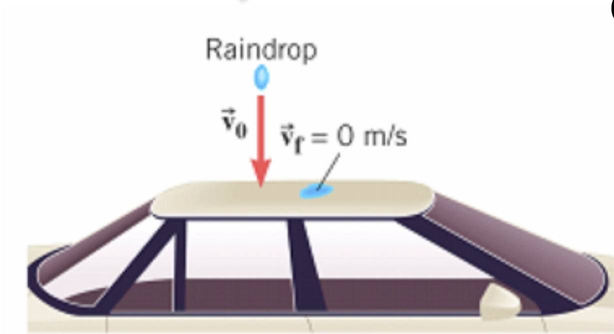
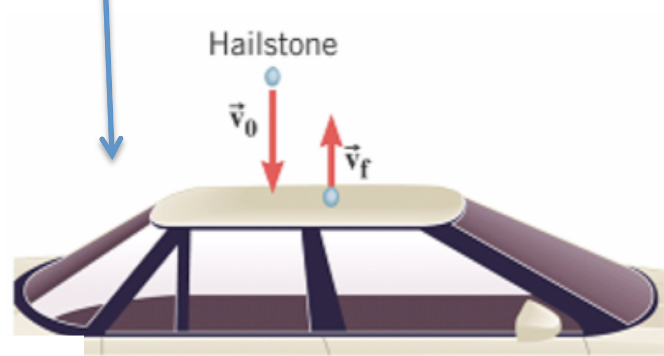
Rain comes down with a velocity of -15 m/s and hits the roof of a car. The mass of rain per second that strikes the roof of the car is 0.060 kg/s. Assuming the rain comes to rest upon striking the car, find the average force exerted by the rain on the roof (neglect the wait of the droplets).

Now hail comes down with a velocity of -15 m/s and hits the roof of a car. The mass of hail per second that strikes the roof of the car is 0.060 kg/s. Assuming the hailstones bounce up from the roof (and leave it) with the velocity of 15 m/s, find the average force exerted by the hail on the roof.



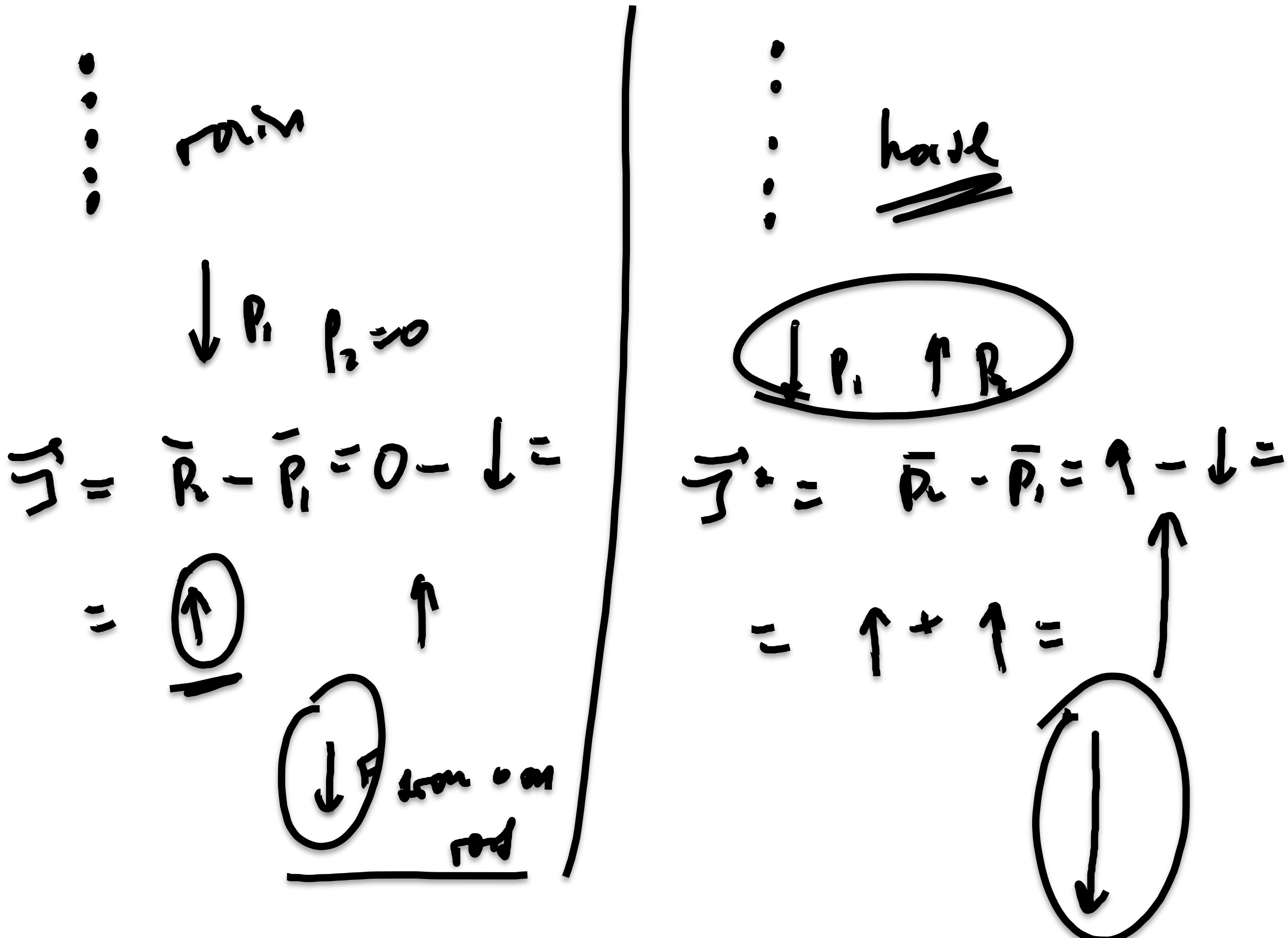
Compare the pressure on the roof:

1. $P_r > P_h$
2. $P_r = P_h$
3. $P_r < P_h$



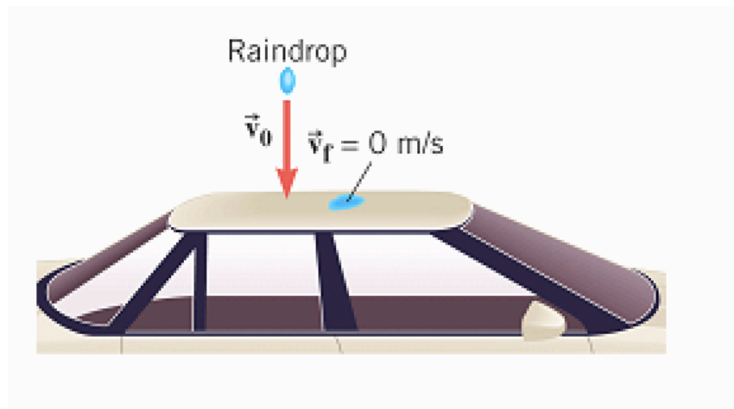
$$\boldsymbol{P} = m\boldsymbol{v}$$

$$F_{net}\Delta t = \boldsymbol{P_2} - \boldsymbol{P_1}$$


main
hard
P1
P2 = 0

Problem

Rain comes down with a velocity of -15 m/s and hits the roof of a car. The mass of rain per second that strikes the roof of the car is 0.060 kg/s. Assuming the rain comes to rest upon striking the car, find the average force exerted by the rain on the roof (neglect the wait of the droplets).



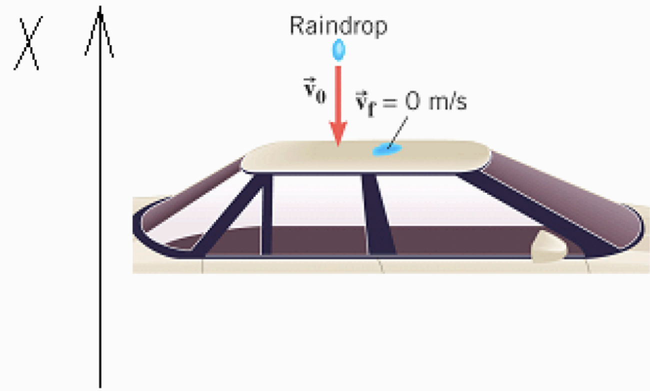
$$\boldsymbol{F}_{net}\Delta t = \boldsymbol{P}_2 - \boldsymbol{P}_1$$

$$\boldsymbol{P} = m\boldsymbol{v}$$

$\frac{m}{\Delta t} = 0.060$ kg/s. $V_0 = -15$ m/s. $V_f = 0$ m/s

$F_{net}\Delta t = P_2 - P_1$

$P = mv$



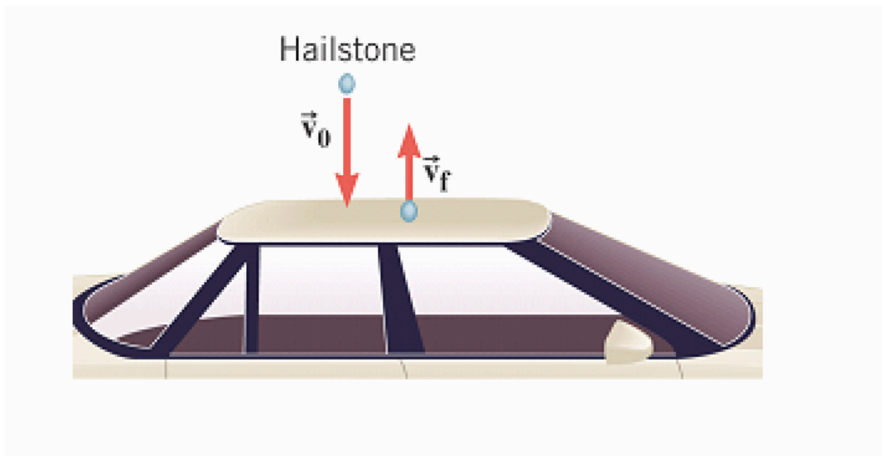
$$\vec{\mathbf{F}}\,\Delta t = \cancel{m\vec{\mathbf{v}}_f}^{\,0} - m\vec{\mathbf{v}}_o \;\longrightarrow\; \vec{\mathbf{F}} = -\left(\frac{m}{\Delta t}\right)\vec{\mathbf{v}}_o$$

$$\vec{\mathbf{F}} = -(0.060\,\text{kg/s})(-15\,\text{m/s}) = +0.90\,\text{N}$$

The average force exerted *by the rain on the roof* is $-\vec{\mathbf{F}} = -0.90$N

Problem

Now hail comes down with a velocity of -15 m/s and hits the roof of a car. The mass of hail per second that strikes the roof of the car is 0.060 kg/s. Assuming the hailstones bounce up from the roof (and leave it) with the velocity of 15 m/s, find the average force exerted by the hail on the roof.



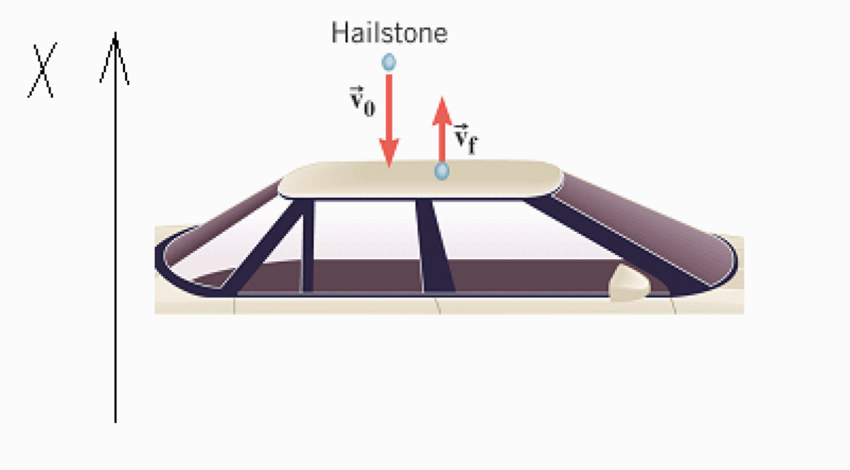
$$F_{net}\Delta t = P_2 - P_1 \qquad P = mv$$

Solution

$$\frac{m}{\Delta t} = 0.060 \text{ kg/s.} \qquad V_i = -15 \text{ m/s.} \qquad V_f = 15 \text{ m/s}$$

$$F_{net}\Delta t = P_2 - P_1$$

$$P = mv$$



The force acting on the hailstones from the roof is

$$\vec{F}\Delta t = m\vec{v}_f - m\vec{v}_o \Longrightarrow \vec{F} = \left(\frac{m}{\Delta t}\right)(\vec{v}_f - \vec{v}_o)$$

$$\vec{F} = 0.060\,(15 - (-15)) = 1.8 \text{ N}$$

The average force exerted *by the hail on the roof* is $-\vec{F} = -1.8$ N

Happy and Sad Balls

Why is it happening?

State the observed difference between the two situations.

Which ball exerts larger force on the block?

Happy Ball	Sad Ball
$F_h \Delta t_h = P_{h\,2} - P_{h\,1}$	$F_s \Delta t_s = P_{s\,2} - P_{s1}$
$F_h \Delta t_h = -P - P = -2P$	$F_s \Delta t_s = 0 - P = -P$

$$|F_h| > |F_s|$$

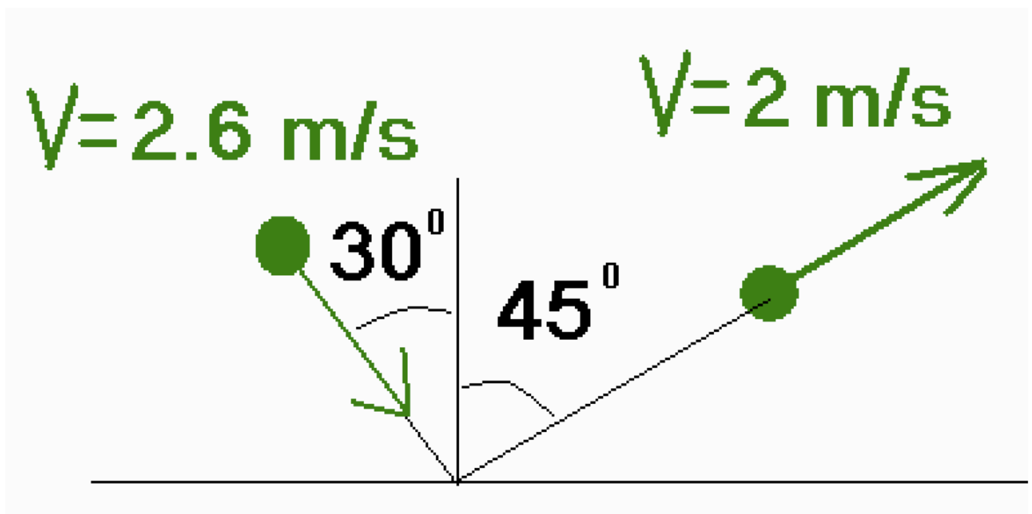
Happy!

Problem

A tennis ball of the mass of 100 g hits the ground and bonuses up again (see the picture).

Find the impulse of the force the ball exerts on the ground *normally* to it when it hits it.

If the contact time is 0.03 s, what is the magnitude of the force the ball exert on the ground?



Problem

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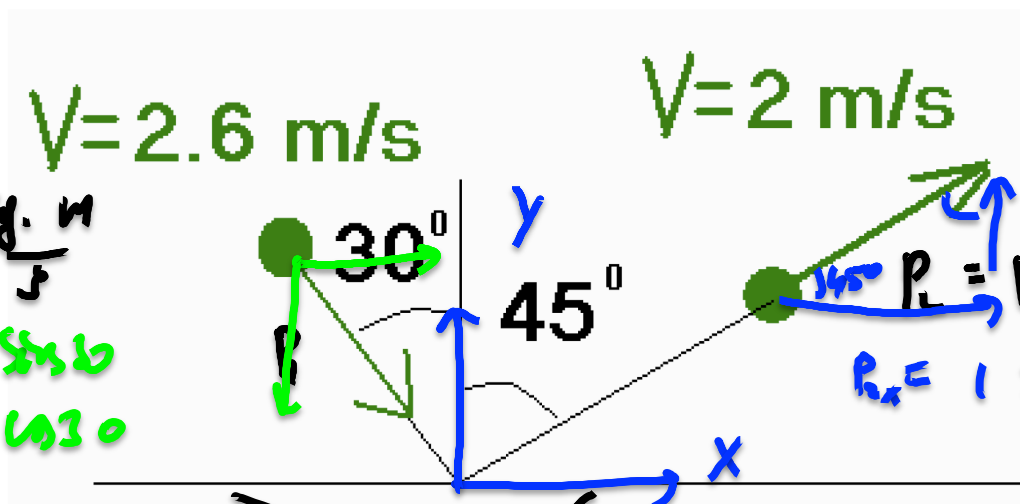


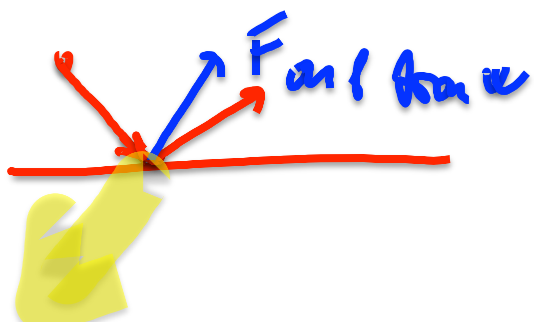
$F_{net}\Delta t = P_2 - P_1$

$P = mv$

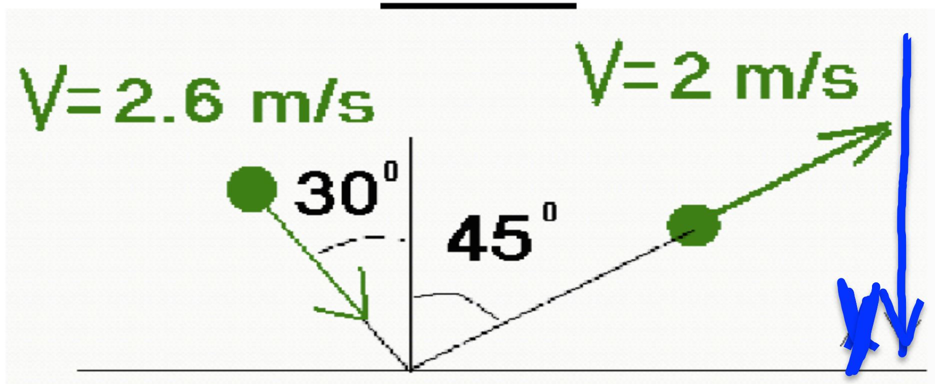
$$F_x \Delta t = J_x = P_{2x} - P_{1x} = 0.2 \cdot \frac{\sqrt{2}}{2} - 0.26 \cdot \frac{1}{2} = \_\_\_$$

$$F_y \Delta t = J_y = P_{2y} - P_{1y} = 0.2 \cdot \frac{\sqrt{2}}{2} - \left(-0.26 \cdot \frac{\sqrt{3}}{2}\right) = \_\_\_$$

$$F_x = \frac{\_\_\_}{0.03} = N \qquad F_y = \frac{\_\_\_}{0.03} = N$$



Solution



The force acting from the ball on the ground has the same magnitude as the force acting on the ball from the ground (that force is the reason the ball changes its direction). For the impulse of the force acting on the ball we can write $\boldsymbol{J} = \boldsymbol{F}_{net}\Delta t = \boldsymbol{P}_2 - \boldsymbol{P}_1 = m\mathbf{v}_2 - m\mathbf{v}_1$. We need to use the x-component of this equation only.

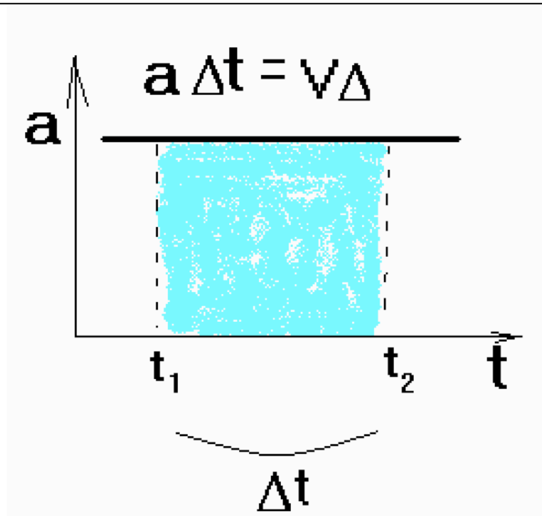
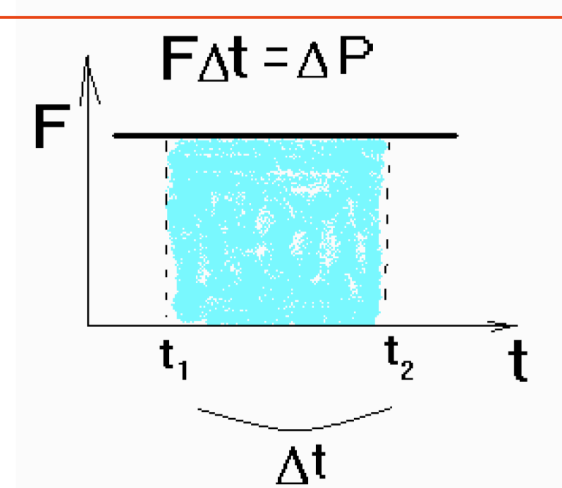
$J_x = -mv_2\cos 45 - mv_1\cos 30^0 = -0.1*2*\cos 45^0 - 0.1*2.6*\cos 30^0 = -\ 0.367$ Ns
(It is a negative number as it is supposed to be)

The normal component of the force acting on the ground is 0.367 Ns

The average force is $F_y = J/\Delta t = 0.367/0.03 = 12.22$ N

<u>Graphical Representation</u>

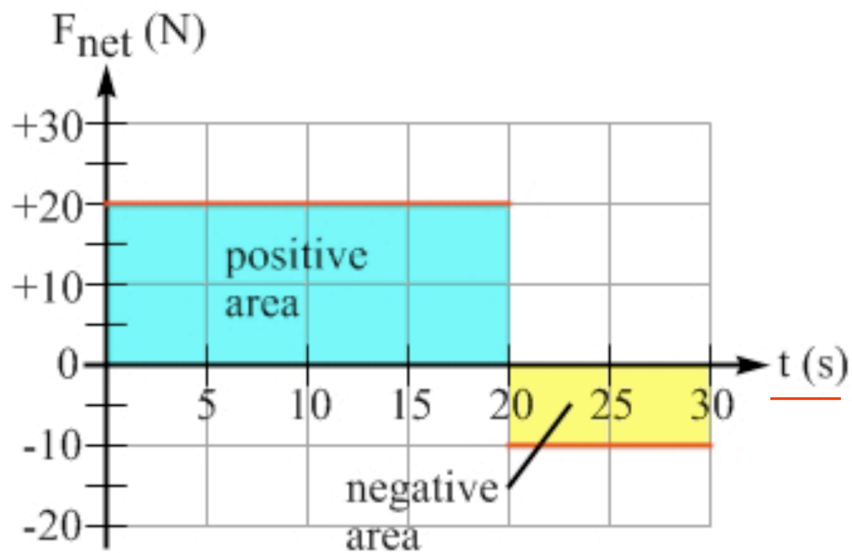
Use an analogy:

<table>
<tr><td>$\boldsymbol{a}\, \Delta t = \mathbf{v}_2 - \mathbf{v}_1$</td><td>$\boldsymbol{F}_{net}\Delta t = \boldsymbol{P}_2 - \boldsymbol{P}_1$</td></tr>
<tr><td>
</td><td>
</td></tr>
</table>

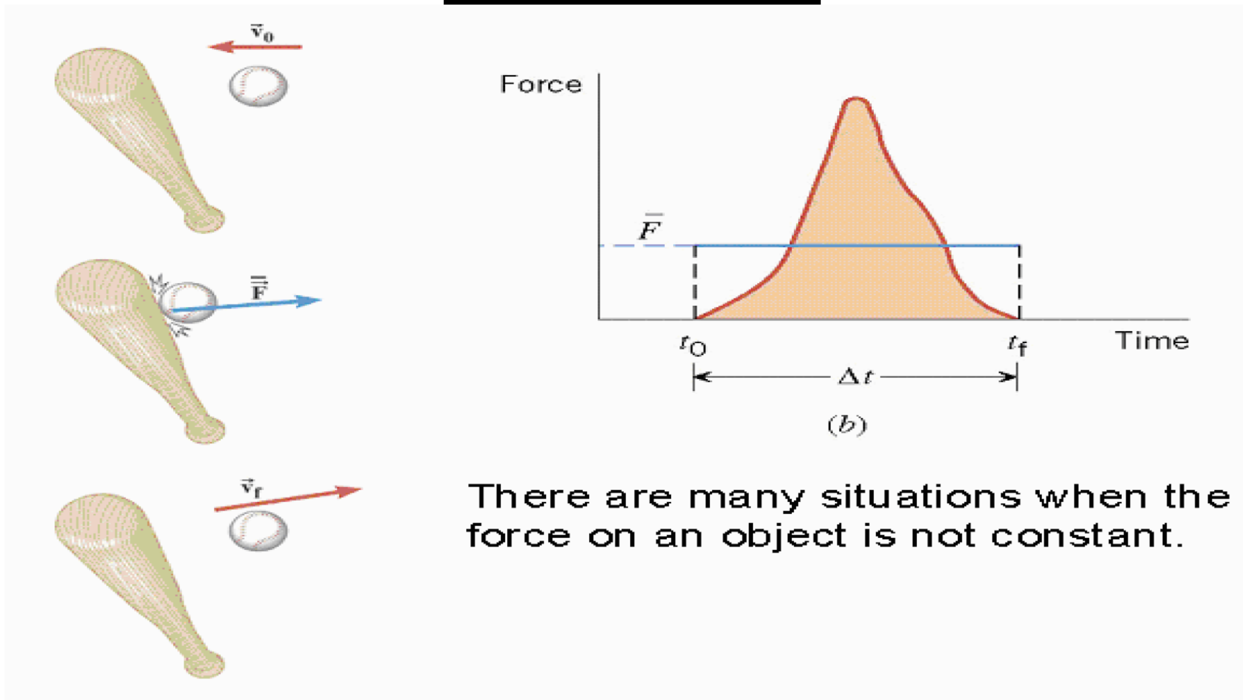
$\boldsymbol{J} = \boldsymbol{F}_{net}\Delta t = \boldsymbol{P}_2 - \boldsymbol{P}_1 = \Delta \boldsymbol{P} =$ The Area

The net force vs. time graph

The area under the net force vs. time graph represents the change in momentum (also known as the impulse).



Changing Force



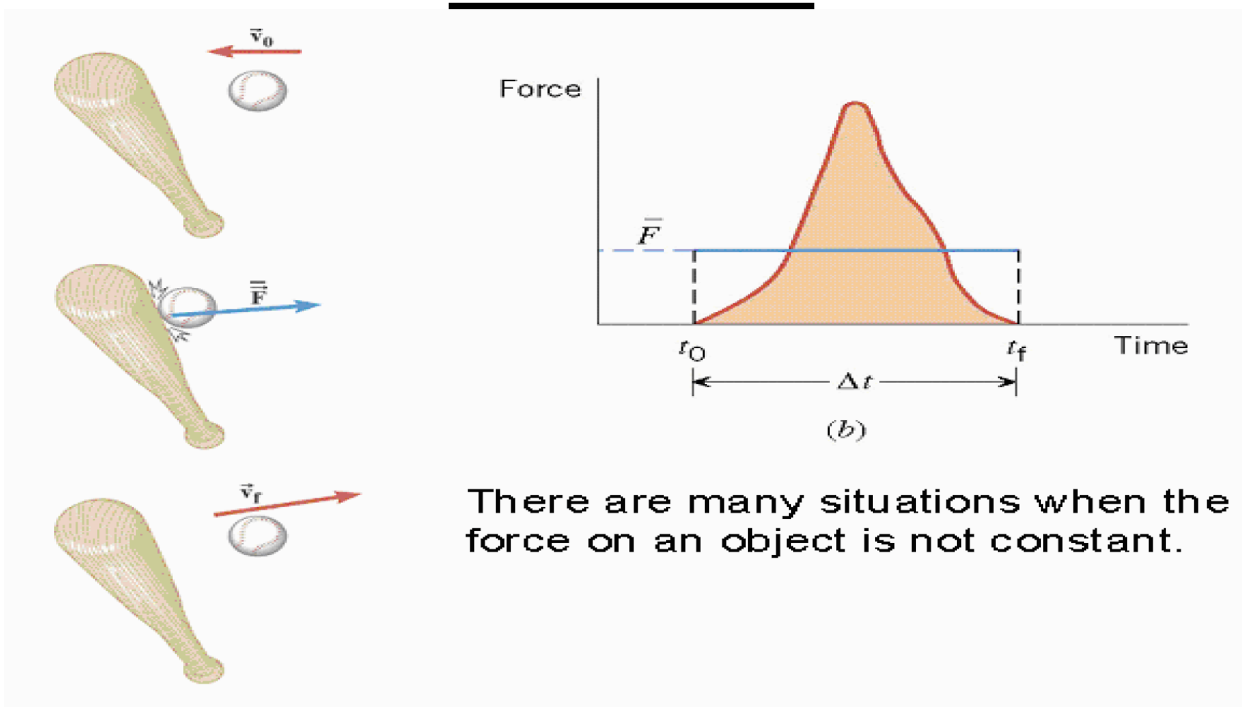
It does not matter! We still can use the fact that

$$\boldsymbol{J} = \boldsymbol{P_2} - \boldsymbol{P_1} = \Delta \boldsymbol{P} = \text{The Area}$$

But now we have to use the average value for the net force

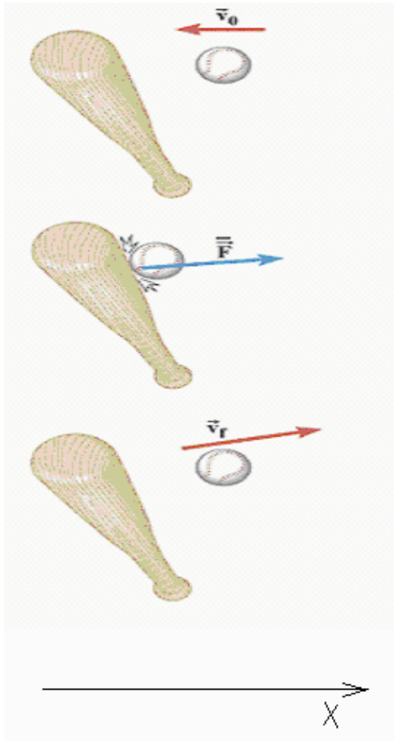
$$\boxed{\boldsymbol{J} = \boldsymbol{F}_{\text{net ave}}\,\Delta t = \Delta \boldsymbol{P} = \text{The Area}}$$

More Problems



A ball of the mass of 120 g was flying at the speed of –120 m/s when the bat hits it. The ball reverses the velocity and flies back at the same speed.

What is the impulse the bat transfers to the ball during the hit?



$m = 0.120$ kg
$v_1 = -120$ m/s
$v_2 = 120$ m/s

$$J = P_2 - P_1$$

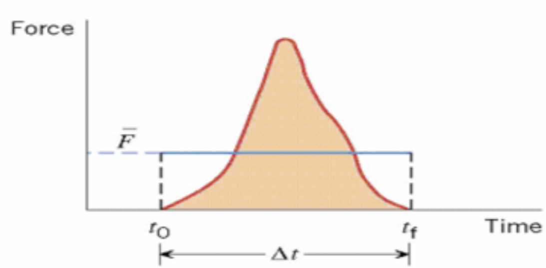
$$J = mv_2 - mv_1 =$$
$$= 0.12*120 - 0.12*(-120)$$
$$= 28.8 \text{ N s}$$

If the duration of the hit (when the bat was touching the ball) is 2.5 ms, what was the average force exerted on the ball?

$\Delta t = 2.5$ ms $= 0.0025$ s

$$F \Delta t = J$$

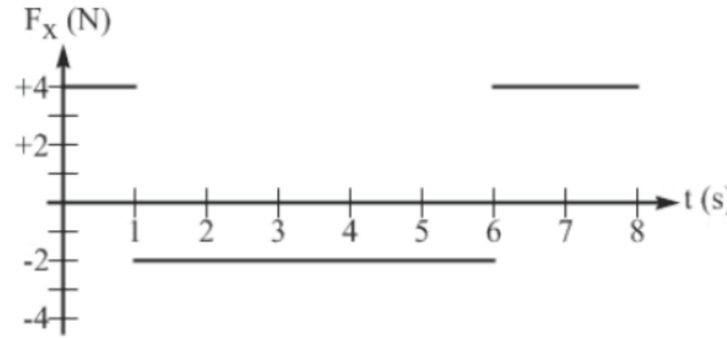
$$F = J/\Delta t = 28.8/0.0025 = 11520 \text{ N}$$



(b)

Example 3

A cart with a mass of 500 g is subjected to the net force shown in the graph. At $t = 0$ the cart has a velocity of 4.0 m/s in the positive x-direction.



What is the cart's velocity at $t = 5.0$ s?

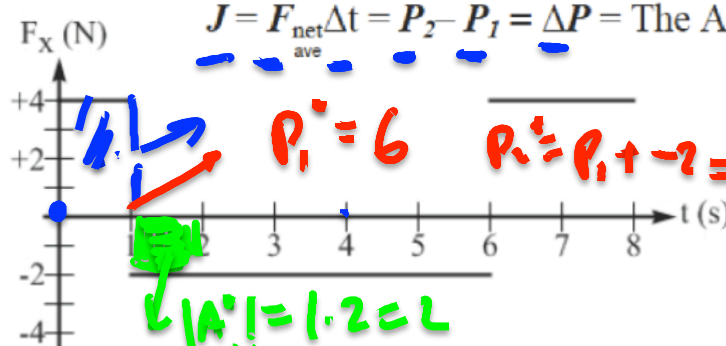
Does the cart change direction at any time? If so, at what time(s)? Create a table showing the cart's momentum. Plot a graph of the cart's momentum at 1-second intervals as a function of time.

Time (s)	Momentum (kg m/s)
0	
1	
2	
3	
4	
5	
6	
7	
8	

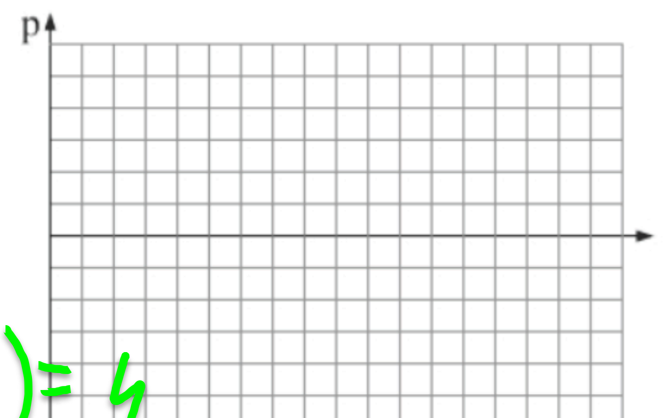
Example

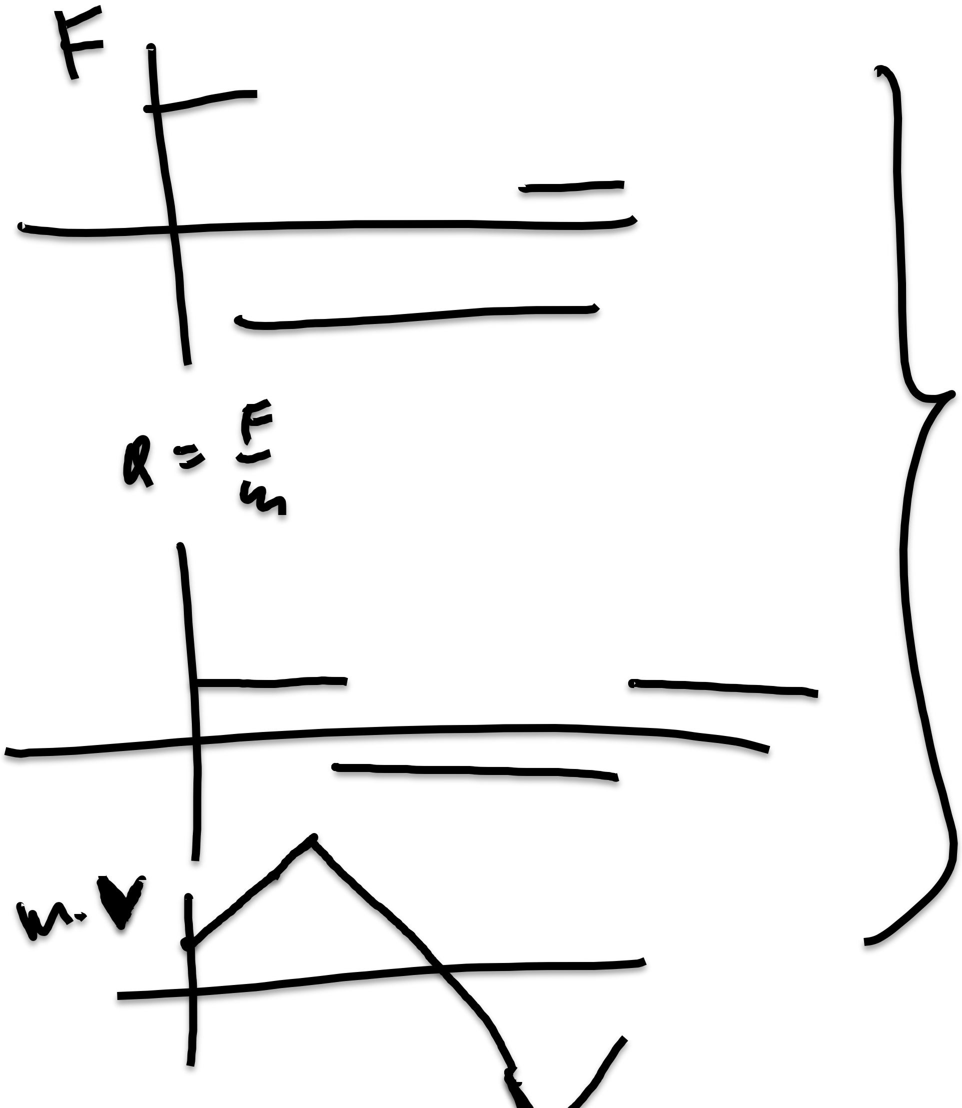
A cart with a mass of 500 g is subjected to the net force shown in the graph. At $t = 0$ the cart has a velocity of 4.0 m/s in the positive x-direction. What is the cart's velocity at $t = 5.0$ s? Does the cart change direction at any time? If so, at what time(s)? Create a table showing the cart's momentum. Plot a graph of the cart's momentum at 1-second intervals as a function of time.

$J = F_{\underset{ave}{net}}\Delta t = P_2 - P_1 = \Delta P$ = The Area



Time (s)	Momentum (kg m/s)
0	
1	6
2	4
3	
4	
5	
6	
7	
8	





not Nex

Two carts

The moving cart of the mass 5 m collides with the stationary cart of mass m.

Which of the two carts experiences the largest average force during the collision? Consider the magnitude of the forces only.

A) The cart of mass m experiences a larger magnitude force

B) The cart of mass 5m experiences a larger magnitude force

C) The carts experience forces of equal magnitude

D) Impossible to answer

Two carts

The moving cart of the mass 5 m collides with the stationary cart of mass m.

Which of the two carts experiences the largest average force during the collision? Consider the magnitude of the forces only.

It does not matter what mass the carts have.

According the Newton's III Law the forces acting between carts are equal in magnitude (on opposite in direction)!

C) The carts experience forces *of equal magnitude*

$$\vec{F}_{12} = -\vec{F}_{21}$$

$$\frac{\vec{p}_{1f} - \vec{p}_{1i}}{\Delta t} = -\frac{\vec{p}_{2f} - \vec{p}_{2i}}{\Delta t}$$

$$\vec{p}_{1f} - \vec{p}_{1i} = -(\vec{p}_{2f} - \vec{p}_{2i})$$

$$\vec{p}_{1f} + \vec{p}_{2f} = \vec{p}_{1i} + \vec{p}_{2i}$$

$$\vec{p}_{system} = const$$

Law of conservation of linear momentum!
(there are NO *outside* forces acting on the system, or they are canceled out!)

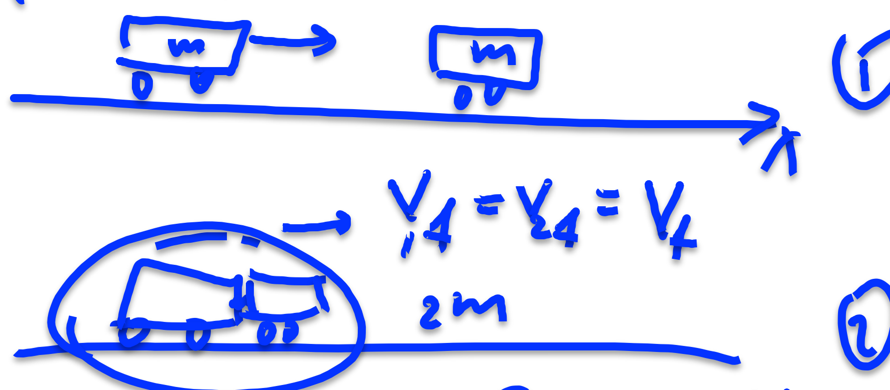
$$\vec{p}_{system} = const$$

Two identical carts collide. One cart is initially at rest. They get stuck during the collision and after the collision the carts are moving together. During the collision the impulse of the moving cart …

1. Does not change
2. Doubles
3. Halves
4. None of he above

$V_i = V_{1i} \neq 0 \qquad V_{2i} = 0 \qquad P_i = V_i \cdot m + 0$

$V_{1f} = V_{2f} = V_f$

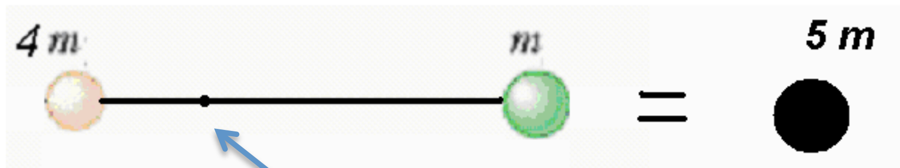


$P_i = P_f \longrightarrow V_i \cdot m = 2m \cdot V_f \qquad P_f = 2m \cdot V_f$

$V_f = \frac{1}{2} V_i$

The meaning of CM

The center-of-mass is the point that moves as though *all the mass of the system* is concentrated there.



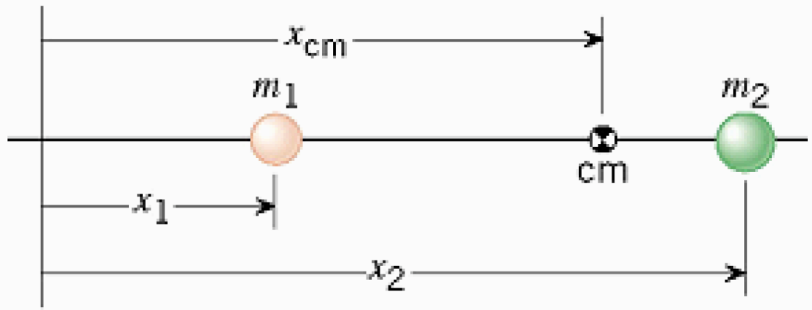
(look at the system from afar!)

If we do not need to know the behavior of the individual parts of the system we can replace the system with a point-like object of the mass of the system and apply the Newton's II Law to it.

$$\sum_{\text{all forces on the system}} F = M_{system} \bullet a_{CM}$$

Center of Mass

The center of mass is a point that represents the average location for the total mass of a system.

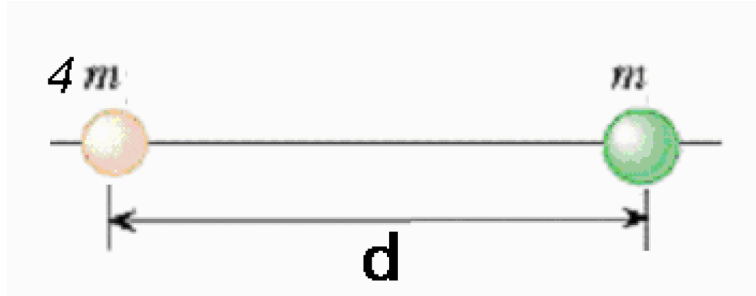


$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

(A similar equation for Y – axis)

Here, cm (or CM) stands for a Center of Mass (not a centimeter)

CM characterizes the distribution of the mass of the system over the parts of the system.



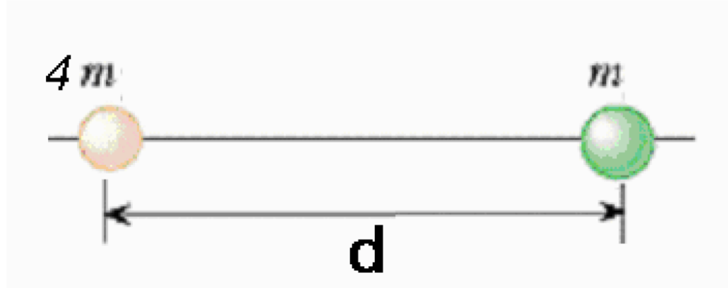
Two balls, of mass *m* and 4*m*, are separated by a distance *d*. Where is their center of mass located?

A) In the middle between the balls

B) Closer the red ball

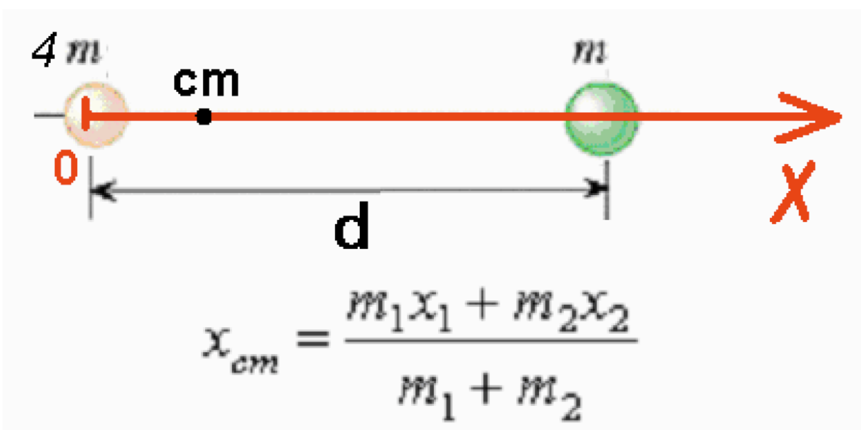
C) Closer to the green ball

D) Impossible to tell



Two balls, of mass m and $4m$, are separated by a distance d. Where is their center of mass located?

Let's introduce the reference frame (x-axiz)



$$x_{cm} = \frac{4m \bullet 0 + m \bullet d}{4m + m}$$

$$x_{cm} = \frac{1}{5} d$$

Center of Mass

The center-of-mass of an object, or a collection of objects, can be found using:

$$X_{cm} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3 + \ldots}{m_1 + m_2 + m_3 + \ldots} \quad ; \quad Y_{cm} = \frac{y_1 \cdot m_1 + y_2 \cdot m_2 + y_3 \cdot m_3 \ldots}{m_1 + m_2 + m_3}$$

That tells us the x-coordinate of the center of mass.

The y-coordinate and z-coordinate can be found from equivalent expressions.

Important! The location of the CM does not depend on the choice of the reference frame.

Example

$$X_{cm} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$$

Three balls have masses and positions as follc

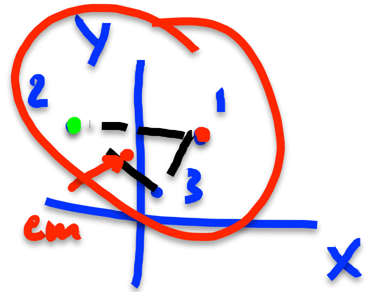
- Ball 1: $m_1 = 1.2$ kg, $x_1 = 0.4$ m, $y_1 = 0.7$ m.
- Ball 2: $m_2 = 2.1$ kg, $x_2 = -0.4$ m, $y_2 = 0.7$ m.
- Ball 3: $m_3 = 1.7$ kg, $x_3 = 0.1$ m, $y_3 = 0.2$ m.

Find the location of the center of mass.

Example

Three balls have masses and positions as follows:

- Ball 1: $m_1 = 1.2$ kg, $x_1 = 0.4$ m, $y_1 = 0.7$ m.
- Ball 2: $m_2 = 2.1$ kg, $x_2 = -0.4$ m, $y_2 = 0.7$ m.
- Ball 3: $m_3 = 1.7$ kg, $x_3 = 0.1$ m, $y_3 = 0.2$ m.



The center-of-mass of this system is given by:

$$X_{com} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3}{m_1 + m_2 + m_3} \qquad X_{com} = \frac{0.4 * 1.2 + (-0.4) * 2.1 + 0.1 * 1.7}{1.2 + 2.1 + 1.7}$$

$X_{com} = -0.038$ m

$$Y_{com} = \frac{y_1 m_1 + y_2 m_2 + y_3 m_3}{m_1 + m_2 + m_3} \qquad Y_{com} = \frac{0.7 * 1.2 + 0.7 * 2.1 + 0.2 * 1.7}{1.2 + 2.1 + 1.7}$$

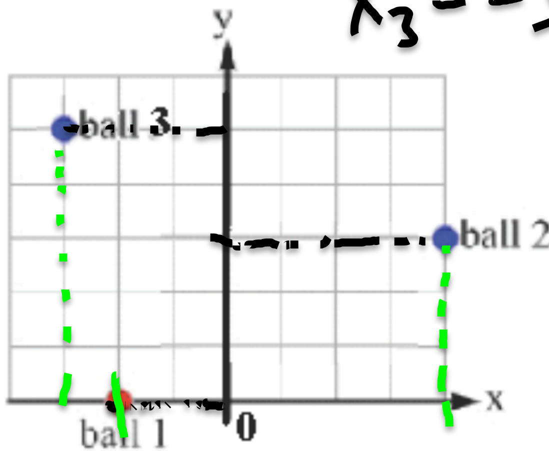
$Y_{com} = 0.53$ m

$X_1 = -2$ $X_2 = 4$

$X_3 = -3$

Example

$m_2 = 2 \cdot m_1$



The ball 2 is twice heavier of the ball 1; the ball 3 is three times heavier of the ball 1; each square of the grid has the size of 1m x 1m.

$m_3 = 3 \cdot m_1$

Try to find the X and Y coordinate of the Center of mass of the system.

$Y_1 = 2.0$

$$Y_{cm} = \frac{m_1 Y_1 + m_2 Y_2 + m_3 Y_3}{m_1 + m_2 + m_3} =$$

$Y_2 = 3$

$Y_3 = 5$

$$= \frac{m_1 \cdot 0 + 2m_1 \cdot 3 + 3 \cdot m_1 \cdot 5}{m_1 + 2m_1 + 3m_1} = \frac{21 \cdot m_1}{6m_1} = \frac{21}{6}$$