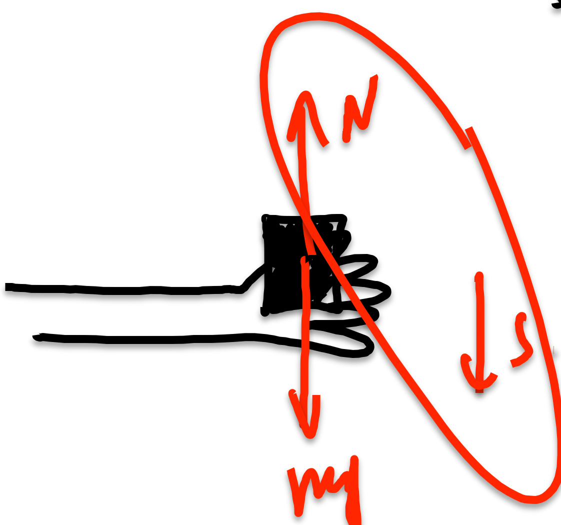
$W_N < 0$



1. ↑
2. →
3. ←
4. ↓

> 0

$W = F \, s \cos\theta < 0$

Review

***Energy* is an ability of an object to do work.**

***Work* is an ability of a force to change energy.**

***Kinetic energy* shows how much work can be done by (or should by done on) an object to bring it to rest.**

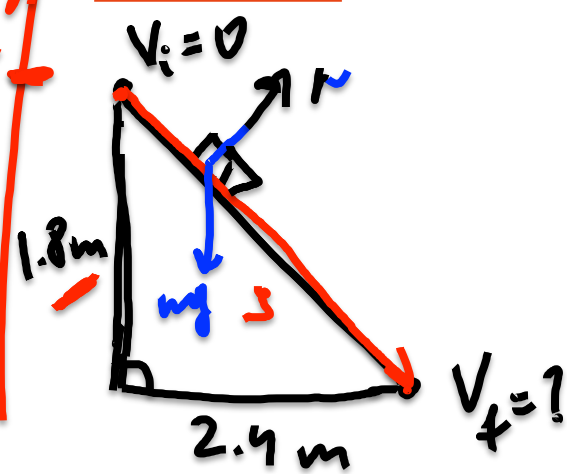
***Potential energy* shows how much work should be done to bring an object from one location to another (*no matter what is its trajectory*).**

Conservative force is the force the work of which can be calculated as the difference between the initial and final value of the potential energy.

Non-conservative force is the reason for mechanical energy being changed.

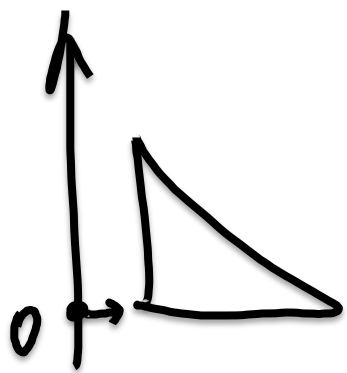
A block sliding down a ramp

A block with a mass of 1.0 kg is released from rest from the top of a ramp that has the shape of a 3-4-5 triangle. The ramp measures 1.8 m high by 2.4 m wide, with the hypotenuse of the ramp measuring 3.0 m. What is the speed of the block when it reaches the bottom, assuming there to be no friction between the block and the ramp?



$$\frac{m \cdot v_f^2}{2} = \underline{K_{\sim}} = V_1 - V_2 = -100 - (-118) =$$

$$= \underline{18 \text{ J}}$$



1. Yes
2. No
3.

$v_f = 6 \text{ m/s}$

A block sliding down a ramp

$$\Rightarrow v_f = \sqrt{2gh_o} = \sqrt{2\times(10\ \text{m/s}^2)\times 1.8\ \text{m}} = 6.0\ \text{m/s}$$

A block sliding down a ramp

$U = mgh = mgy$

For the same problem discussed above, suppose friction is not negligible, and the final speed of the block is 4 m/s (i.e., 2 m/s less than the case without friction). Find the numerical value for the work done by friction on the block and the coefficient of kinetic friction.

$K_1 + U_1 + W_{nc} = K_2 + U_2$

$W_{nc} = \frac{mv_f^2}{2} - mgh = \ldots$

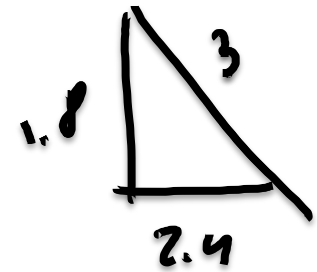
1. $W_{nc} > 0$
2. $= 0$
3. < 0

A block sliding down a ramp

Solution

$K_i + U_i + W_{nc} = K_f + U_f$

$\Rightarrow \Delta K + \Delta U = W_{nc}$ = Work done by friction



	Initial	Final
Speed	$v_0 = 0$	$v_f = 4$ m/s
Height	$h_0 = 1.8$m	$h_f = 0$

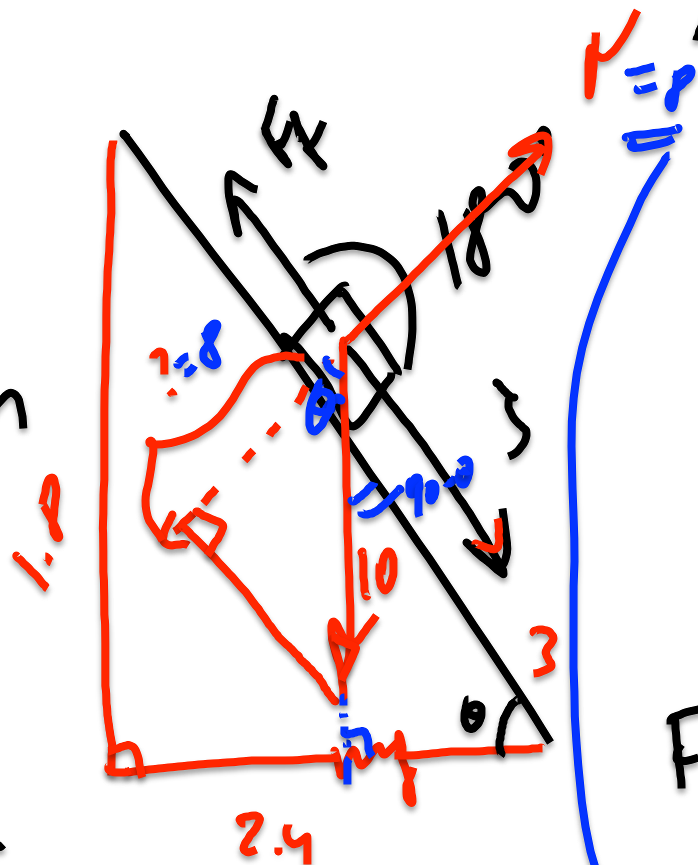
$$\Rightarrow W_{nc} = \frac{1}{2}mv_f^2 - 0 + mg(0 - h_o)$$

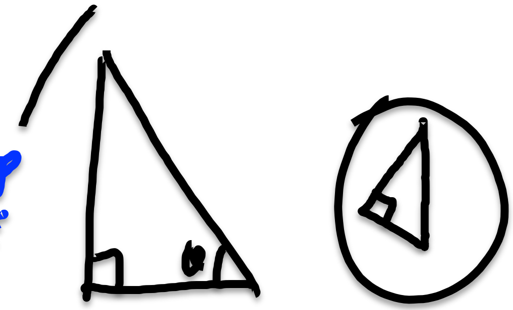
$$= \frac{1}{2}(1kg)(4m/s)^2 + (1kg)(10m/s^2)(-1.8m) = -10J$$

Negative W_{nc} means that the friction (non-conservative force) acts in a direction opposite to the displacement.

$\mu_k = ?$

~~NFmg~~





$F_g \cdot 3 \cdot (-1) = -10$

$? = 10 \cos\theta$

$\cos\theta = \frac{2.4}{3} =$

$F_{fr} = \mu \cdot N$

$= \frac{10 \cdot 2.4}{3} = 8N$

A block sliding down a ramp

$W_{nc} = -f_k s$

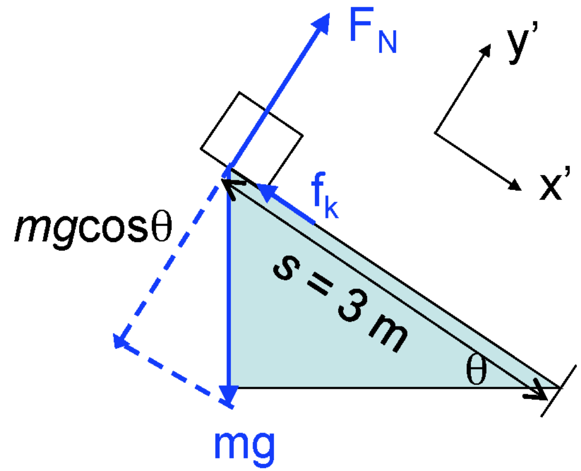
Note the negative sign

$\Rightarrow -10\ \text{J} = -f_k \times (3\ \text{m})$

$\Rightarrow f_k = 10/3\ \text{N}$

Since $\Sigma F_{y'} = 0$, we have:

$F_N = mg\cos\theta = 4mg/5 = 0.8mg$



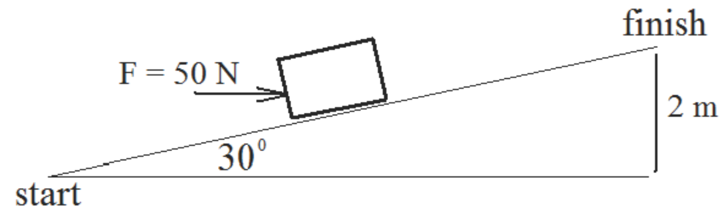
o : a : h = 3 : 4 : 5

$$\mu_k = \frac{f_k}{F_N} = \frac{10}{3} N \times \frac{1}{(0.8 \times 1kg \times 10m/s^2)} = \frac{5}{12} \simeq 0.42$$

$$W = F \cdot s\cos\theta$$

Example

The box was pushed up an incline by the means of the horizontal force of 50 N.



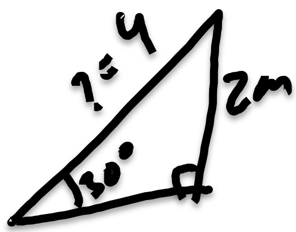
Find the work done by the force on the box.

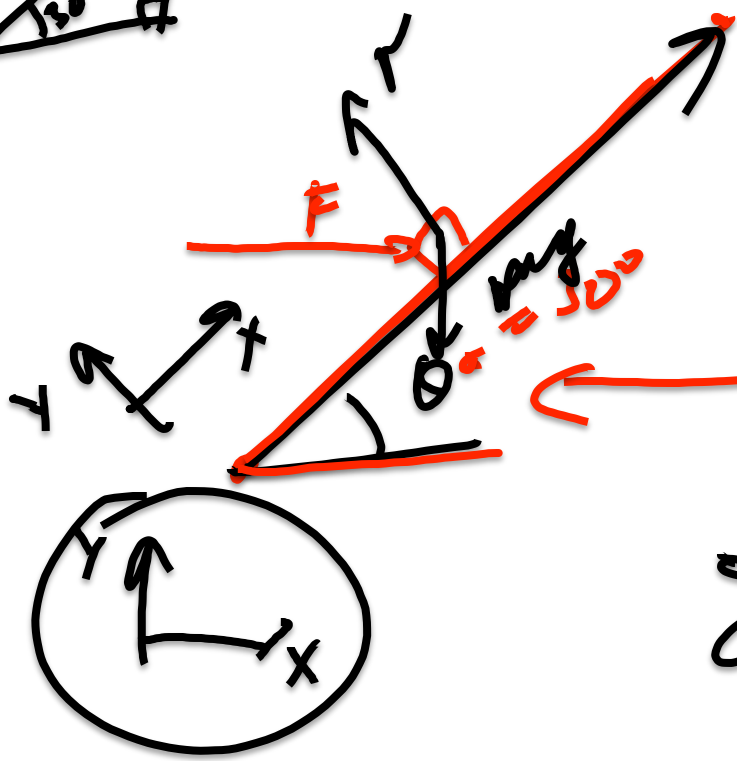
If the ramp is frictionless and 50 N is the minimal force needed to bring the box up the ramp, find the work done by the net force on the box, as well as the work done by all individual forces. Find he mass of the box.

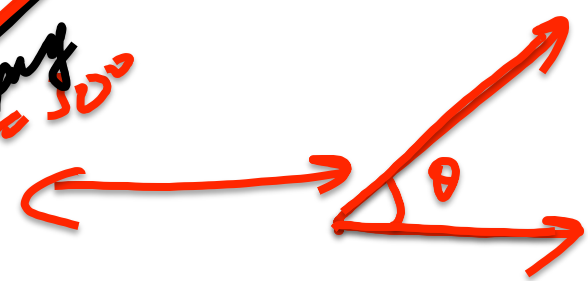
Now, if the coefficient of friction is 0.5, find all the work done by all the forces on the box if it was moving with constant velocity.

Is F = 50 N again?

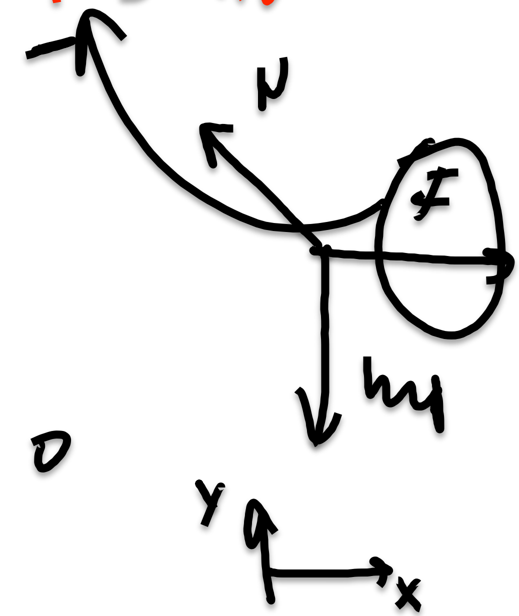
$2m = ?$; $\sin 30°$





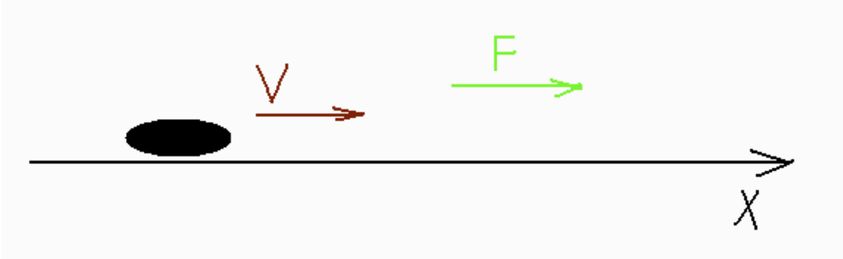


$W = F \cdot S \cos\theta$

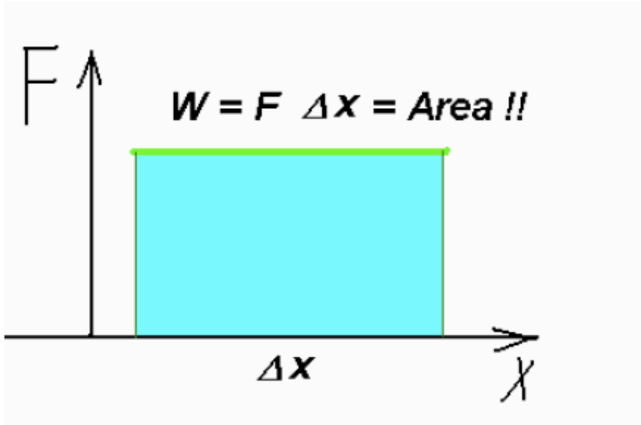


$\sum F = m \vec{a}$

One – dimensional motion



$$W = F \cdot \Delta x$$

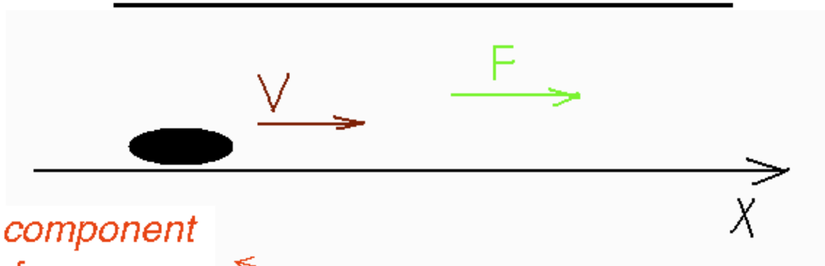


Work done by a force can be calculated through the area under the graph!

W = Area

(Works for ANY force!)

One – dimensional motion

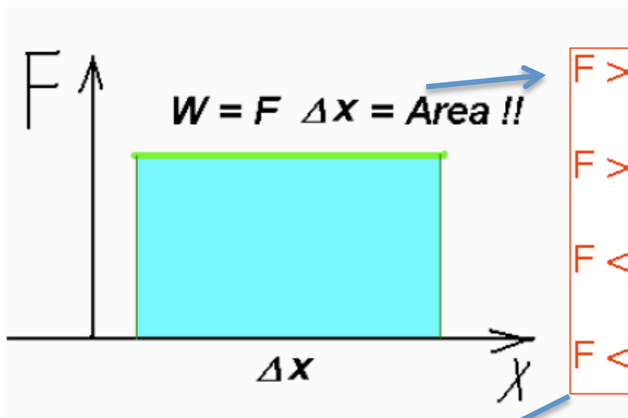


Negative area does not always mean negative work!

the x *component* of the force

$$W = F \cdot \Delta x$$

the x *displacement* of the object



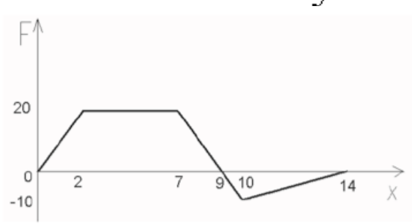
$F > 0 \quad \Delta X > 0 \quad \Rightarrow \quad W > 0$

$F > 0 \quad \Delta X < 0 \quad \Rightarrow \quad W < 0$

$F < 0 \quad \Delta X > 0 \quad \Rightarrow \quad W < 0$

$F < 0 \quad \Delta X < 0 \quad \Rightarrow \quad W > 0$

Work done by a force can be calculated through the area under the graph!



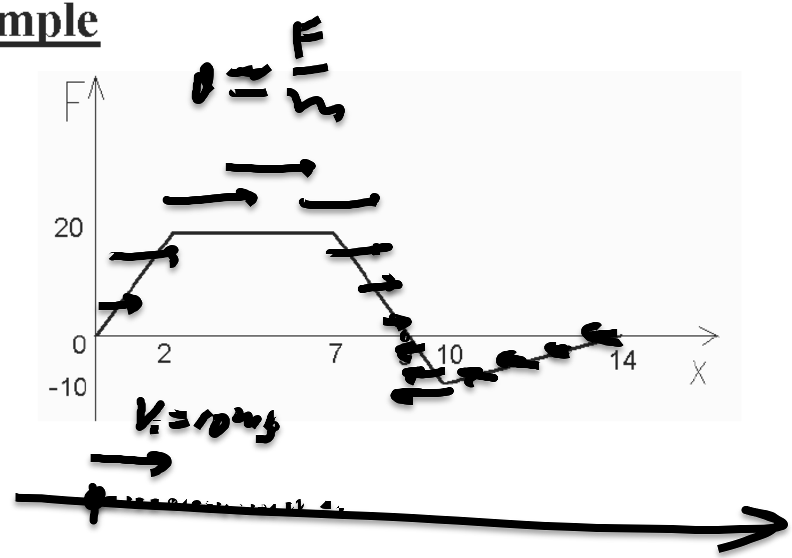
$|W| = |Area|$

(Works for ANY force!)

Example

At t = 0 an object with mass of 3 kg is located

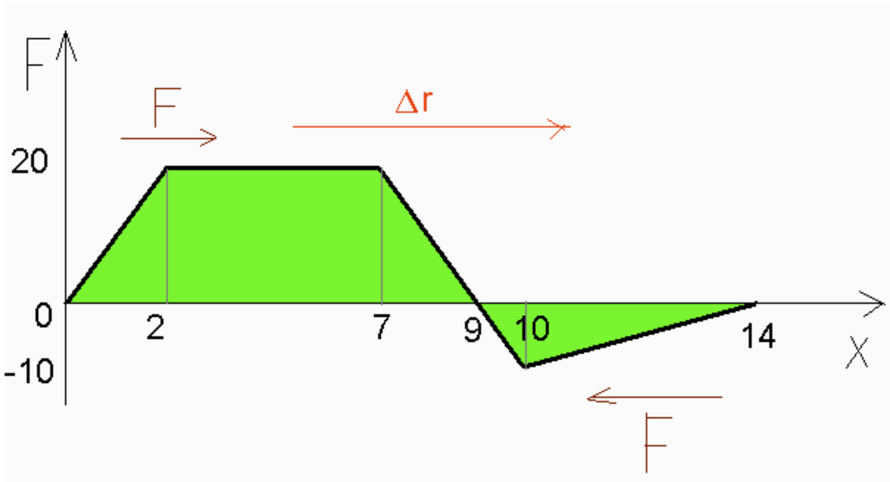
at the origin and has velocity of 10 m/s. The object is under an influence of the force, which is changing with the location of the object (see the graph).



Explain the motion of the object, what is happening to its velocity?

Find the maximum speed of the object.

Find its speed at t = 14 s. Does it ever reverse its direction?



$x_i = 0$ m

$m = 3$ kg

$v_i = 10$ m/s

What is happening to the velocity of the object? The graph F vs. x lets us to find the work W done by the force.

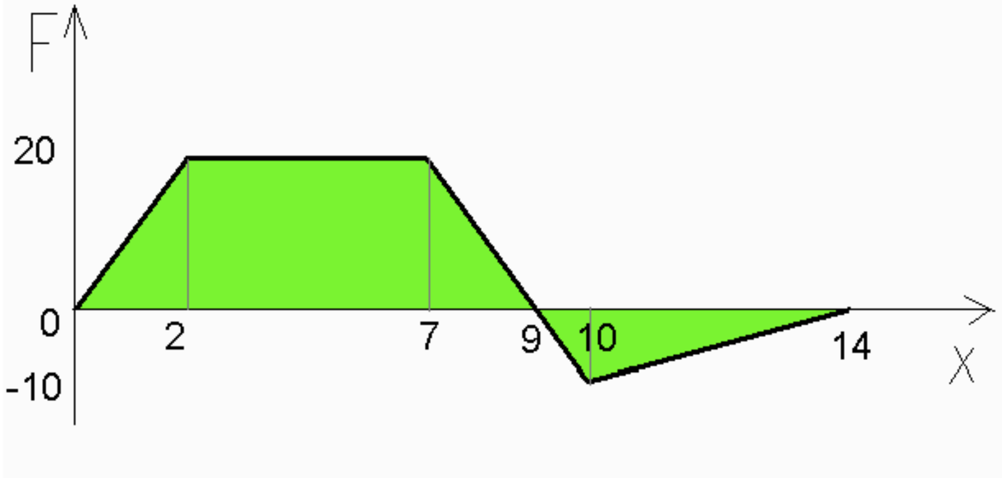
The work is related to the change in the kinetic energy $K = \frac{1}{2}mv^2$

According to the Work-Kinetic energy Theorem
(and only one force is acting)

$$K_2 - K_1 = W_{total} = W_F = Area$$

This leads to

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_i^2 = Area$$



$m = 3$ kg

$v_i = 10$ m/s

What is happening to the velocity of the object?

$$\tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_i^2 = \text{Area}$$

Hence, $\tfrac{1}{2}mv^2 = \tfrac{1}{2}mv_i^2 + \text{Area}$

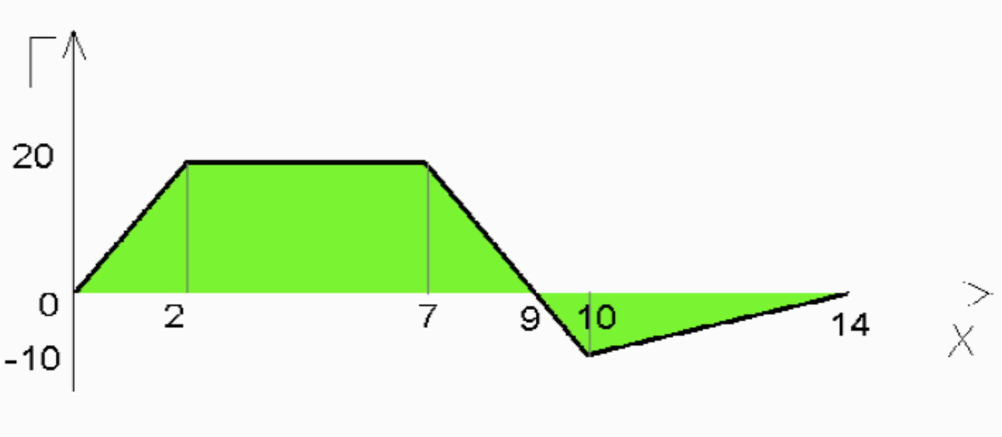
Between x = 0 m and x = 9 m the area is increasing, hence the speed is increasing too.

The maximum value of the speed is reached at x = 9 m.

$$\tfrac{1}{2}mv_{max}^2 = \tfrac{1}{2}mv_i^2 + \text{Area}_{\underline{0\ 9}}$$

$$\tfrac{1}{2}\cdot 3\cdot v_{max}^2 = \tfrac{1}{2}\cdot 3\cdot 10^2 + \tfrac{1}{2}\cdot 2\cdot 20 + (7-2)\cdot 20 + \tfrac{1}{2}\cdot(9-7)\cdot 20$$

Just solve this equation now to get v_{max}



$m = 3$ kg

$v_i = 10$ m/s

What is happening to the velocity of the object?

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}mv_i^2 + \text{Area}$$

After the maximum is reached (after x = 9 m) the force does negative work on the object, hence its speed is decreasing.

We can find the speed of the object at x = 14

$$\tfrac{1}{2}\cdot 3\cdot v_{14}^2 = \tfrac{1}{2}\cdot 3\cdot 10^2 + \tfrac{1}{2}\cdot 2\cdot 20 + (7-2)\cdot 20 + \tfrac{1}{2}\cdot(9-7)\cdot 20 + \tfrac{1}{2}\cdot(10-9)\cdot(-10) + \tfrac{1}{2}\cdot(14-10)\cdot(-10)$$

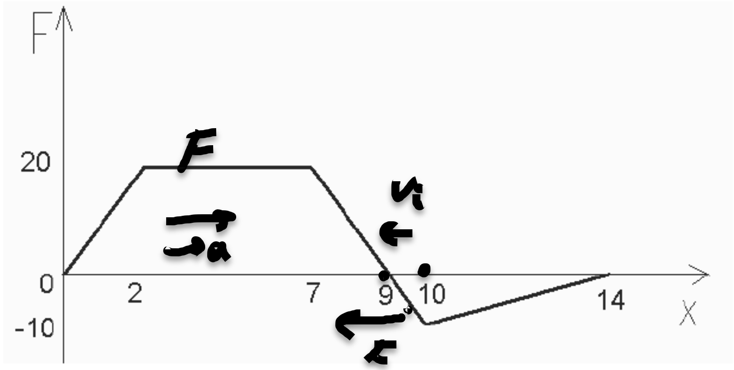
<u>or</u>

$$\tfrac{1}{2}\cdot 3\cdot v_{14}^2 = \tfrac{1}{2}\cdot 3\cdot v_{max}^2 + \tfrac{1}{2}\cdot(10-9)\cdot(-10) + \tfrac{1}{2}\cdot(14-10)\cdot(-10)$$

Example

Now the same object with m = 3 kg is located at x = 10 m with initial velocity directed to the left.

Find the minimum initial speed the object has to have in order to reach the origin.



W = Area

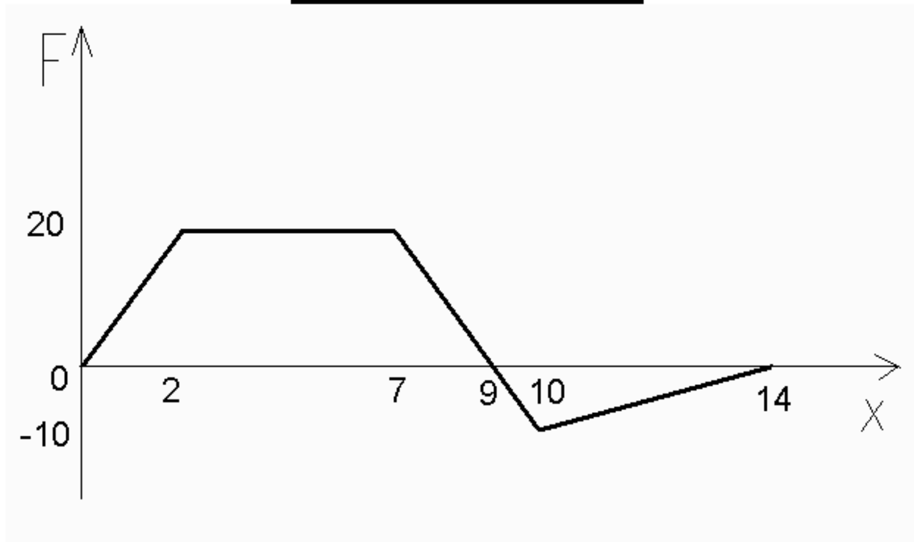
$F > 0 \quad \Delta X > 0 \quad \Rightarrow \quad W > 0$

$F > 0 \quad \Delta X < 0 \quad \Rightarrow \quad W < 0$

$F < 0 \quad \Delta X > 0 \quad \Rightarrow \quad W < 0$

$F < 0 \quad \Delta X < 0 \quad \Rightarrow \quad W > 0$

Tricky Example

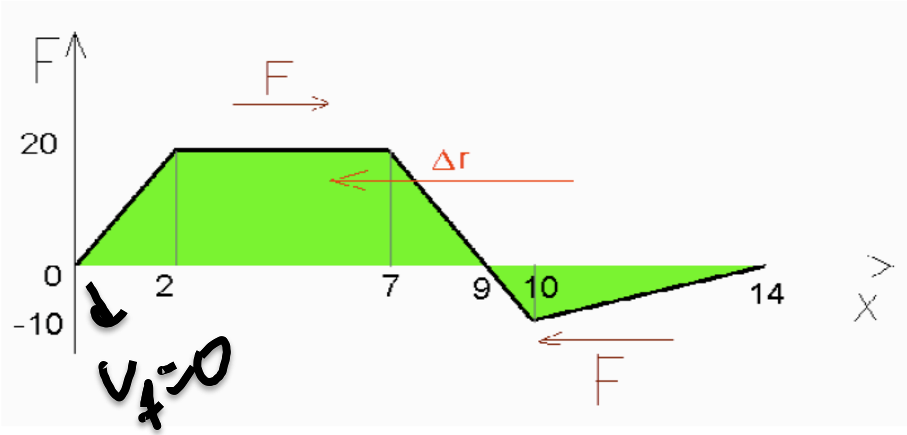


An object of mass 3 kg is initially located at the x = 10 m moving initially to the left. The object is under an influence of the force, which is changing with the location of the object (see the graph).

What is happening to the velocity of the object?

The object is moving initially in the *negative* x- direction; hence its displacement is directed to the left.

Hence, when the force is directed to the right (positive value), it is directed against the displacement and is doing negative work (to the left =>positive)!



$x_i = 10$ m

$m = 3$ kg

$K_2 - K_1 = W$

$K_2 = K_1 + W$

0

Now, the force does positive work when $9 < x < 10$.

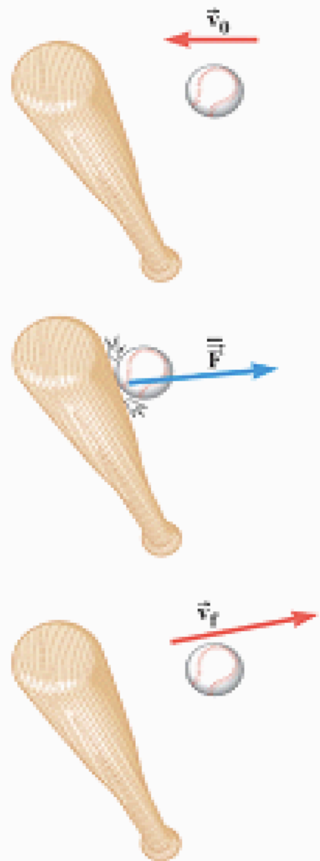
But it does negative work when $0 < x < 9$ (when the object is moving to the left)

Now the object should be eventually brought to a stop at x_f ($v_f = 0$).

If happens when $\frac{1}{2}mv^2 + W = 0$ and we can find the initial kinetic energy (and initial speed) and object needs to have to stop at the origin ($x_f = 0$).

$$0 = K_i + -[\tfrac{1}{2}\cdot(10 - 9)\cdot(-10)] + -[\tfrac{1}{2}\cdot(9 - 7)\cdot 20] + + -[(7 - 2)\cdot 20] + -[\tfrac{1}{2}\cdot 2\cdot 20]$$

The Impulse-Momentum Theorem



$$\bar{\vec{\mathbf{a}}} = \frac{\vec{\mathbf{v}}_{\mathbf{f}} - \vec{\mathbf{v}}_{\mathbf{o}}}{\Delta t}$$

$$\sum \bar{\vec{\mathbf{F}}} = m\,\bar{\vec{\mathbf{a}}}$$

$$\sum \bar{\vec{\mathbf{F}}} = \frac{m\vec{\mathbf{v}}_{\mathbf{f}} - m\vec{\mathbf{v}}_{\mathbf{o}}}{\Delta t}$$

$$\left(\sum \bar{\vec{\mathbf{F}}}\right)\Delta t = m\vec{\mathbf{v}}_{\mathbf{f}} - m\vec{\mathbf{v}}_{\mathbf{o}}$$

IMPULSE-MOMENTUM THEOREM

When a net force acts on an object, the impulse of this force is equal to the change in the momentum of the object

impulse

$$\left(\sum \vec{\mathbf{F}}\right)\Delta t = m\vec{\mathbf{v}}_{\mathbf{f}} - m\vec{\mathbf{v}}_{\mathbf{o}}$$

final momentum

initial momentum

Impulse and Momentum

Any change takes some time!
Let's set acceleration being a constant

$$\boldsymbol{F}_{net} = m\boldsymbol{a} = m\frac{\Delta V}{\Delta t} = m\frac{V_2 - V_1}{\Delta t} \quad \text{hence} \quad \boxed{\boldsymbol{F}_{net}\Delta t = m\boldsymbol{v}_2 - m\boldsymbol{v}_1}$$

Two new physical quantities:

Impulse of the force $\boldsymbol{J} = \boldsymbol{F}_{net}\Delta t$	Linear Momentum of the object $\boldsymbol{P} = m\boldsymbol{v}$

The unit is the same: N s = kg*m/s

Relationships for impulse and momentum:

$$\boldsymbol{J} = \boldsymbol{F}_{net}\Delta t$$

$$\boldsymbol{P} = m\boldsymbol{v}$$

$$\boldsymbol{F}_{net}\Delta t = \boldsymbol{P}_2 - \boldsymbol{P}_1$$

$$\vec{J} = \Delta\vec{P}$$