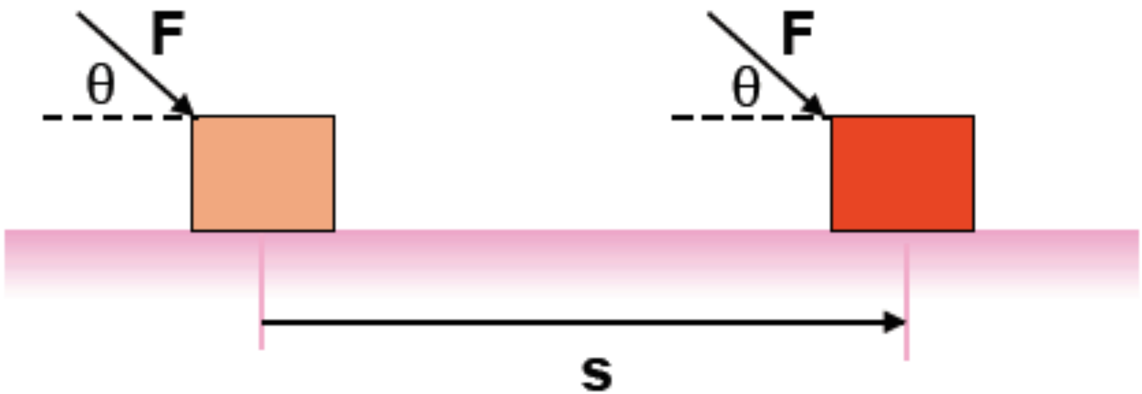
This week topics

Kinetic energy, work, work-force connection, force-position graph, power, power-force connection, work-kinetic energy theorem, conservative force, potential energy, gravitational potential energy, mechanical energy, non-conservative force, law of conservation of energy, impulse of a force, linear momentum, force-tine graph, closed (isolated) system, law of conservation of linear momentum, a collision, elasticity, four types of collisions, methods for solving collision problems, center of mass (COM), calculating COM, Circular motion (CM), circumference, radius, uniform circular motion (UCM), period, frequency, centripetal acceleration, properties of horizontal UCM, properties of vertical UCM, properties of vertical CM.

Work Done by a Constant Force



In general, if the net force, **F**, makes an angle θ, with the displacement vector, **s**, the work done W by **F** is:

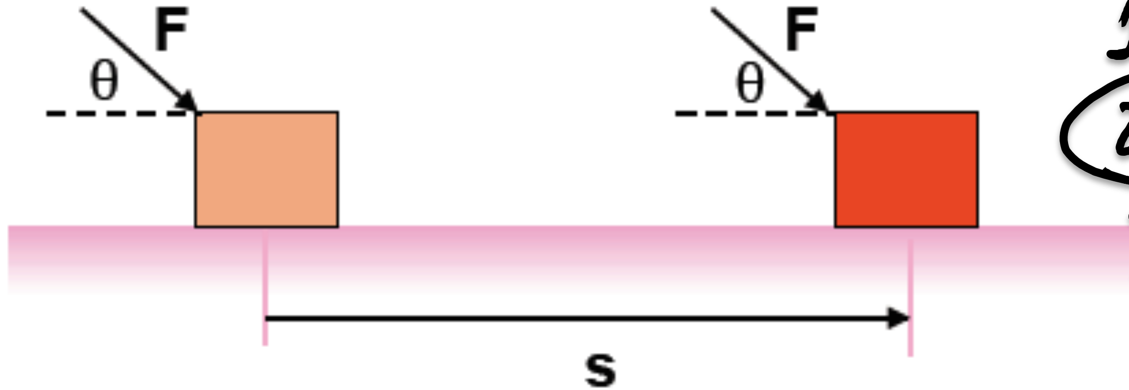
(a vector!)

(For a *constant* force!)

$$W = F \cdot s \cos\theta$$

(a magnitude!)

Work Done by a Constant Force



W
1. < 0
2. = 0
3. > 0

In general, if the net force, **F**, makes an angle θ, with the displacement vector, **s**, the work done W by **F** is:

Abs. v. of the Force

(a vector!)

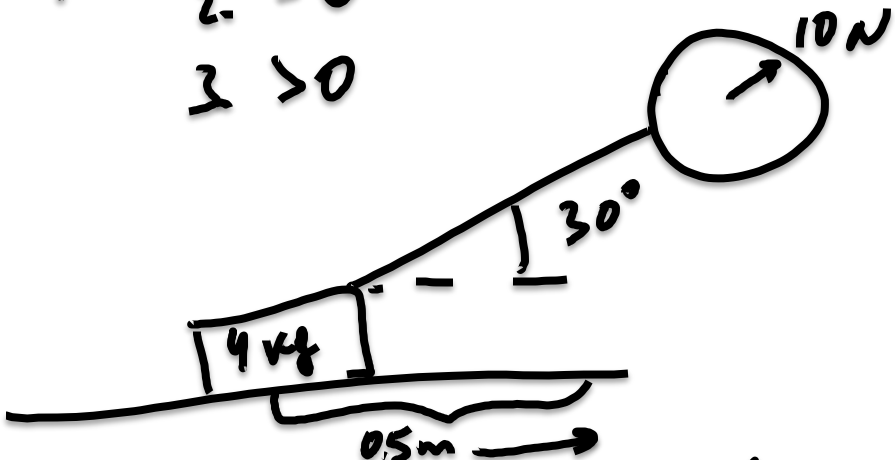
(For a *constant* force!)

$$W = F\,s\cos\theta$$

(a magnitude!)

$W_{NF} = ?$

1. < 0
2. $= 0$
3. > 0



$v = const = 10\,sm / 3s$

$W = F \cdot S \cdot \cos\theta$

1. Read

2. FBD

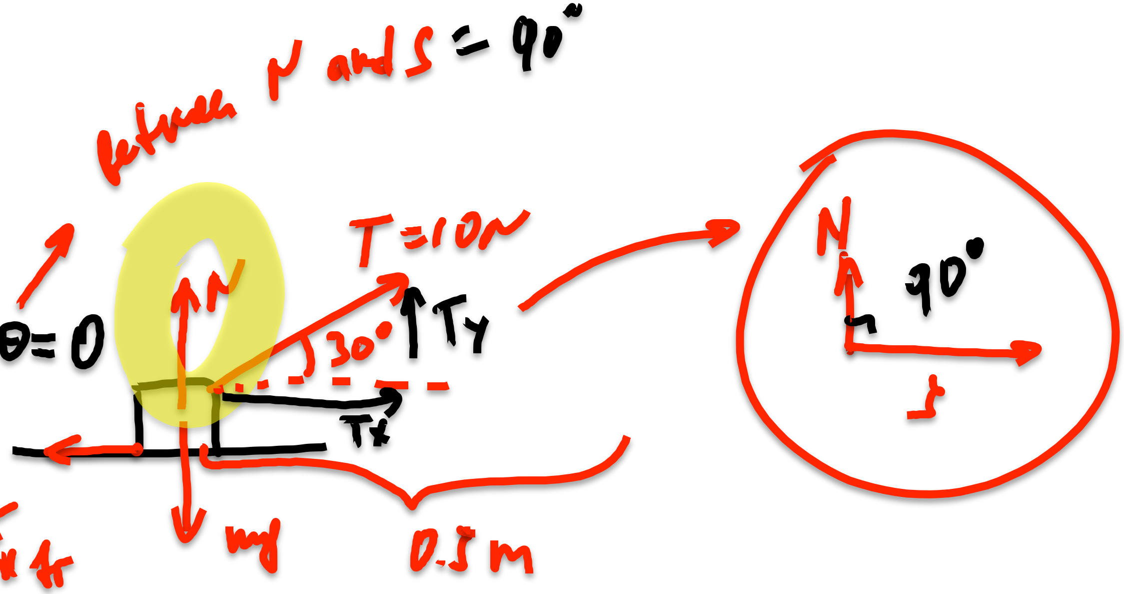
3. $W = F \cdot S \cos\theta = 0$

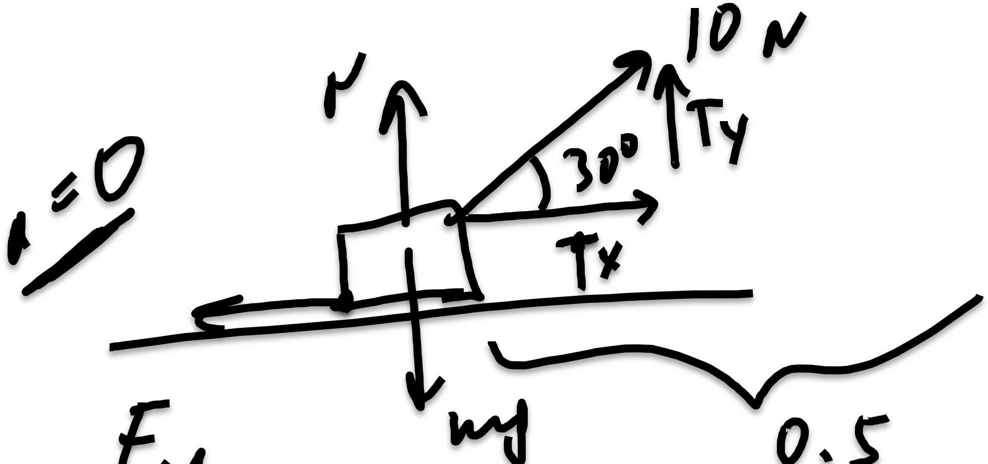
$F \rightarrow N$, $S \rightarrow 0.5$

$N = mg - T_y$

$= 4 \cdot 10 = 40\,N - T_y$

$\hookrightarrow T_y = T \cdot \sin 30^\circ$





F_{kfr}

$V = const \Rightarrow a = 0$

y

$N + T_y = mg$

$N = mg - T_y =$

$= 40 - 10 \cdot \sin 30°$

x

$W = F_{kfr} \cdot 0.5 \cdot \cos 180°$

$10 \cos 30° = T_x = F_{kfr}$

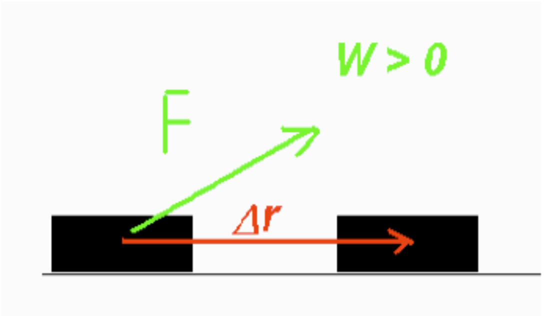
$\mu = \frac{F_{kfr}}{N}$

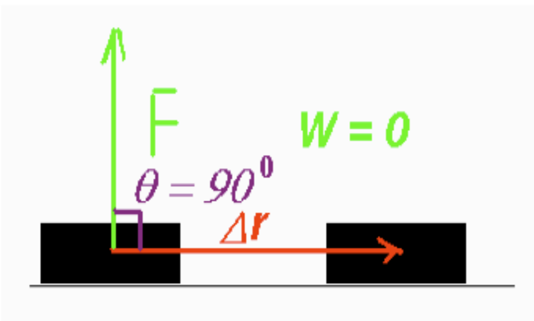
$|W| = 10 \cos 30° \cdot 0.5 \cdot (-1) = -0.865 =$

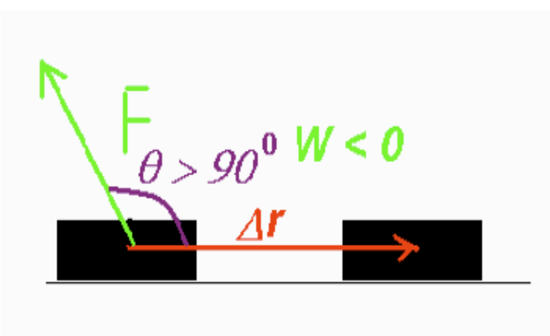
$= -4.33$

Work for different forces

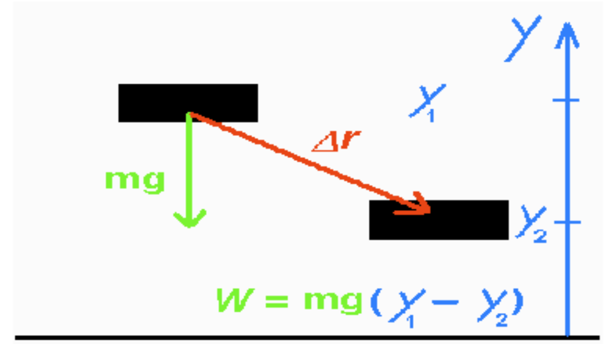
Any forces

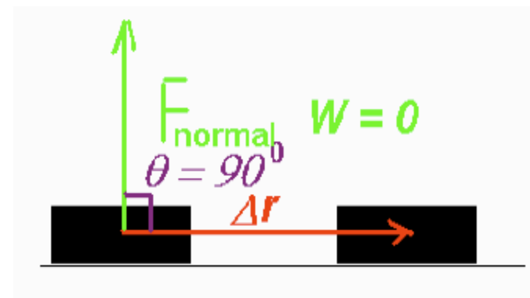


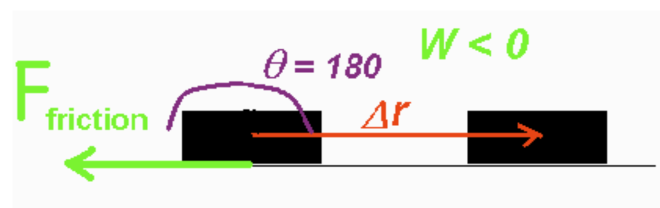




Specific forces



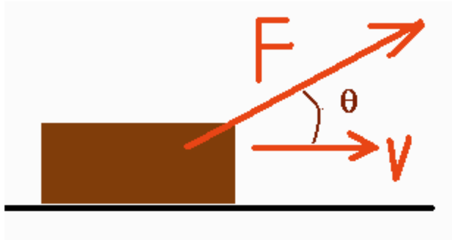




Power is the rate at which work is done.

Power shows how fast the work is done (how fast the energy is changing).

$$P = \frac{\Delta W}{\Delta t}$$



The MKS unit for power is the watt (W). 1 W = 1 J/s

(*Note*: 1 hp = 746 W 1 cal = 4.186 J)

When the force is *constant*, work is given by $W = F \bullet \Delta r \bullet \cos\theta$,

the power equation becomes ($\Delta r/\Delta t = v$). This equation works for ANY force.

$$P = F \bullet v \bullet \cos\theta$$

θ is the angle between the force (which is doing the work) and the velocity of the object.

Kinetic Energy

The kinetic energy K of an object with mass *m* and velocity *v* is defined as:

$$K = \frac{1}{2} mv^2$$

$$K_2 - K_1 = W$$ ← **Net Wrok !**

When a force is acting on an object (system), the work done by the force on the object is equal to the change in the kinetic energy of the object (system).

What if many forces are acting on an object (system)?

Each force does the own work!
(all forces are constant)

$$W_1 = F_1 \cdot \Delta r \cdot \cos\theta_1$$
$$W_2 = F_2 \cdot \Delta r \cdot \cos\theta_2$$
$$W_3 = F_3 \cdot \Delta r \cdot \cos\theta_3$$
etc.

Two ways to calculate the net work.

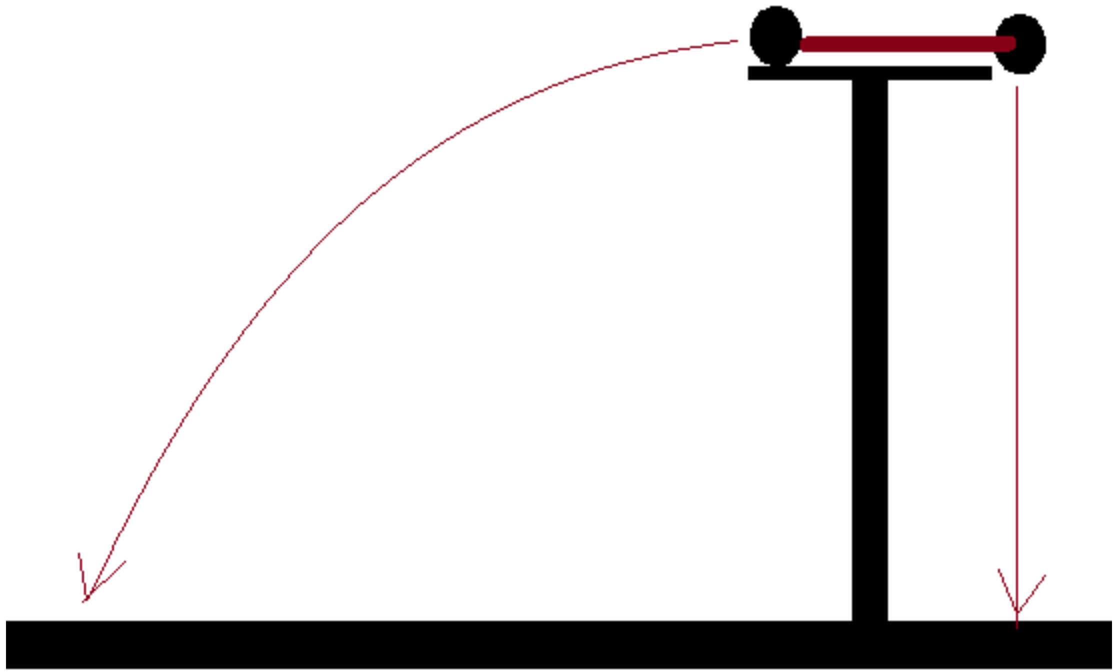
The total work $W_{total} = W_1 + W_2 + W_3 + \ldots = F_{Net} \cdot \Delta r \cdot \cos\theta_{Net}$

$K_2 - K_1 = W_{total}$ (Work – Kinetic Energy Theorem)

(Works for any forces!!!)

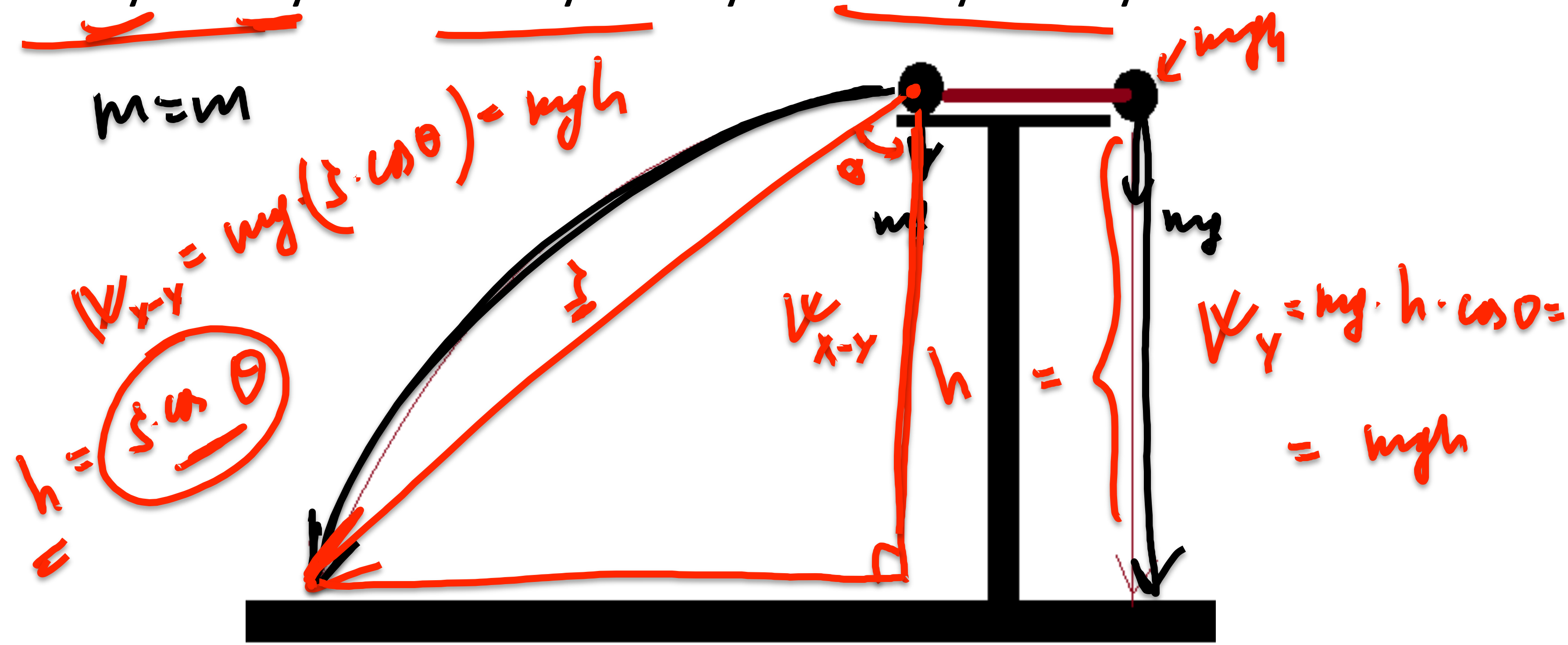
Compare the work done by the force of gravity on each ball:

1. $W_y > W_{x\text{-}y}$
2. $W_y < W_{x\text{-}y}$
3. $W_y = W_{x\text{-}y}$
4. ???



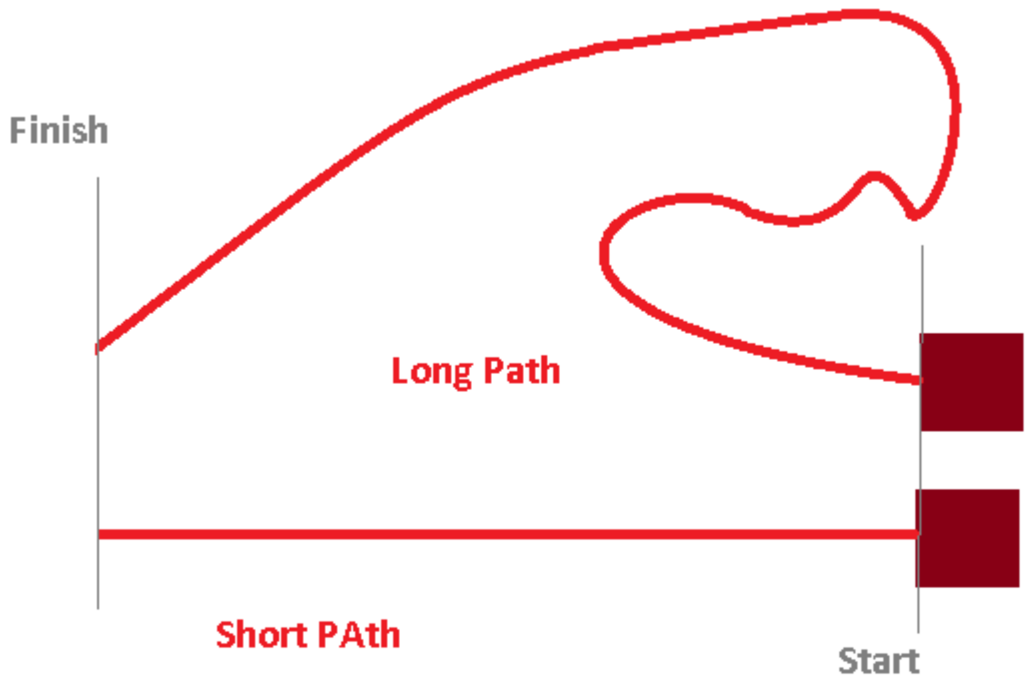
Compare the work done by the force of gravity on each ball:

1. $W_y > W_{x\text{-}y}$ 2. $W_y < W_{x\text{-}y}$ 3. $W_y = W_{x\text{-}y}$ 4. ???



Compare the work done by the force of friction on each box:

1. $W_{short} > W_{long}$
2. $W_{short} < W_{long}$
3. $W_{short} = W_{long}$
4. ???



$\vec{F}_{fr}$ is always at 180° to $\vec{d}$

All the forces in the nature can be divided into two large categories.

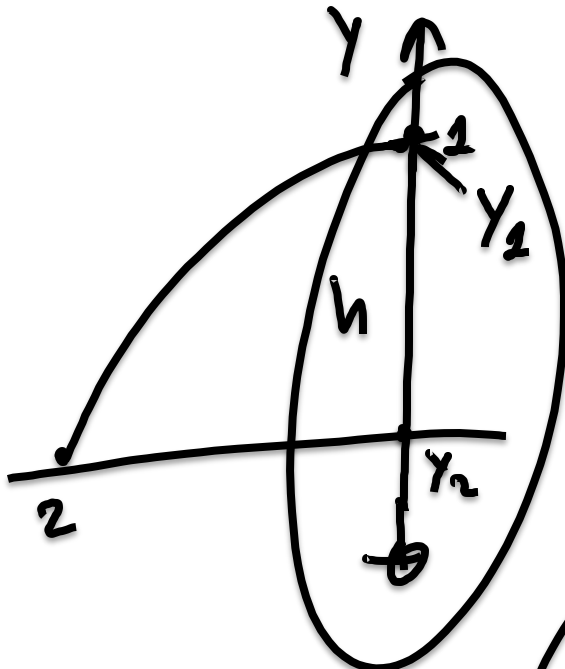
Conservative forces and Non-Conservative forces.

Conservative forces: Forces which give an input in the mechanical energy in the form of potential energy (for example, force of gravity). For *any* conservative force the work done by the force is equal to the difference between the initial and final values of the potential energy.

$$\mathbf{W_{cons} = U_1 - U_2}$$

(for example: $W_{mg} = mgy_1 - mgy_2$)

The work of a conservative force depends only on the initial and final location of the object (no matter what was happening in between!)



$$W_{gr} = mgh = mg(y_1 - y_2) =$$

$$= mgy_1 - mgy_2$$

$$h = y_1 - y_2$$

$$U = mgy$$

$$W = 0.5 \cdot 10 \cdot 0.7 - 0.5 \cdot 10 \cdot 1 =$$

All the forces in the nature can be divided into two large categories.

Conservative forces and Non- Conservative forces.

Non-Conservative forces: Forces which work depends on the path the object makes (for example, a frictional force). The work of a non-conservative force has to be calculated every time with the definition of Work:

$$\mathbf{W}_{\textbf{non-cons}} = \mathbf{F}\,\Delta r \cos\theta$$

Now we can write the **Law of Conservation of Mechanical Energy**

Change in the mechanical energy of the system is equal to the work done on the system by all *non*-conservative forces

E = K + U
mechanical energy

Master Equation $W_{total} = W_1 + W_2 + W_3 + \ldots$

cons non-cons

$$\mathbf{E_2 - E_1 = W_{non\text{-}cons}}$$

$$\mathbf{E_2 = E_1 + W_{non\text{-}cons}} \quad \textbf{or} \quad \mathbf{K_2 + U_2 = K_1 + U_1 + W_{non\text{-}cons}}$$

$$K_2 - K_1 = W_{net} = W_{cons} + W_{non\_cons} = U_1 - U_2 + W_{non\_cons}$$

Summary

Kinetic energy of an object: $K = \frac{mv^2}{2}$

Work done by a constant force: $W = F \cdot \Delta r \cdot \cos\theta$

In general: $W = \text{Area}_{\text{under-the-graph-F-vs.-}\Delta r}$

The total work: $W_{total} = W_1 + W_2 + W_3 + \ldots$ or $W_{total} = F_{Net} \cdot \Delta r \cdot \cos\theta_{Net}$

Work – Kinetic Energy Theorem: $K_2 - K_1 = W_{total}$

$E = K + U$ is ***mechanical energy***

U is ***potential energy***

$U = mgy$ **when an object is under an influence of the force of gravity**

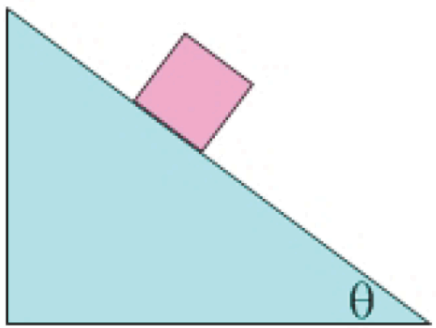
Work done by a conservative force: $W_{cons} = U_1 - U_2$

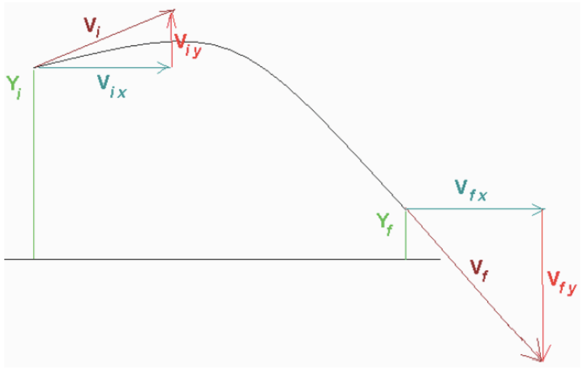
Master Equation: $E_2 - E_1 = W_{non\text{-}cons}$ or $E_2 = E_1 + W_{non\text{-}cons}$

Master Equation (full form): $K_2 + U_2 = K_1 + U_1 + W_{non\text{-}cons}$

Conservation of mechanical energy: If there are conservative forces only (no friction!) $E_2 = E_1$

Example



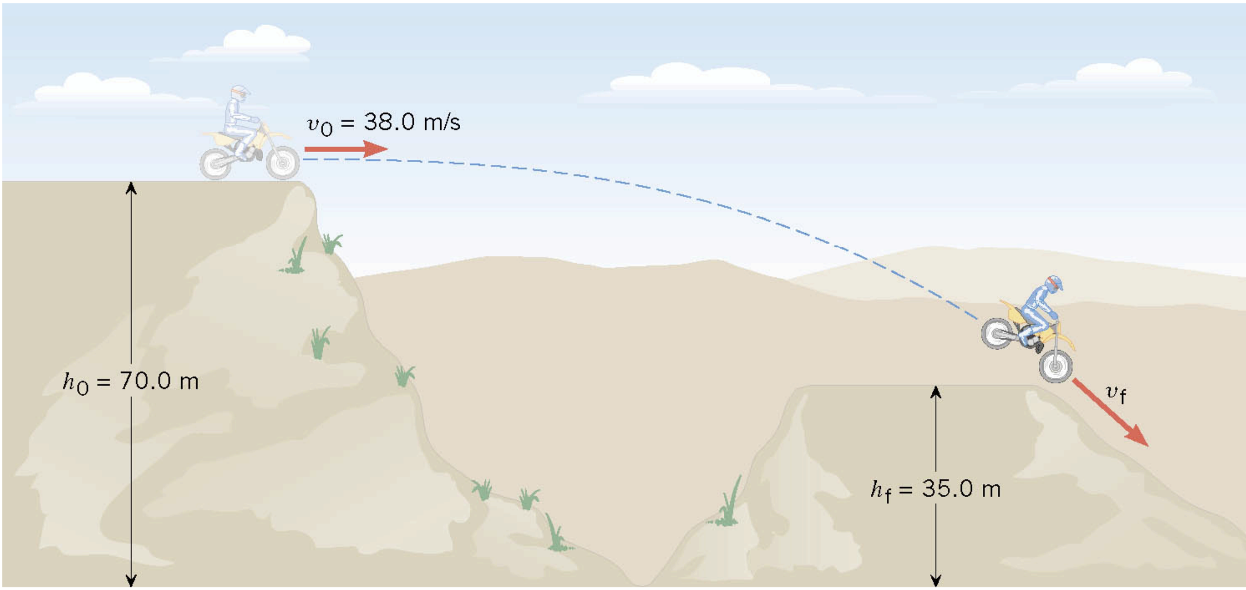


$$K_1 + U_1 + W_{nc} = K_2 + U_2$$

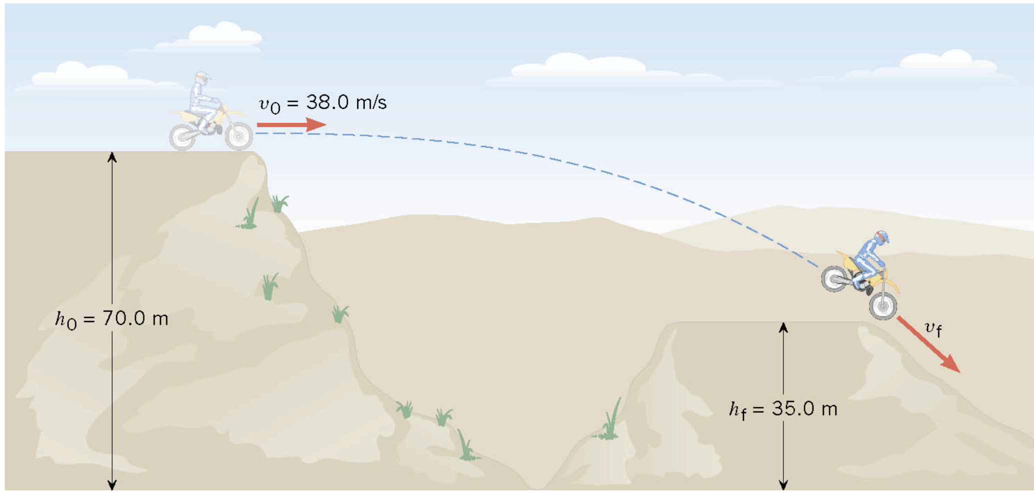
How does L C E work for the well-known situations?

A Daredevil Motorcyclist

A motorcyclist is trying to leap across the canyon by driving horizontally off a cliff 38.0 m/s. Ignoring air resistance, find the speed with which the cycle strikes the ground on the other side.



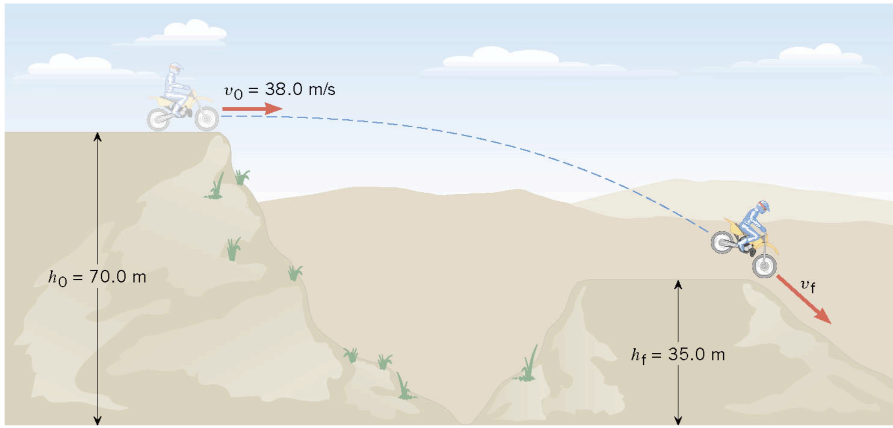
A Daredevil Motorcyclist



$$mgh_f + \frac{1}{2}mv_f^2 = mgh_i + \frac{1}{2}mv_i^2$$

$$gh_f + \frac{1}{2}v_f^2 = gh_i + \frac{1}{2}v_i^2$$

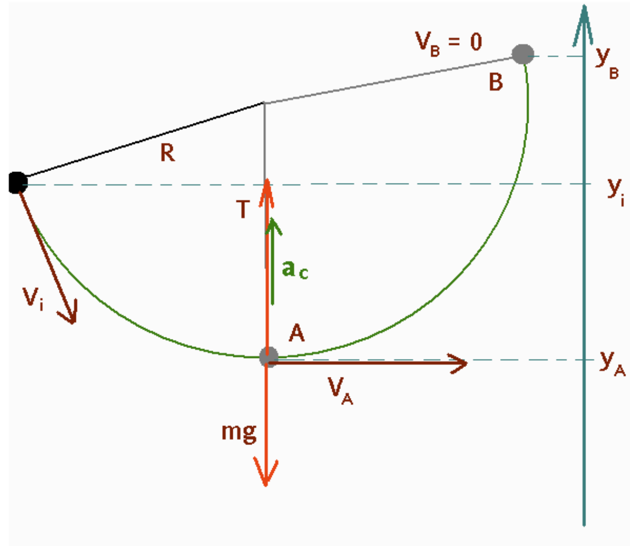
A Daredevil Motorcyclist



$$gh_f + \tfrac{1}{2}v_f^2 = gh_i + \tfrac{1}{2}v_i^2$$

$$v_f = \sqrt{2g\left(h_i - h_f\right) + v_i^2}$$

$$v_f = \sqrt{2\left(9.8\,\text{m/s}^2\right)\left(35.0\,\text{m}\right) + \left(38.0\,\text{m/s}\right)^2} = 46.2\,\text{m/s}$$



Two forces do the work on the ball: force of gravity and the tension force of the string.

The work done by the tension is:

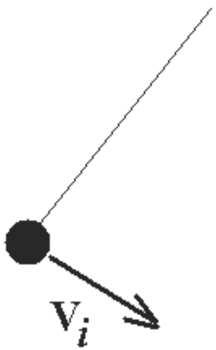
A) Negative

B) Zero

C) Positive

D) Impossible to answer

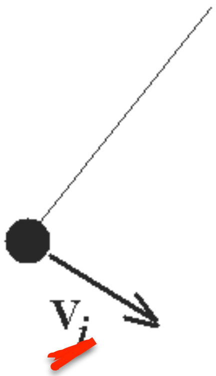
Example 4



We give a toss to a ball on a long string.

Try to find what initial speed the ball should have to reach the maximum height 20 cm higher its initial position. Use g = 10 m/s^2.

Example



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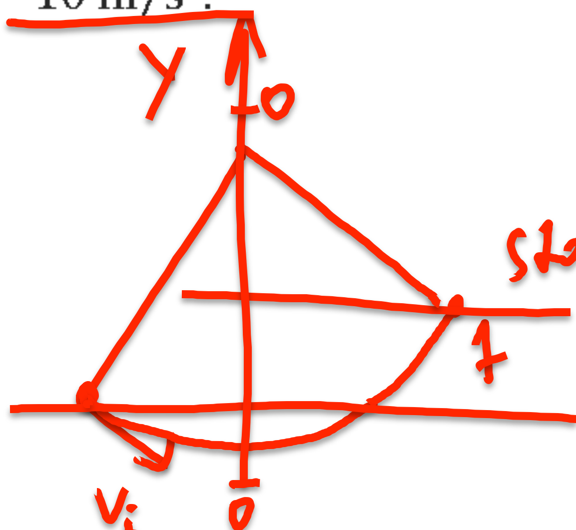


Stop: $V_f = 0$

$y_f = 0.2\,m$

$y_i = 0$

$$\frac{mV_i^2}{2} + mg\cdot 0 + 0 = \frac{m0^2}{2} + m\cdot 10\cdot 0.2$$

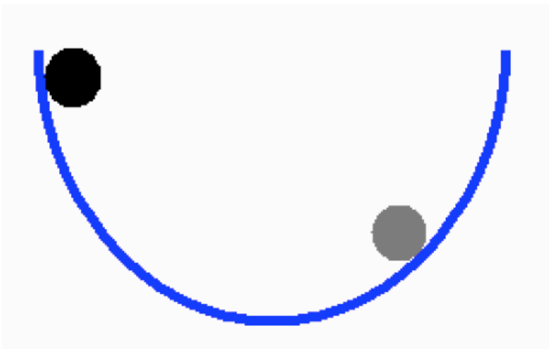
$= 0$

$$\frac{mV_i^2}{2} = m\cdot 2$$

$V_i^2 = 4$ $\quad V_i = 2\,m/s$

More Examples

Potential well



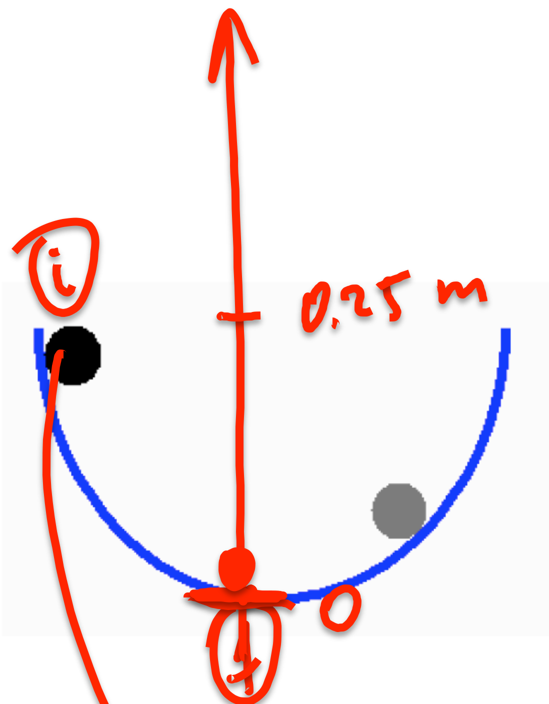
$$E_2 = E_1 + W_{non\text{-}cons}$$

If the cart starts from the height y_i it stops eventually because of friction, hence the whole potential energy stored in the cart transfers into the work done by friction.

$$0 = mgy_i + W_{friction} \qquad W_{friction} = -mgy_i$$

More Examples

Potential well



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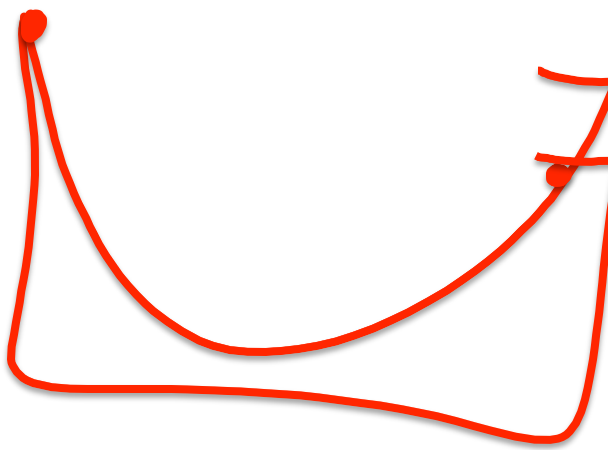


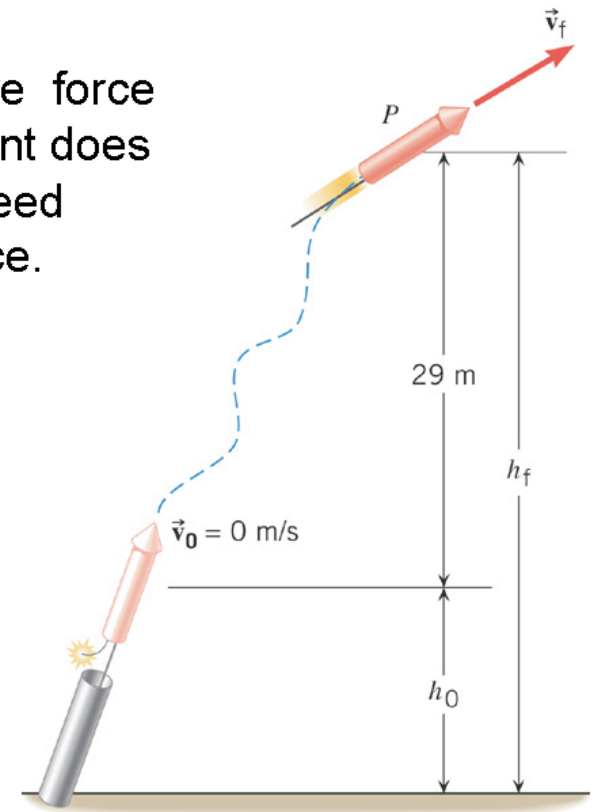
#3oh → W_t in 1 move

how many moves

Fireworks

Assuming that the nonconservative force generated by the burning propellant does 425 J of work, what is the final speed of the rocket? Ignore air resistance.

$$\left(mgh_o + \tfrac{1}{2}mv_o^2\right) + W_{nc} = \left(mgh_f + \tfrac{1}{2}mv_f^2\right)$$



Fireworks

$$W_{nc} = \left(mgh_f + \tfrac{1}{2}mv_f^2\right) - \left(mgh_o + \tfrac{1}{2}mv_o^2\right)$$

$$= mgh_f - mgh_o + \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_o^2$$

$$= mg\left(h_f - h_o\right) + \tfrac{1}{2}mv_f^2$$

Put in the numbers:

$$425\,\mathrm{J} = (0.20\,\mathrm{kg})(9.80\,\mathrm{m/s^2})(29.0\,\mathrm{m}) + \tfrac{1}{2}(0.20\,\mathrm{kg})v_f^2$$

$$\Rightarrow \quad v_f = 61\,\mathrm{m/s}$$