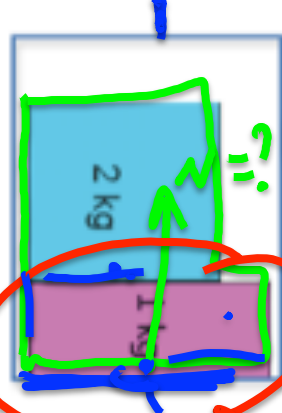


When acceleration of the elevator is up,
the velocity points ...

1. Up 2. Down
3. To the left 4. To the right
1. Impossible to say

Calculate the net force for each block, the normal forces
acting on each block, when $|a| = 2 \text{ m/s}^2$.

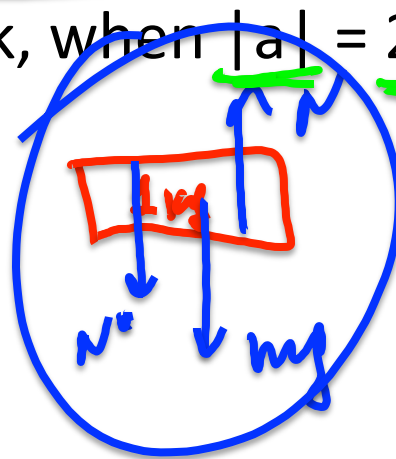


When acceleration of the elevator is up, the velocity points ...

1. Up 2. Down
3. To the left 4. To the right
1. Impossible to say

$$g = 10 \text{ m/s}^2$$

Calculate the net force for each block, the normal forces acting on each block, when $|a| = 2 \text{ m/s}^2$.

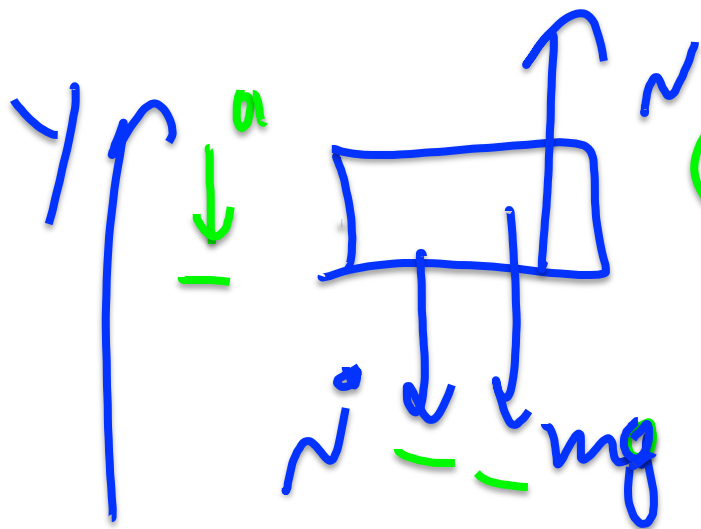


$$mg + \bar{N} + \bar{N}^* = \bar{F}_{\text{net}} = m\bar{a}$$

Handwritten calculations:

$$1 \times 10 + 2 + 2 = 1 \times 2$$

$$12 = 2$$



$$\bar{N} + \bar{N}^* + m\bar{g} = m\bar{a}$$

(Y:) $N - N^* - mg = -ma$?

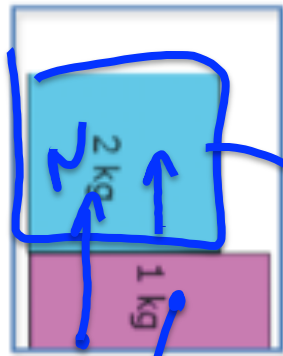
$$N = \underline{mg + N^* - ma}$$

$$N = 1 \cdot 10 + N^* - 1 \cdot 2$$

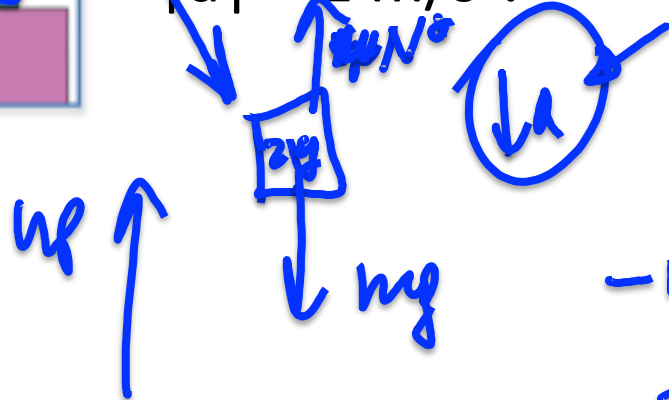
$$N = 24 \text{ N}$$

0
1
2
3
4

5
6
7
8
9



Calculate the net force for each block, the normal forces acting on each block, when $|a| = 2 \text{ m/s}^2$.



$$-mg + N^* = -ma$$

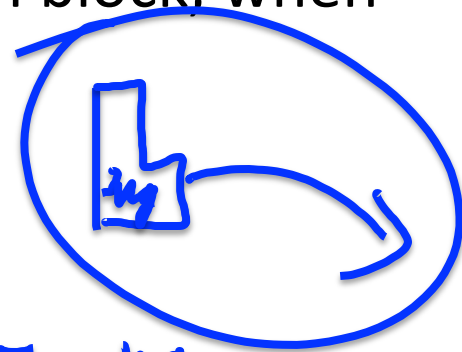
$$-2 \cdot 10 + N^* = -2 \cdot 2$$

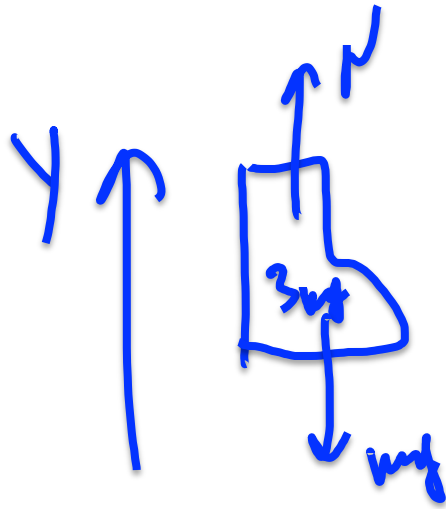
$$N^* = 16$$

$$N^* =$$

$$|F_{\text{net}}| = ma = 1 \cdot 2 = 2 \text{ N}$$

$$\downarrow F_{\text{net}}$$





$\downarrow a$

$$-mg + N = -ma$$

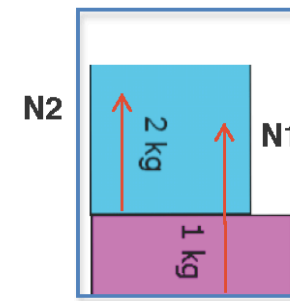
$$-3 \cdot 10 + N = -3 \cdot 2$$

$$N = 24 \text{ N}$$

Example

Physics of elevators.

Let's put the same two blocks in an elevator.



$$g = 10 \text{ m/s}^2$$

(forces of gravity are not shown)

Find all possible forces when the elevator is:

a) moving up with constant velocity

when $v = \text{const} \Rightarrow a = 0 ! \Rightarrow$

(blue) $Mg = 20 \text{ N}$ (magenta) $mg = 10 \text{ N}$

b) moving down with constant velocity

$N1 = (m+M)g = 30 \text{ N}$ $N2 = mg = 20 \text{ N}$

c) moving up with constant acceleration of 2 m/s^2 (what can we say about its velocity?) $\uparrow a = 2 \text{ m/s}^2$ $\uparrow v$ (speeding up)

(blue box) $2 \cdot 2 = N2 - 2 \cdot 10 \Rightarrow N2 = 24 \text{ N}$

(both) $3 \cdot 2 = N1 - 3 \cdot 10 \Rightarrow N1 = 36 \text{ N}$

d) moving up with constant acceleration of -2 m/s^2 (what can we say about its velocity?) $\downarrow a = -2 \text{ m/s}^2$ $\uparrow v$ (slowing down)

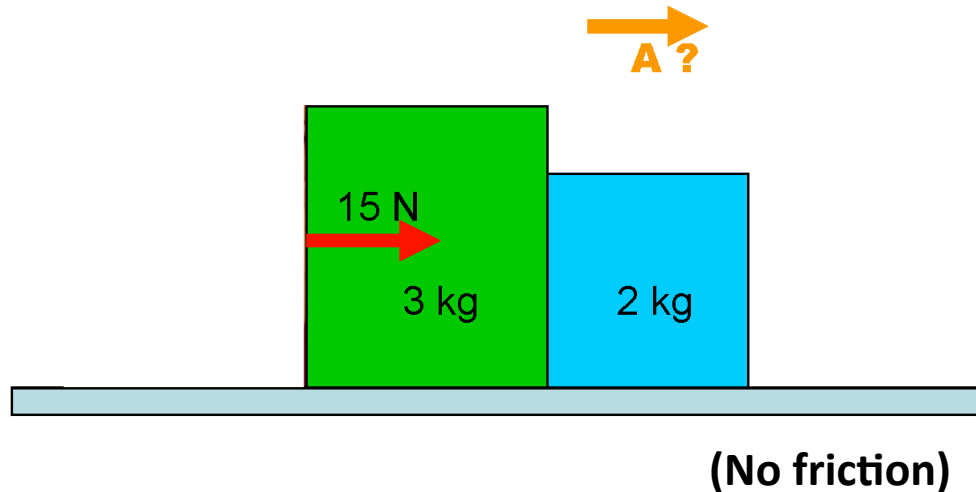
(blue box) $-2 \cdot 2 = N2 - 2 \cdot 10 \Rightarrow N2 = 16 \text{ N}$

(both) $-3 \cdot 2 = N1 - 3 \cdot 10 \Rightarrow N1 = 24 \text{ N}$

(magenta) $-2 \cdot 1 = 24 - 16 - 1 \cdot 10$ (works!)

Work together - Two boxes

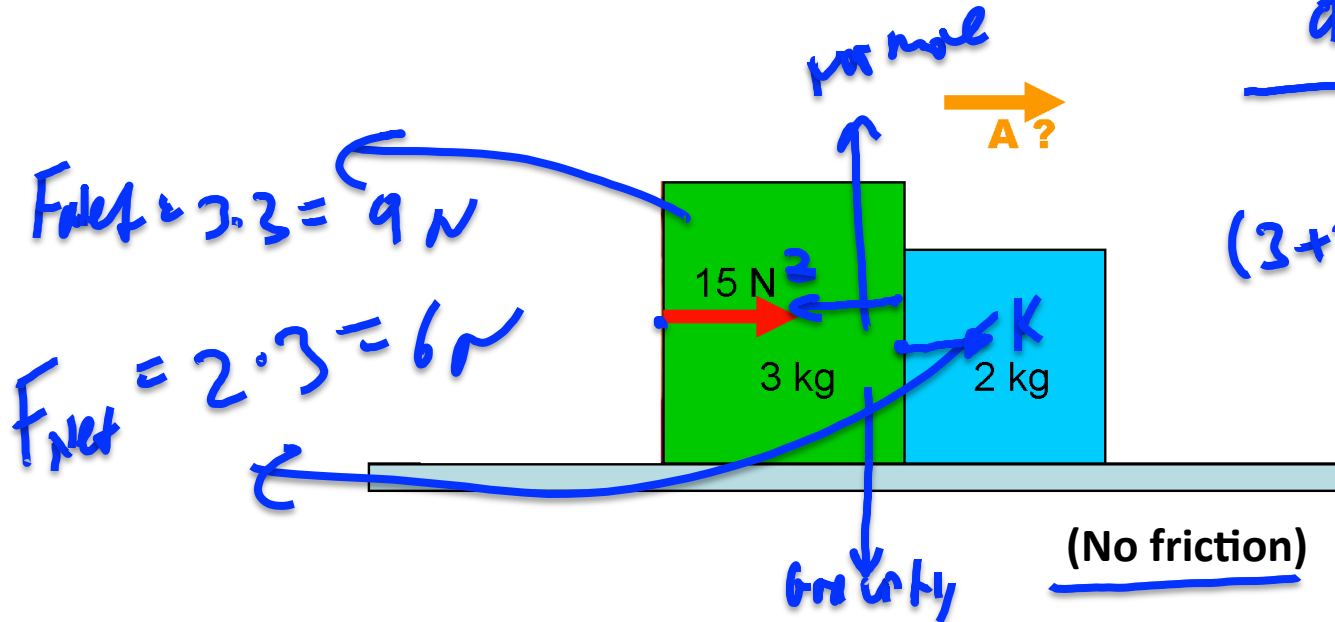
Consider a system of two boxes, with a hand exerting a 15 N force to the right on the green box. The green box has a larger mass. Sketch three free-body diagrams (green box, blue box, combined system)



Work together - Two boxes

0 1 2 3 4 5 6 7 8 9

Consider a system of two boxes, with a hand exerting a 15 N force to the right on the green box. The green box has a larger mass. Sketch three free-body diagrams (green box, blue box, combined system)



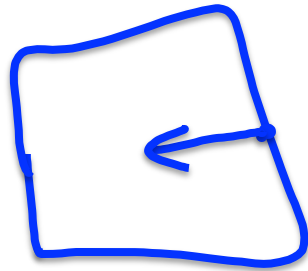
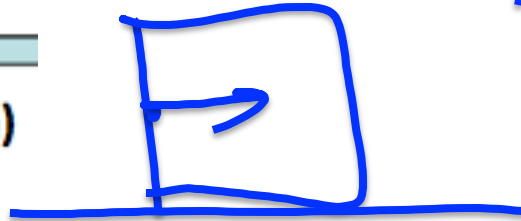
Handwritten equations and notes:

- $a \rightarrow |Z| = |K|$
- $(3+2) \cdot a = 15$
- $a = 3 \text{ m/s}^2$

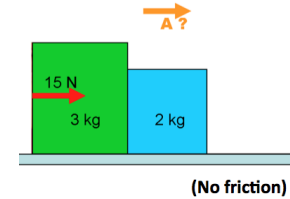


(No friction)

$$|K| = |F_{\text{net}}| = 6 \approx 12$$



Find the acceleration

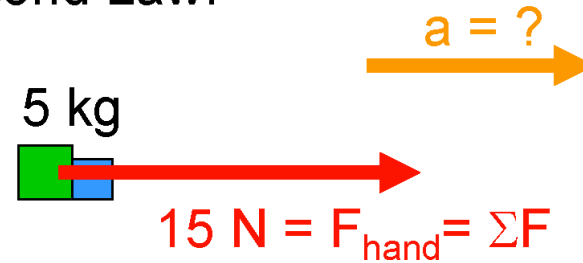


Let's choose positive to be to the right.

Which of the three free-body diagrams (for horizontal components) should we use? (Vertical: $m\mathbf{g}$ just cancels \mathbf{F}_n)

The simplest is the free-body diagram of the two-box system. Apply Newton's Second Law.

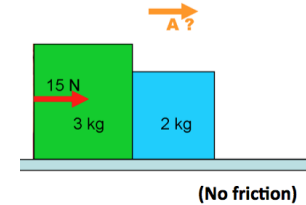
$$\Sigma \vec{F} = (m_g + m_b) \vec{a}$$



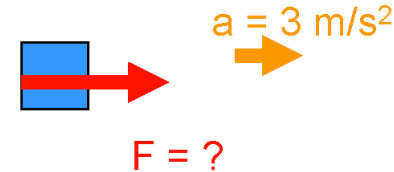
$$\vec{a} = \frac{\Sigma \vec{F}}{(m_g + m_b)} = \frac{+15 \text{ N}}{5.0 \text{ kg}} = +3.0 \text{ m/s}^2 \rightarrow$$

Find the force the green box applies to the blue box.

Which free-body diagram should we use?



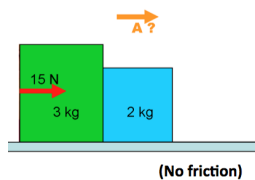
Let's use the free-body diagram of the blue box.



Apply Newton's Second Law.

$$\sum \vec{F} = m_b \vec{a} = 2.0 \text{ kg} \times (+3.0 \text{ m/s}^2) = +6.0 \text{ N}$$

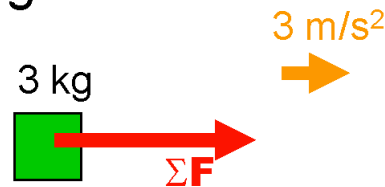
The vertical forces cancel one another, so the net force is the force the green box applies to the blue box, 6.0 N to the right.



Find the force the blue box applies to the green box.

In this case, let's use the free-body diagram of the green box.

Apply Newton's Second Law.

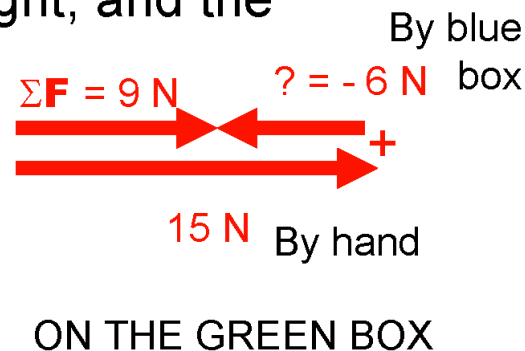


$$\Sigma \vec{F} = m_g \vec{a} = 3.0 \text{ kg} \times (+3.0 \text{ m/s}^2) = +9.0 \text{ N}$$

The vertical forces cancel, and the net force is the vector sum of the 15 N force directed right, and the force the blue box exerts to the left.

$$+15.0 + \vec{F}_{N,bg} = +9.0 \text{ N}$$

$$\vec{F}_{N,bg} = +9.0 \text{ N} - 15.0 \text{ N} = -6.0 \text{ N}$$

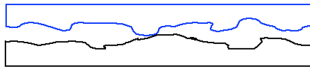


This agrees with Newton's Third Law.

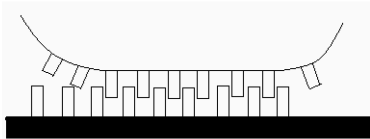
Static and Kinetic Frictional Forces

When the two surfaces are not sliding across one another the friction is called **static friction**.

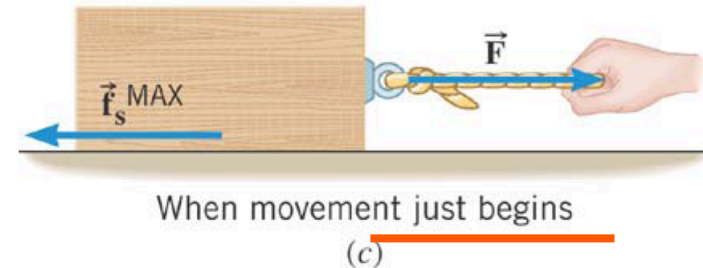
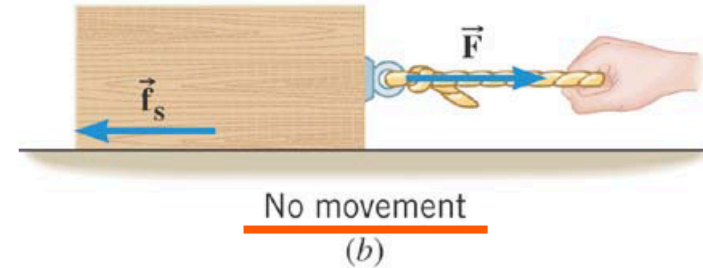
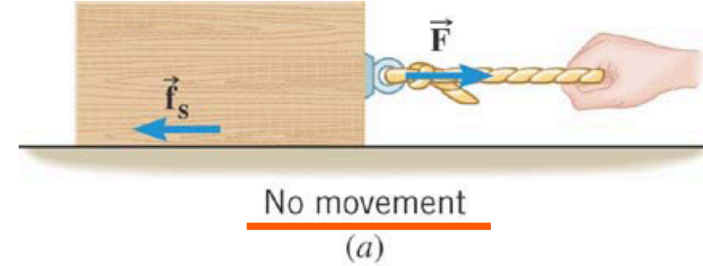
Static friction is the reason we can walk and cars can move!



Static friction between two surfaces works in a similar way with two gears attached to each other.



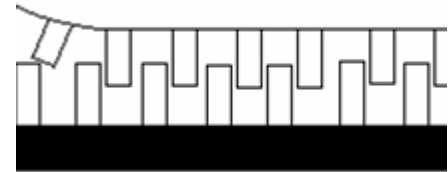
Viewed under a microscope, a surface generally looks rough. Surfaces put together make contact at very few places. When trying to move past each other, the high parts on each surface get stuck on one another.



Lack of area dependence

The normal force is whatever it has to be. The object presses “into” the surface until there is enough microscopic contact area to hold the object up. As long as this is a small fraction of the apparent macroscopic area, it is independent of the apparent area.

The maximum sideways force required to break the object loose is proportional to this microscopic contact area. Hence it is proportional to the normal force.



Static and Kinetic Frictional Forces

The magnitude of the static frictional force can have any value from zero up to a maximum value.

$$f_s \leq f_s^{MAX}$$

$$f_s^{MAX} = \mu_s F_N$$



is called the coefficient of static friction.

Static and Kinetic Frictional Forces

Static friction opposes the *impending* relative motion between two objects.

Kinetic friction opposes the relative sliding motion motions that actually does occur.

$$f_k = \mu_k F_N$$



is called the coefficient of kinetic friction.

Static and Kinetic Frictional Forces

Approximate Values of the Coefficients of Friction
for Various Surfaces*

Materials	Coefficient of Static Friction, μ_s	Coefficient of Kinetic Friction, μ_k
Glass on glass (dry)	0.94	0.4
Ice on ice (clean, 0 °C)	0.1	0.02
Rubber on dry concrete	1.0	0.8
Rubber on wet concrete	0.7	0.5
Steel on ice	0.1	0.05
Steel on steel (dry hard steel)	0.78	0.42
Teflon on Teflon	0.04	0.04
Wood on wood	0.35	0.3

*The last column gives the coefficients of kinetic friction, a concept that will be discussed shortly.

A large box is being pushed across the floor at a constant speed of 4.0 m/s. What can you conclude about the forces acting on the box?

- (A) If the force applied to the box is doubled, the constant speed of the box will increase to 8.0 m/s.
- (B) The amount of force applied to move the box at a constant speed must be more than its weight.
- (C) The amount of force applied to move the box at a constant speed must be equal to the amount of the frictional forces that resist its motion.
- (D) The amount of force applied to move the box at a constant speed must be more than the amount of the frictional forces that resist its motion.
- (E) There is a force being applied to the box to make it move but the external forces such as friction are not “real” forces they just resist motion.

Friction when walking

Let's say you're standing still and you then start to walk forward across a flat floor. Which force of friction is involved? Which direction is it?

1. Static friction, acting opposite to the way you are walking.
2. Static friction, acting in the direction you are walking.
3. Kinetic friction, acting opposite to the way you are walking.
4. Kinetic friction, acting in the direction you are walking.

Friction when walking

Because your foot does not slip on the floor, the frictional force is **static friction**.

To determine the direction think about what would happen if there was no friction.

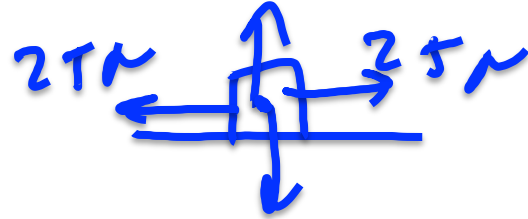
When you pushed against the floor your foot would slide backwards. Friction opposes this, and **acts forwards**.

Static friction is the force that propels you forward.

$$F_{s1} \leq \mu_s \cdot N \quad \text{Problem} \quad F_{k1} = \mu_k \cdot N$$

A box with a weight of $mg = 50 \text{ N}$ is at rest on a flat surface. A force of 25 N directed right is then applied. If the coefficient of static friction between the box and the surface is 0.6 , and the coefficient of kinetic friction between the box and the surface is 0.4 , what is the force of friction acting on the box?

- A. 20 N to the left
- B. 25 N to the left
- C. 30 N to the left
- D. 20 N to the right
- E. It depends on the point of view



(Hint: if the situation with no motion is possible, first check if the static friction is strong enough to prevent an object from moving: **if the applied force is less than maximum value of static friction, the object is not moving**)

Friction on a box

The force of static friction is whatever is needed, up to a limit of $\mu_s F_N = 0.6 (50 \text{ N}) = 30 \text{ N}$.

The ~~applied force is only 25 N~~ to the right, so it can be balanced by a static frictional force of 25 N to the left.

What is the *net* force acting on the box?

1. 0 N

2. 15 N

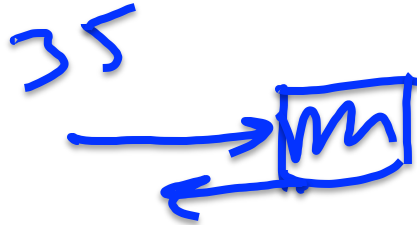
3. 25 N

4. 30 N

$$W = mg$$

$$m = \frac{50}{10}$$

Friction on a box



$$F_{\text{Net}} = 15 = m \cdot a$$

$$F_k = \mu_k \cdot N = 0.4 \cdot 50 = 20 \text{ N}$$

If the applied force is 35 N to the right, then the box breaks loose, and as it slides to the right, the force of kinetic friction is only $\mu_k F_N = 0.4 (50 \text{ N}) = 20 \text{ N}$ to the left, opposing the relative motion.

The net force is $35 \text{ N} - 20 \text{ N} = 15 \text{ N}$ to the right.

If $g = 10 \text{ m/s}^2$, $m = 5 \text{ kg}$, and the box accelerates with
(acceleration) = (net force)/(mass) = 3 m/s^2 to the right.

$$a = \frac{15}{5}$$

If we know the applied force and the masses of the block, and coefficients of friction are 0.2 (static) 0.1 (kinetic) how many forces can we find ($g = 10 \text{ m/s}^2$); what is the acceleration of the green block?



How many systems can we choose?

What is the critical value of the applied force?

If we know the applied force and the masses of the block, and coefficients of friction are 0.2 (static) 0.1 (kinetic) how many forces can we find ($g = 10 \text{ m/s}^2$); what is the acceleration of the green block?

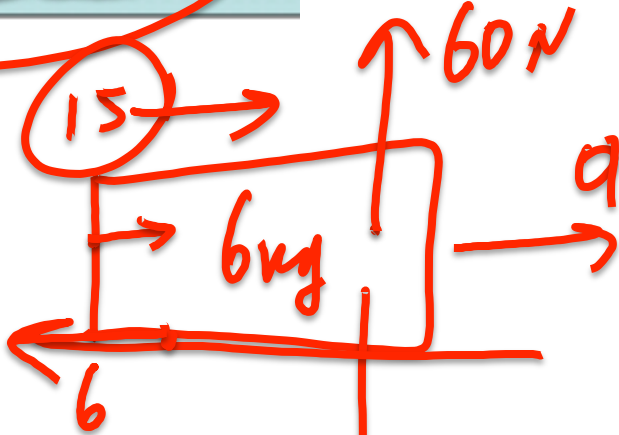


$\rightarrow a$

0 1 2 3...

How many systems can we choose?

What is the critical value of the applied force?



$$F_k = \mu_k \cdot N = 0.1 \cdot 60 = 6 \text{ N}$$

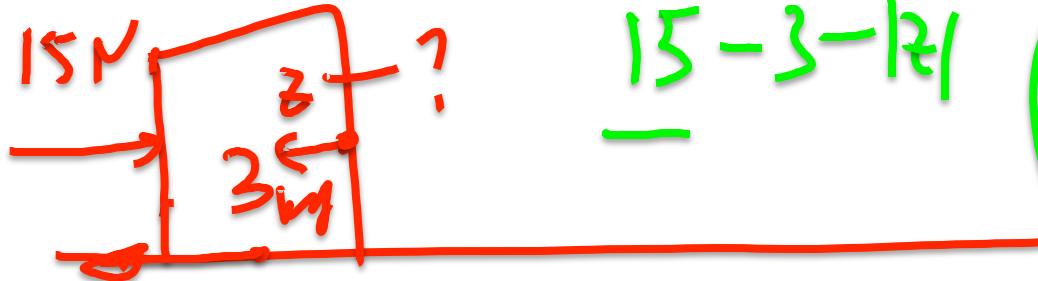
15 VS. max static Fr F_{max}

$$\mu_{\text{st}} \cdot N$$

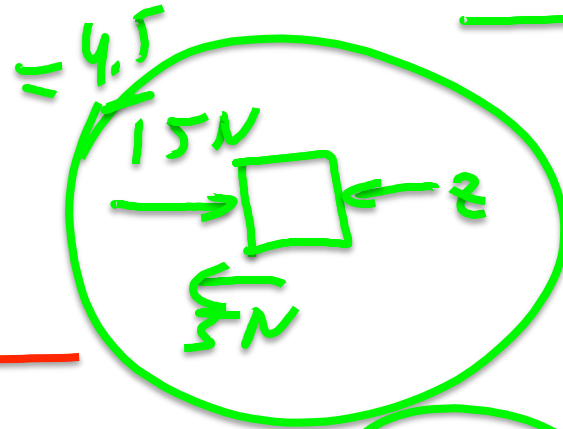
$$0.2 \cdot 60 = \underline{12 \text{ N}}$$

$$F_{\text{Net}} = 15 - 6 = 9 \text{ N} = 6 \cdot a \Rightarrow a = 1.5 \text{ m/s}^2$$

$$a \rightarrow F_{\text{net box 1}} = m_k \cdot a = 3 \cdot 1.5 = 4.5 \text{ N}$$



$$F_{\text{fric}} = \mu_k \cdot N_1 = 0.1 \cdot 30 = 3 \text{ N}$$



$$4.5 \text{ N} \rightarrow = \text{Net}$$

Q. None of the to the right 1 N ~~2 N~~ ~~3 N~~ ~~4 N~~ ~~5 N~~

$$|T| = 15 - 3 - 4.5 = 7.5 \text{ N}$$

How many systems can we choose? 6

If we know the applied force and the masses of the block, and coefficients of friction are 0.2 (static) 0.1 (kinetic) how many forces can we find ($g = 10 \text{ m/s}^2$); what is the acceleration of the green block?

$$15 > 0.2 \cdot (3+2+1) \cdot 10$$



(the whole) $(3+2+1)a = 15 - 0.1 \cdot (3+2+1) \cdot 10 \Rightarrow a = 1.5 \text{ m/s}^2$

(the green) $3 \cdot 1.5 = 15 - 0.1 \cdot 3 \cdot 10 - F_1 \Rightarrow F_1 = 7.5 \text{ N}$

(the blue) $2 \cdot 1.5 = 7.5 - 0.1 \cdot 2 \cdot 10 - F_2 \Rightarrow F_2 = 2.5 \text{ N}$

(the magenta) $1 \cdot 1.5 = 2.5 - 0.1 \cdot 1 \cdot 10 \Rightarrow \text{works!}$