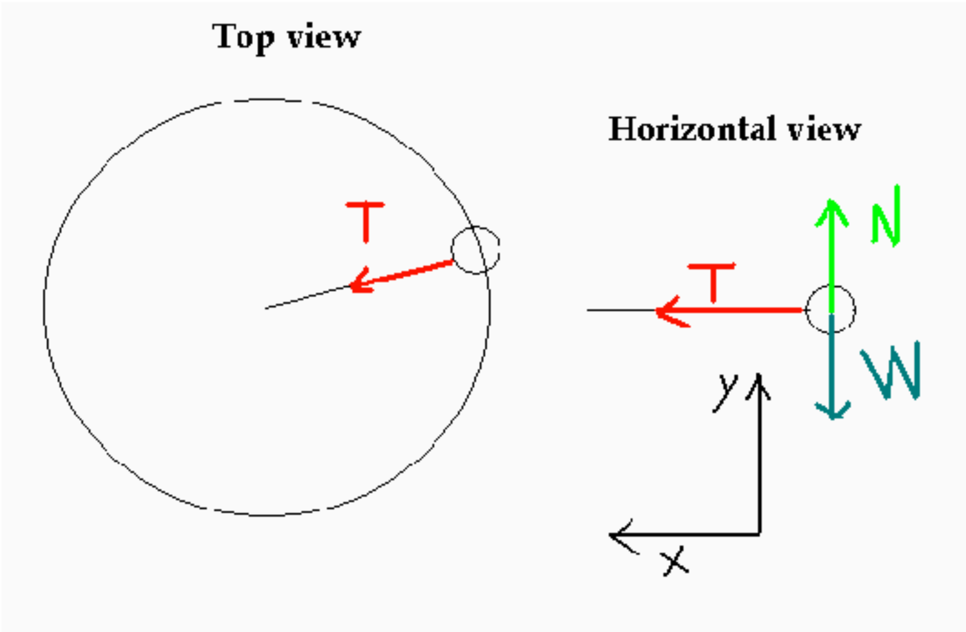
What can we tell about acceleration in this case?

There is non=zero acceleration, $a \neq 0$

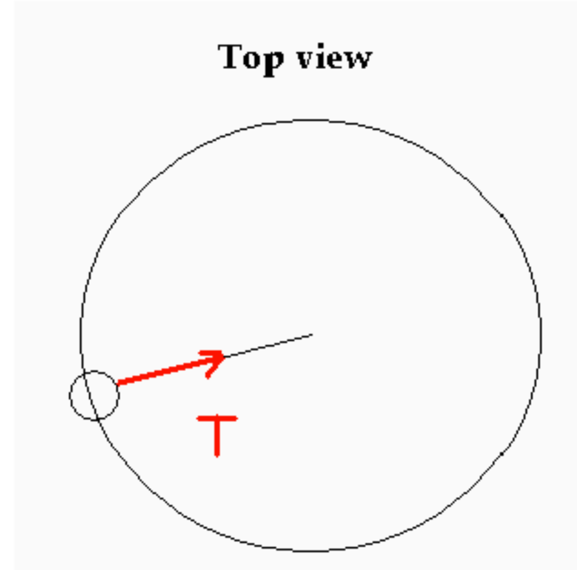
Let's use an example to investigate the uniform circular motion (a billiard ball on a string running on the surface of the table)

How many forces are acting on the ball (neglect friction)?

FBD at one moment of time



FBD at another moment of time



What is the direction of the acceleration?

To the center of the circle

Conclusion

When an object undergoes a Uniform Circular Motion the object has acceleration which is directed to the center to the circular path!

If **V** is the speed (!) of the object and **R** is the radius of the circle, *the magnitude* of the acceleration can be found as $\mathbf{a = V^2/R}$ (see the textbook for the proof). **The name is:**

Centripetal (or normal) acceleration!

Summary

Centripetal acceleration

The centripetal acceleration is the special form the acceleration has when an object is experiencing uniform circular motion. It is:

$$a_c = \frac{v^2}{r}$$

v = ANY !!

and is directed toward the center of the circle.

Newton's second law can then be written as:

$$\Sigma \mathbf{F} = ma = \frac{m\,v^2}{r}$$

v = const !!

A magical force "centripetal force" *do not exists!!!*

Inertia is the reason for a circular motion!

However, as a *term* "centripital force" just means ma_c

$$\frac{m\,v^2}{r}$$

A magical force "centripetal force" *do not exists!!!*

Inertia is the reason for a circular motion!