1. Two balls are launched straight up into the air. The green ball has an initial velocity twice as large as the red ball's initial velocity. Compare the maximum heights reached by the two balls (neglect air resistance).

A. The red ball goes 4 times higher
B. The red ball goes 2 times higher
C. The balls reach the same height
D. The green ball goes 2 times higher
E. The green ball goes 4 times higher

Maximum Height

The maximum height, y_{max}, is determined solely by the initial velocity in the y direction and the acceleration due to gravity.

It can be found from the equation:

$v_y^2 = v_{oy}^2 - 2\,g\,(y_{max} - y_o)$ ← **Let's prove this!**

We use that:

1) $v_{oy} = v_o$ (we shoot it straight up)

2) when the projectile is at the maximum height, $v_y = 0$.

3) $y_o = 0$ (the natural choie)

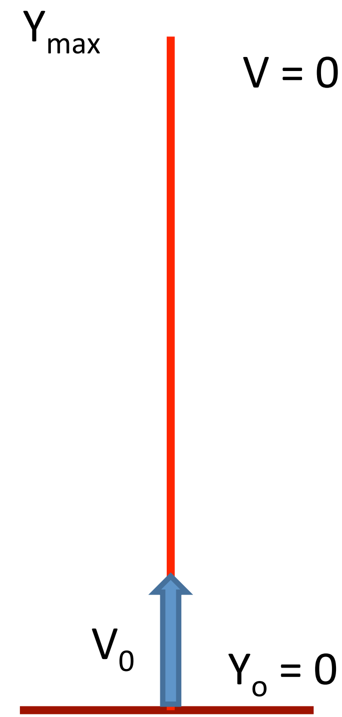
Solving the equation for y_{max} gives:

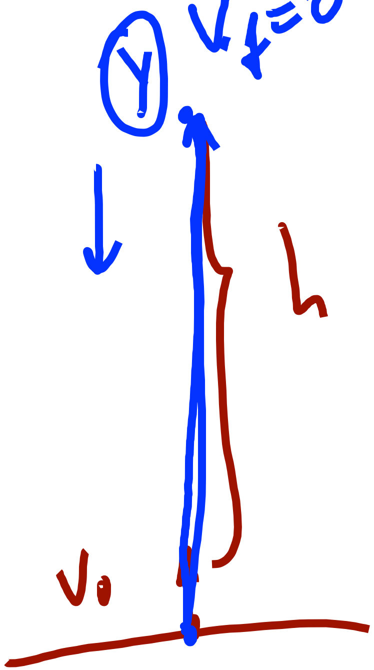
$$y_{max} = \frac{v_o^2}{2\,g}$$

where $g = 9.8\ m/s^2$

$h \sim V_0^2$

Y_{max} is measured from the initial location, NOT from the ground!





$\vec{g} = 9.8\ m/s^2$

$$V_f = V_i + a \cdot t \rightarrow 0 = V_i - g \cdot t$$

$$t = \frac{V_i}{g}$$

$$\Delta Y = V_i \cdot t + \frac{a t^2}{2}$$

$$h = V_i \cdot t - \frac{g \cdot t^2}{2}$$

$$v_i - gt = 0$$

$$\frac{v_i}{g} = \frac{gt}{g}$$

$$\frac{v_i}{g} = t$$

Free fall example

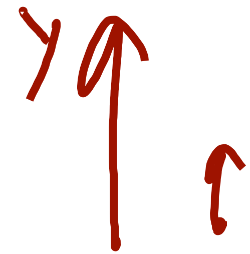
You throw a ball straight up. It leaves your hand at 12.0 m/s.

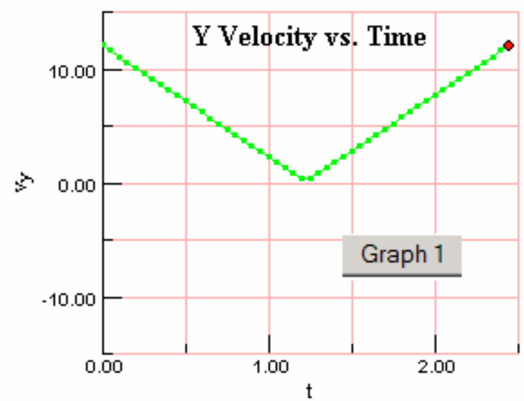
(a) How high does it go?

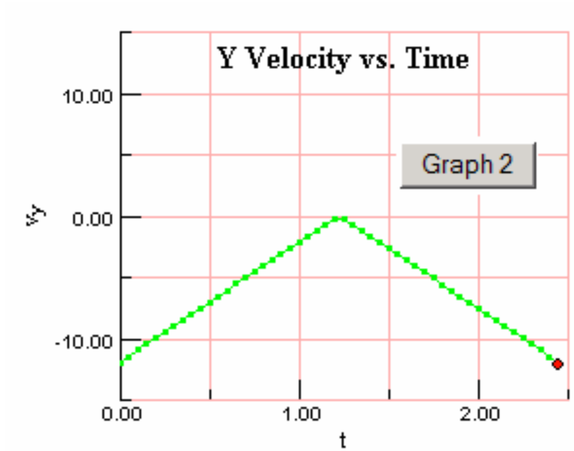
(b) If, when the ball is on the way down, you catch it at the same height at which you let it go, how long was it in flight?

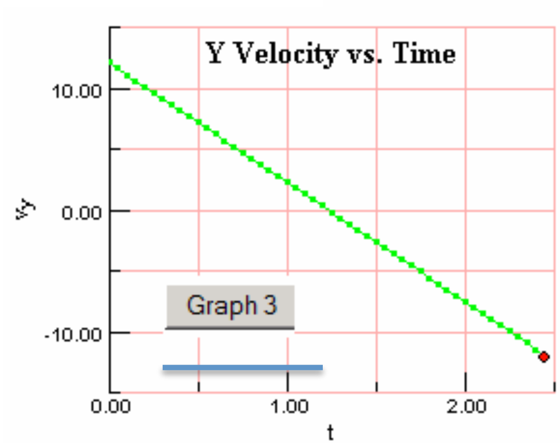
(c) How fast is it traveling when you catch it?

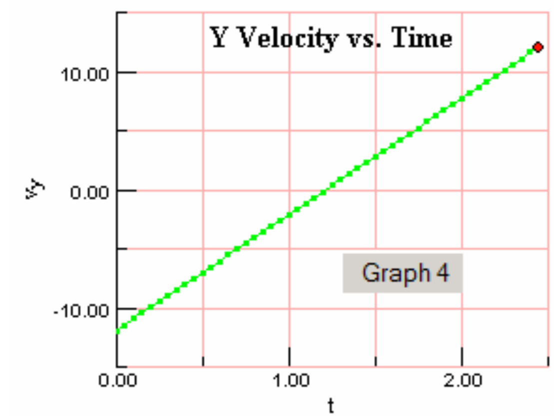
Which graph correctly shows the *velocity* as a function of time for the ball?



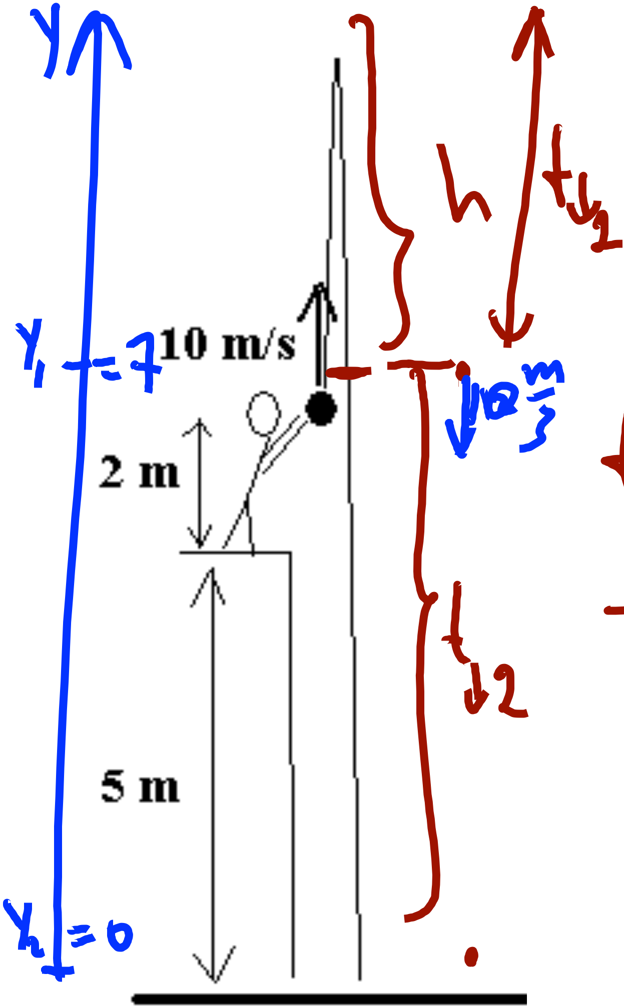








A man standing on the roof of a 5 m high barn shoots a basketball straight upward with the initial speed of 10 m/s, releasing the ball when it is 2 m above the roof. Try to find (use g = 10 m/s^2) how much time is needed for the ball to reach the highest point of the trajectory? The height of the ball from the ground at its highest point. The total time the ball was flying. The total distance traveled by the ball. The total displacement traveled by the ball. The average speed of the ball over the total time the ball was in the air. The average velocity of the ball over the total time the ball was in the air. The speed of the ball just before it hits the ground.



$$v = v_i - gt$$

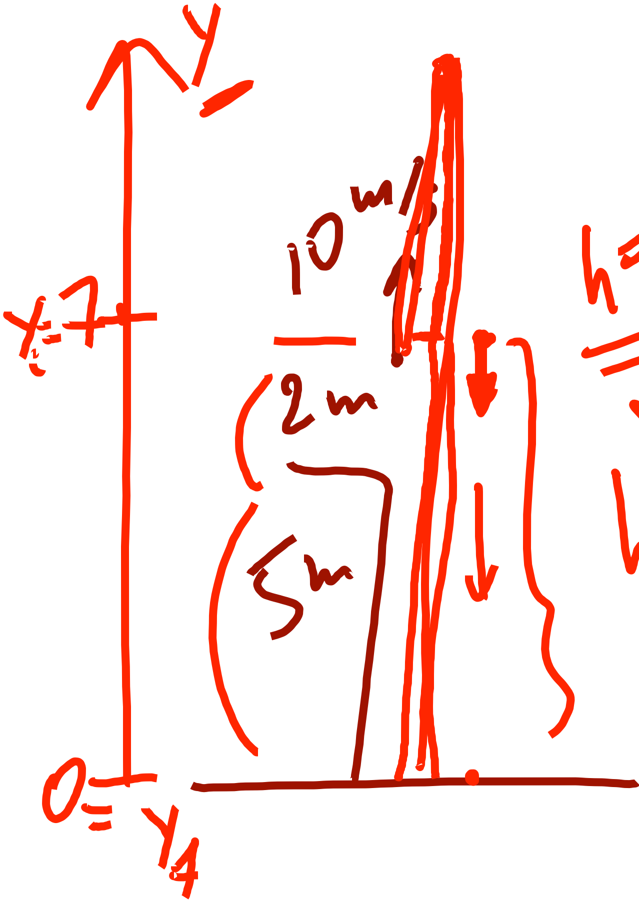
$$\Delta y = v_i t - \frac{gt^2}{2}$$

$$t_\uparrow = \frac{v_i}{g} \qquad \Delta h = \frac{v_i^2}{2g}$$

$$t_\uparrow = \frac{10}{10} = 1s \qquad t_{\downarrow 2} = 1$$

$$\Delta y = v_i t + \frac{at^2}{2};$$

$$0 - 7 = -10 \cdot t - \frac{10 \cdot t^2}{2}$$



$h = 7\,m$

$h = 10 \cdot t + \frac{10 t^2}{2}$

$\Delta y = v_i t + \frac{a t^2}{2}$

$0 - 7 = -10 \cdot t - \frac{10 \cdot t^2}{2} \quad \times (-1)$

$7 = 10t + 5t^2$

$$5t^2 + 10 \cdot t - 7 = 0$$

$$t = \frac{-10 \pm \sqrt{10^2 - 4 \cdot 5 \cdot (-7)}}{2 \cdot 5} =$$

Practice at home

You release a ball from a rest from a 5 m high barn. How much time will it take for the ball to reach the ground? Find the lending speed of the ball.

Answer the question below.

1. What is the direction of the y-axes in the experiment?

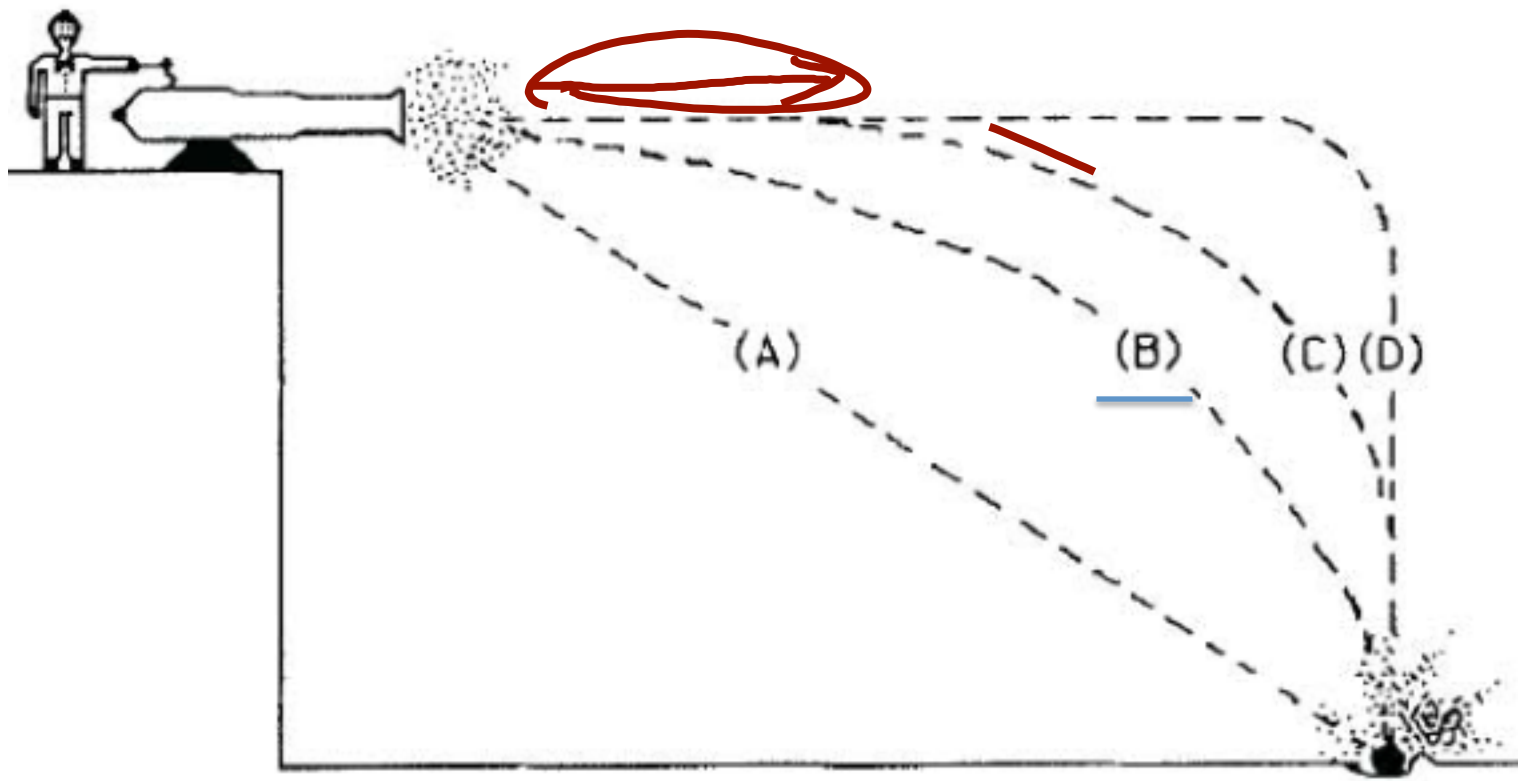
2. Clearly explain the reason for your choice.

3. If the origin is chosen at the initial location of the ball, write the motion equation for its position as a function of time.

4. Now write the motion equation for its velocity as a function of time.

5. Let's say that now instead of letting the ball falling down you throw it upward from the same location and with initial velocity of 1 m/s. Draw the picture for this situation, and write the motion equations x(t) and v(t) for this case (you do NOT change your reference frame).

Which of the paths in the picture best represents the path of the cannon ball?



The list of concepts, definitions, laws and relations to memorize when taking PY 105 course

For each physical quantity a students must be able to answer the following questions:

1. what is its name;

2. what is a usual symbol for the quantity;

3. what is its unit;

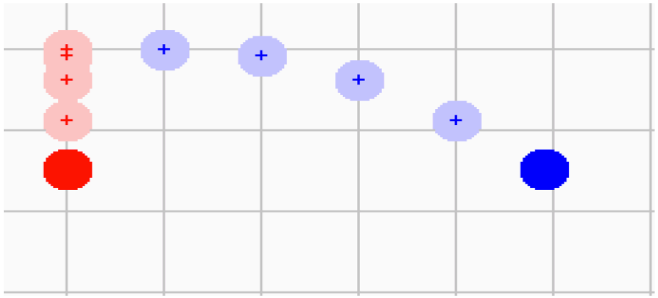
4. how to measure the quantity;

5. to what other physical quantities is this quantity related;

6. how is this physical quantity algebraically related to other physical quantities?

Projectile motion

Projectile motion (PM), properties of PM, range, maximum height, flight time.



A race

Two balls are launched simultaneously from the same height. Ball A is released from rest, and drops straight down. Ball B is given an initial *horizontal* velocity.

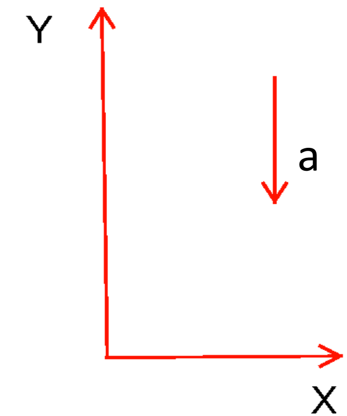
Which ball hits the ground first?

A. This is too early for the morning class.
B. Ball A
C. Ball B
D. Both balls hit the ground at the same time
E. It depends on the mass of the balls.

Projectile Motion

Under the influence of gravity alone, an object near the surface of the Earth will accelerate downwards at $9.80 m/s^2$.

$$a_y = -9.80\,\mathrm{m/s^2} \qquad a_x = 0$$



$g = |a| = 9.8\ m/s^2$

$a_y = -g$

$$v_x = v_{ox} = \text{constant}$$

Equations for 2-D motion

Add an x or y subscript to our usual equations of 1-D motion (appropriate for constant acceleration…).

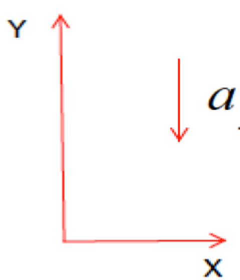
$$v_x = v_{0x} + a_x t$$

$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

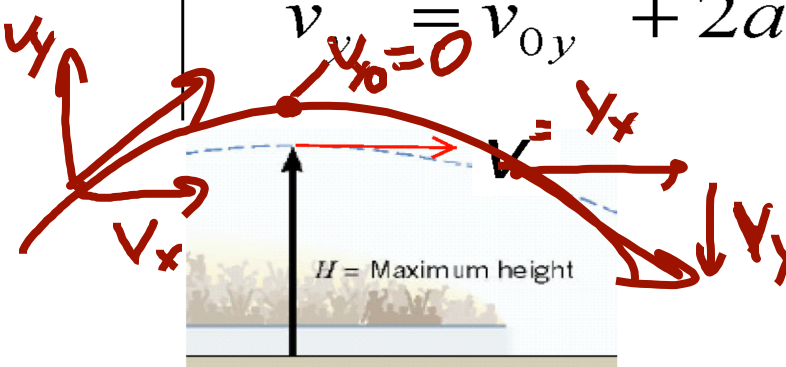
$$v_y = v_{0y} + a_y t$$

$$y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$$

$$v_y^{\,2} = v_{0y}^{\,2} + 2a_y (y - y_0)$$



$$a_y = -9.80\,\mathrm{m/s^2}$$



IMPORTANT: When solving problems always keep the x-component data separate from the y-component data. **The only thing that can be used in both sets of equations is time.**

When solving problems on projectile motion; for the natural choice of the x – and y (up) – coordinates we have this:

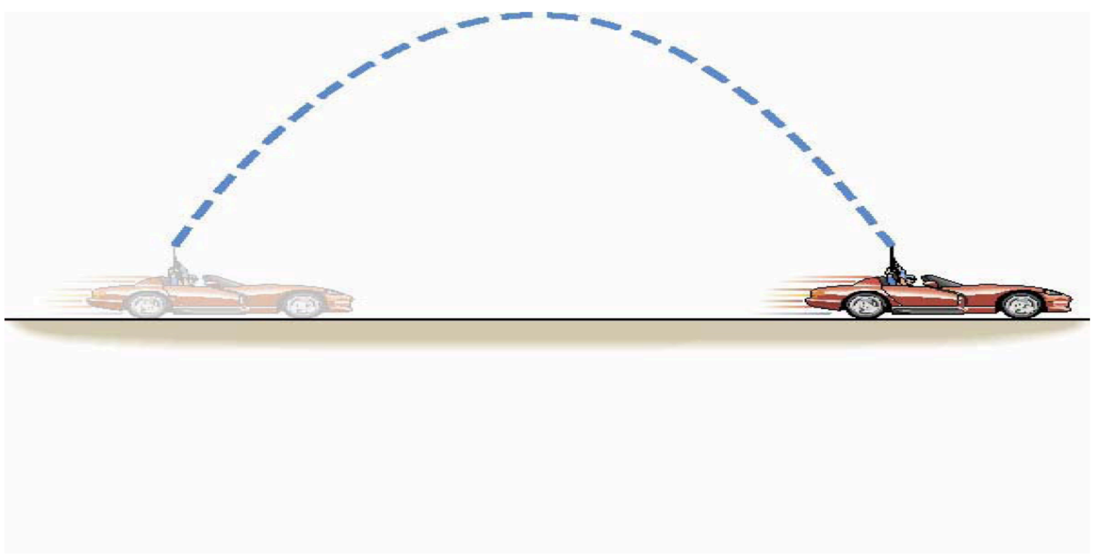
The x-component equations	**The x-component equations**
$v_x = v_{ox}$	$v_y = v_{oy} - g\,t$
$x = v_{ox}\,t$	$y = h + v_{oy}\,t - \frac{1}{2}\,g\,t^2$

In general, $h \neq 0$!

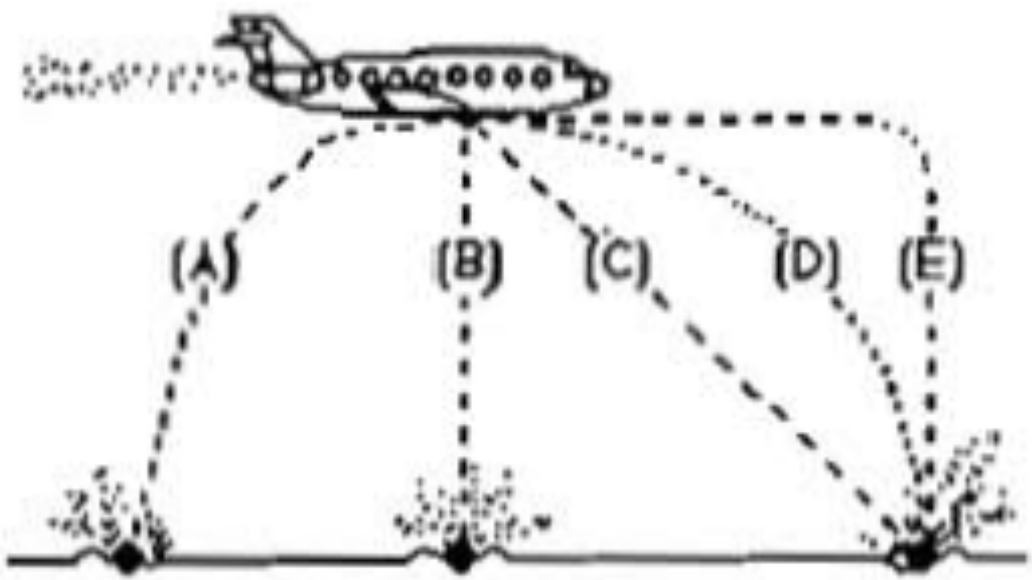
I Shot a Bullet into the Air...

Suppose you are driving a convertible with the top down. The car is moving to the right at constant velocity. You point a rifle straight up into the air and fire it. In the absence of air resistance, where would the bullet land –

(1) behind you, (2) ahead of you, or (3) in the barrel of the rifle?



A bowling ball accidently falls out of the cargo bay of an airliner as it flies along in a horizontal direction. As seen from the ground, which path would the ball most closely follow after falling the airplane?

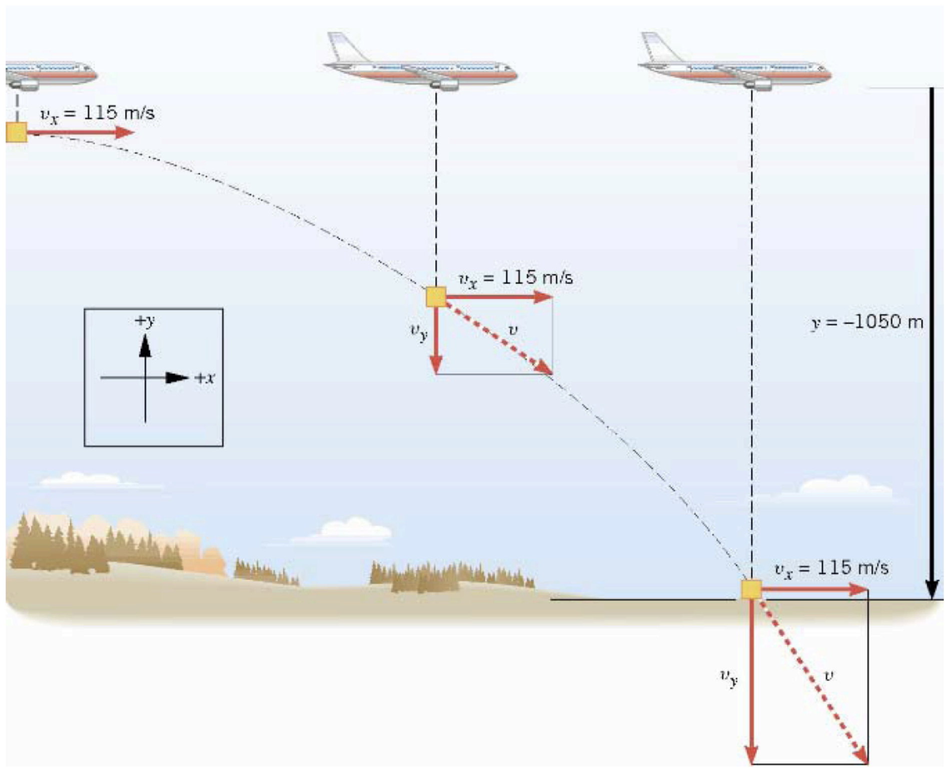


Two steel balls, one of which weighs twice as much as the other, roll off of a horizontal table with the same speeds. In this situation:

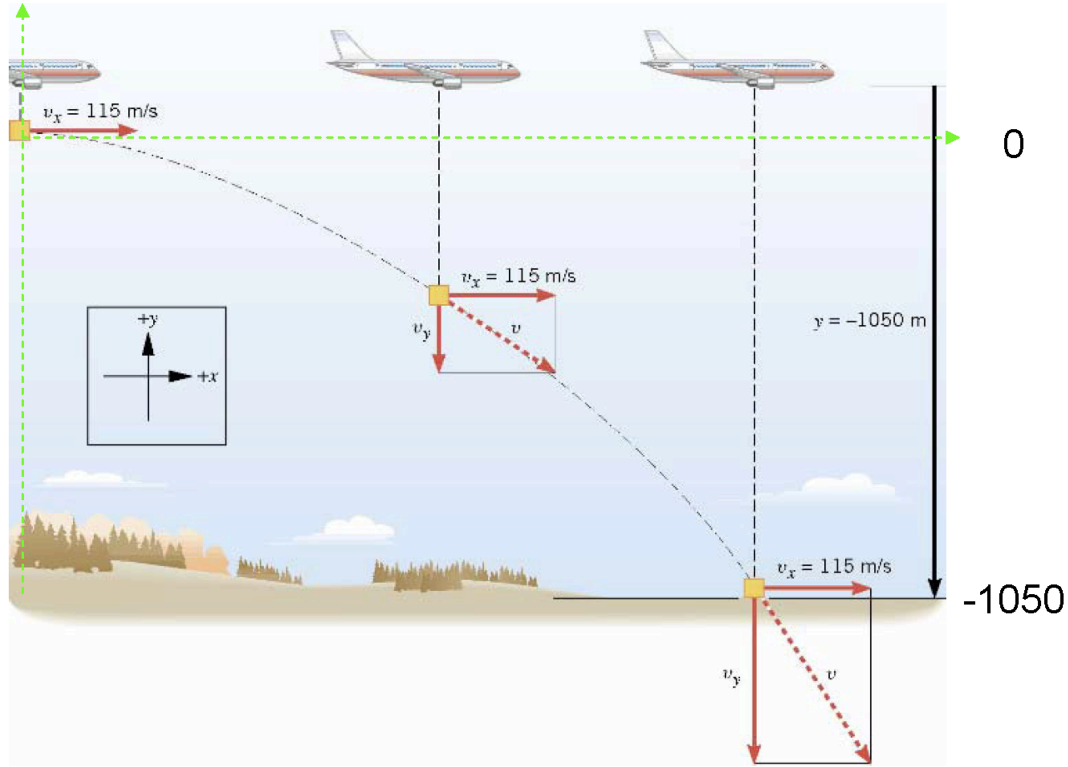
(A) both balls Impact the floor at approximately the same horizontal distance from the base of the table.
(B) the heavier ball impacts the floor at about hall the horizontal distance from the base of the table than does the lighter.
(C) the lighter ball impacts the floor at about half the horizontal distance from the base of the table than does the heavier.
(D) the heavier ball hits considerably closer to the base of the table than the lighter, but not necessarily half the horizontal distance.
(E) the lighter ball hits considerably closer to the base of the table than the heavier, but not necessarily half the horizontal distance.

Example A Falling Care Package

The airplane is moving horizontally with a constant velocity of +115 m/s at an altitude of 1050m. Determine the time required for the care package to hit the ground.



Projectile Motion



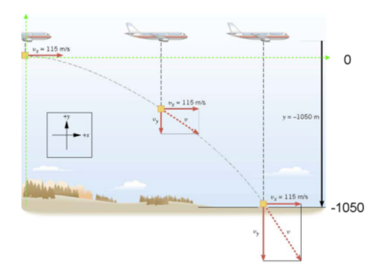
y	a_y	v_y	v_{oy}	t
-1050 m	-9.80 m/s^2		0 m/s	?

Projectile Motion

y	a_y	v_y	v_{oy}	t
-1050 m	-9.80 m/s^2		0 m/s	?

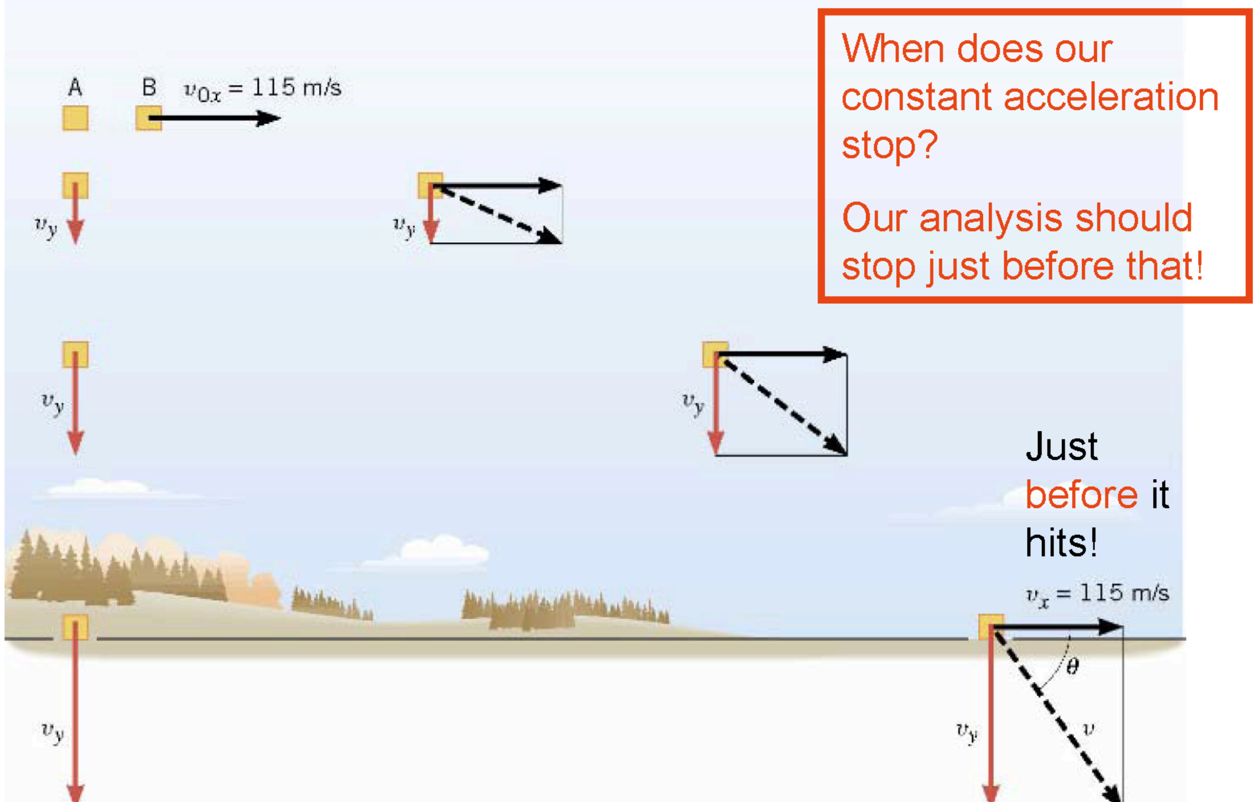
$$y = v_{oy}t + \tfrac{1}{2}a_y t^2 \quad \Rightarrow \quad y = \tfrac{1}{2}a_y t^2$$

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-1050\ \mathrm{m})}{-9.80\ \mathrm{m/s^2}}} = 14.6\ \mathrm{s}$$



Example The Velocity of the Care Package

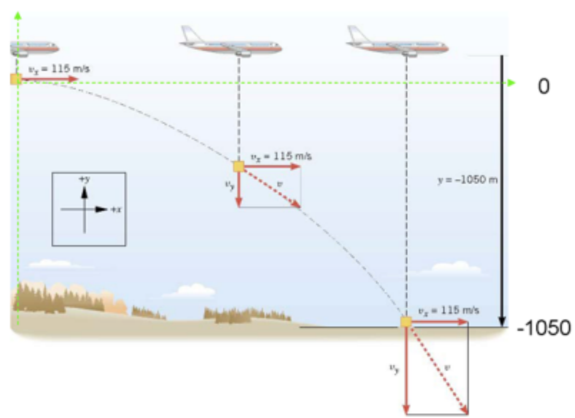
What are the magnitude and direction of the final velocity of the care package?



y	a_y	v_y	v_{oy}	t
-1050 m	-9.80 m/s^2	?	0 m/s	14.6 s

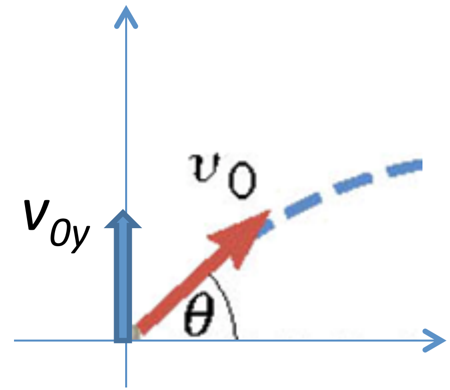
$$v_y = v_{oy} + a_y t = 0 + \left(-9.80\,\mathrm{m/s^2}\right)\left(14.6\,\mathrm{s}\right)$$

$$= -143\,\mathrm{m/s}$$



Maximum Height

The maximum height, y_{max}, is determined solely by the initial velocity in the y direction and the acceleration due to gravity. It's not affected by what's happening in the x direction.



It can be found from the equation:

$v_y{}^2 = v_{oy}{}^2 - 2\,g\,(y_{max} - y_o)$

We use that:

1) $v_{oy} = v_o \sin(\theta)$

2) when the projectile is at the maximum height, $v_y = 0$.

3) $y_o = 0$ (the natural choie)

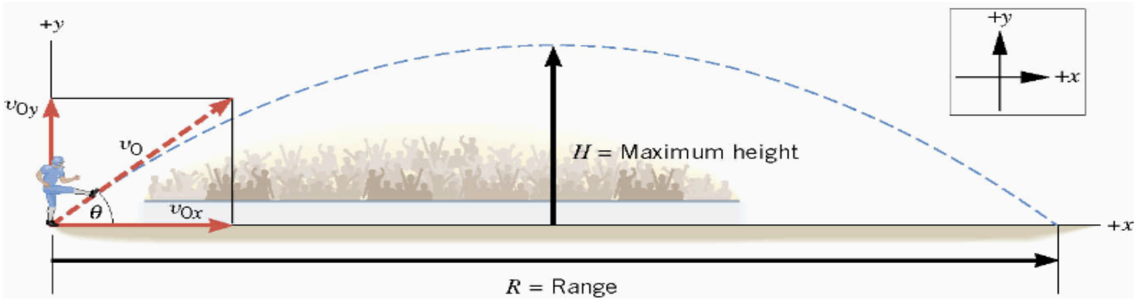
Solving the equation for y_{max} gives:

$$y_{max} = \frac{v_o{}^2 \sin^2(\theta)}{2\,g}$$

where $g = 9.8\ m/s^2$

Y_{max} is measured from the initial location, NOT from the ground!

A special case



Let us consider a projectile that is launched from ground level at a particular angle. The most convenient choice for the system of reference is this: $\mathbf{x_o} = 0$ $\mathbf{y_o} = 0$ $\mathbf{y} = 0$

The only acceleration is the acceleration due to gravity.

Determine general equations that give:

- the maximum height
- the time of flight
- the range (maximum horizontal distance)

The equations:

1) $v_x = v_{ox}$	3) $v_y = v_{oy} - g\,t$
2) $x = v_{ox}\,t$	4) $y = v_{oy}\,t - \frac{1}{2}\,g\,t^2$

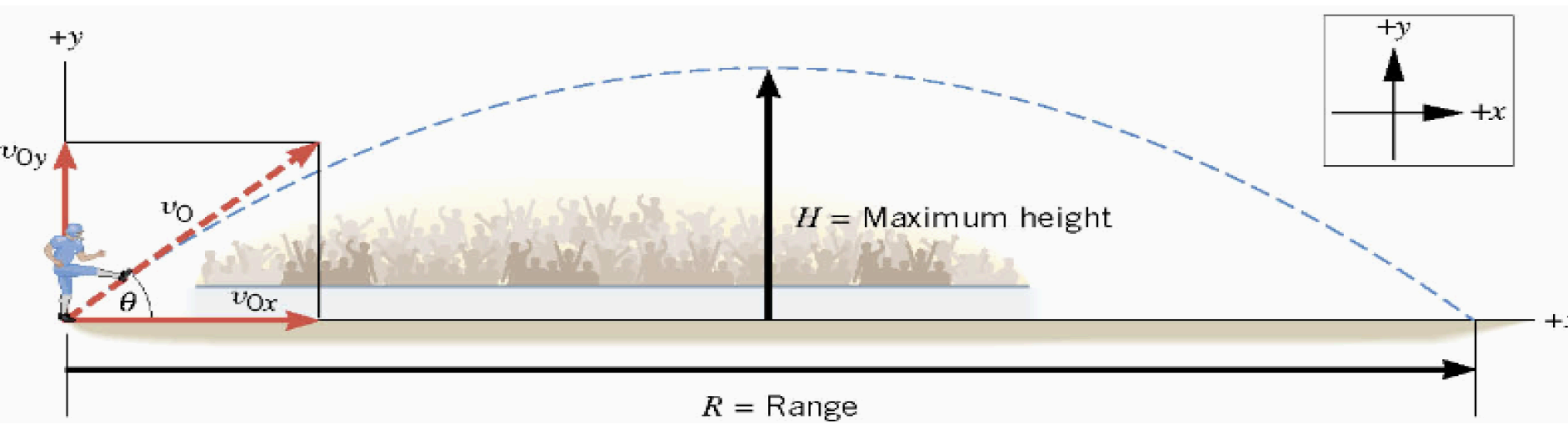
The Height of a Kickoff

A special case: h = 0

A placekicker kicks a football at an angle of 40.0 degrees and the initial speed of the ball is 22 m/s. Ignoring air resistance, analyze various points in the motion.

$$v_{ox} = v_o \cos\theta = (22\text{ m/s})\cos(40°) = 16.85\text{ m/s}$$

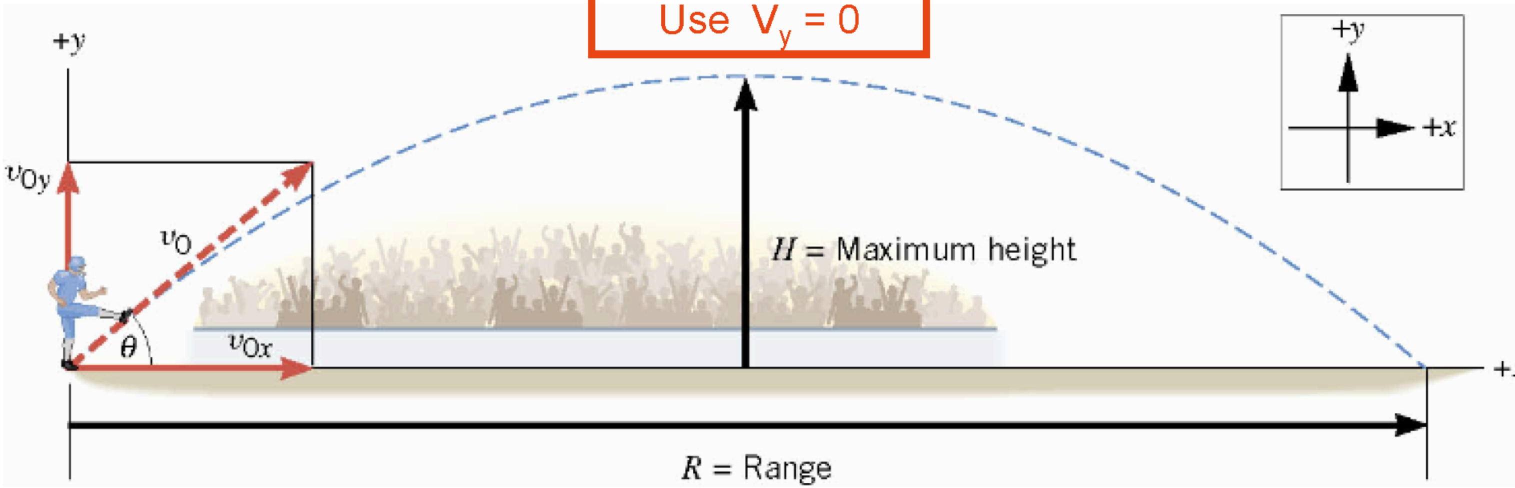
$$V_{oy} = v_y \sin\theta = (22\text{ m/s})\sin(40°) = 14.14\text{ m/s}$$



Projectile Motion

Find time to top.

Use $V_y = 0$



y	a_y	v_y	v_{oy}	t
?	-9.80 m/s^2	0	14 m/s	?

$0 = v_y = v_{oy} + a_y t = (14\text{ m/s}) + (-9.8\text{ m/s}^2)\,t$

$(9.8\text{ m/s}^2)\,t = (14\text{ m/s})$

$t = (14\text{ m/s}) / (9.8\text{ m/s}^2) = 1.428\text{ s} = 1.43\text{ s}$ time to top

Projectile Motion

Find time to hit ground (hang time).
Use y = 0

y	a_y	v_y	v_{oy}	t
0	-9.80 m/s^2		14 m/s	?

$$y = v_{oy}t + \tfrac{1}{2}a_y t^2$$

$$0 = (14\,\mathrm{m/s})t + \tfrac{1}{2}(-9.80\,\mathrm{m/s^2})t^2$$

"Cancel" t

$$0 = 2(14\,\mathrm{m/s}) + (-9.80\,\mathrm{m/s^2})t$$

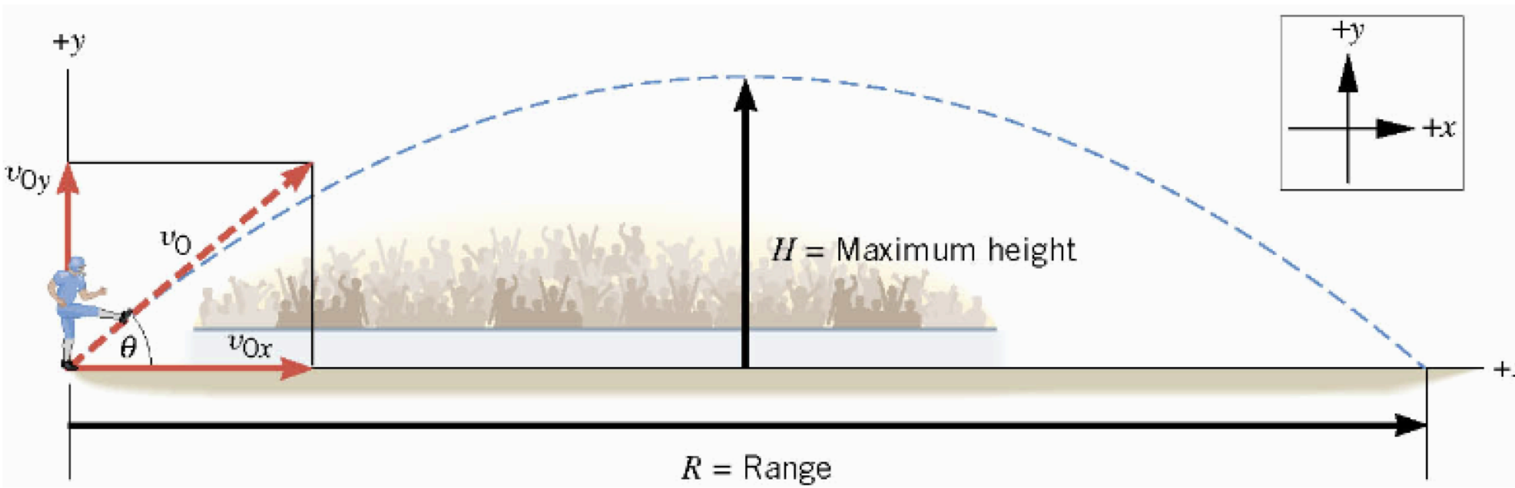
t = 0 is also correct mathematically, but doesn't answer the physical question.

$$t = 2.9\,\mathrm{s}$$

= twice the time to top!

Example The Range of a Kickoff

Calculate the range R of the projectile.



$$x = v_{ox}t + \tfrac{1}{2}a_x t^2 = v_{ox}t$$

$$= (17\,\text{m/s})(2.9\,\text{s}) = +49\,\text{m}$$

Monkey and Hunter

The list of concepts, definitions, laws and relations to memorize when taking PY 105 course

For each physical quantity a students must be able to answer the following questions:

1. what is its name;

2. what is a usual symbol for the quantity;

3. what is its unit;

4. how to measure the quantity;

5. to what other physical quantities is this quantity related;

6. how is this physical quantity algebraically related to other physical quantities?

Relative motion

Relative motion, velocity addition, “crossing a river”, 2-D motion, projectile motion, properties of projectile motion, the range, the maximum height, the time of the fight.

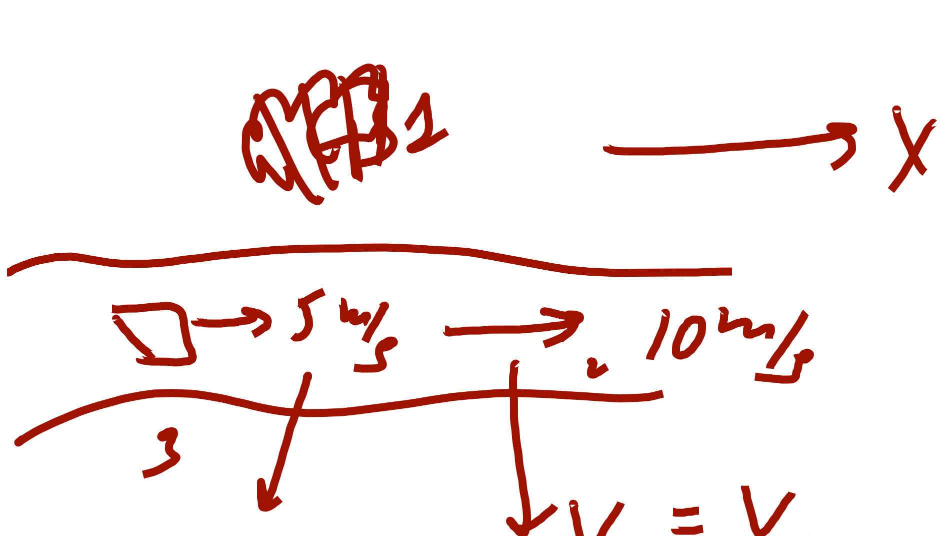
2

1

3

$$\vec{V}_{31} = \vec{V}_{32} + \vec{V}_{21}$$

1)

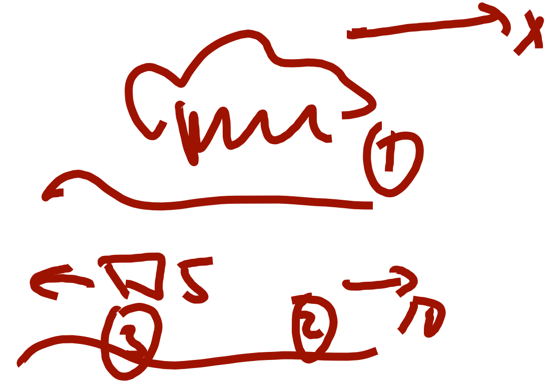


$V_{RB} = V_{21}$

$V_{obs\,R} = V_{32}$

$\vec{V}_{31} = \vec{V}_{32} + \vec{V}_{21}$

$V_? = 5 + 10 = 15$

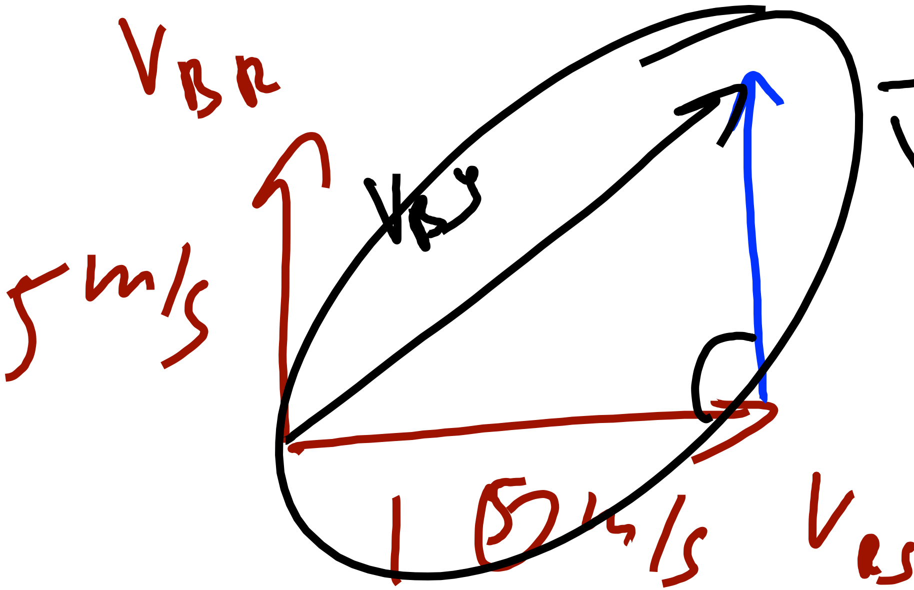


(1)

(2)

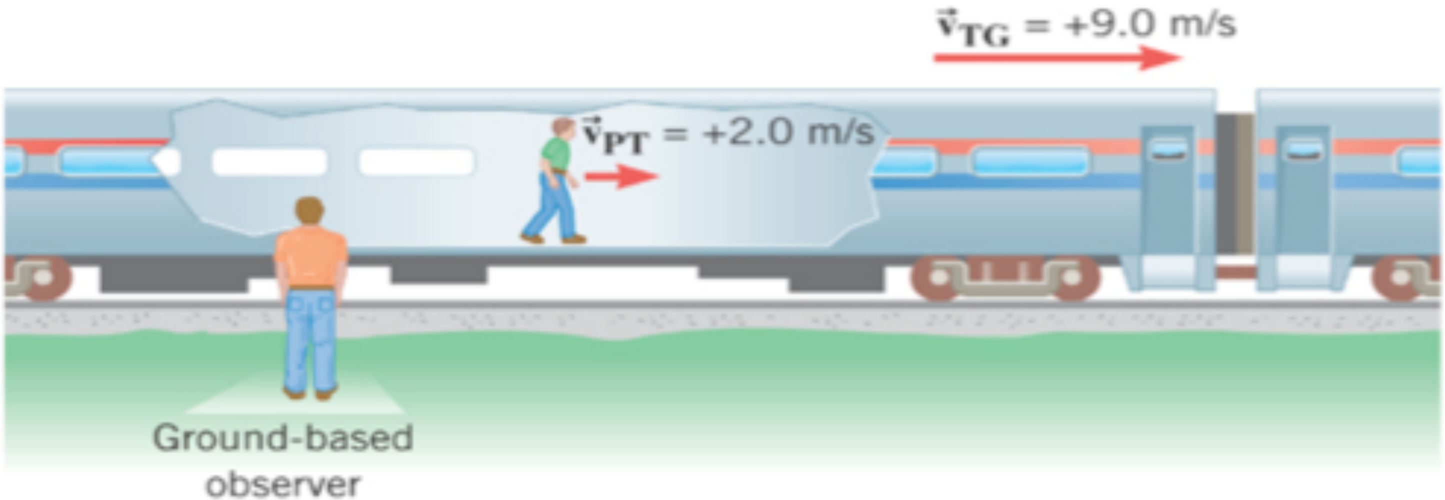
$\vec{V}_{31} = \vec{V}_{32} + \vec{V}_{21}$

$V_{31} = -5 + 10 = 5$



$$\vec{V}_{BJ} = \vec{V}_{BR} + \vec{V}_{RS}$$

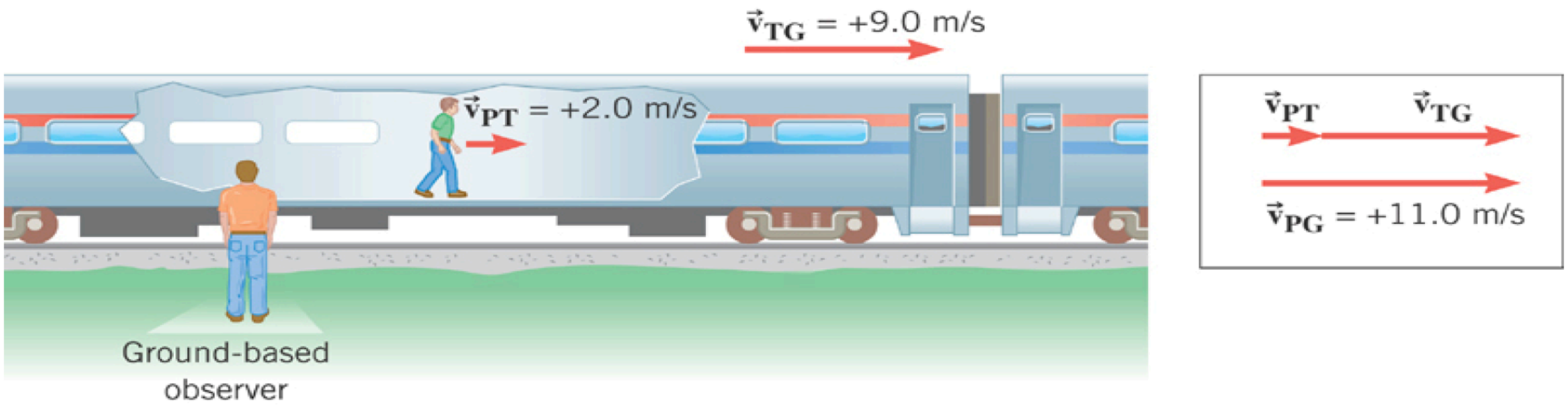
$$= \sqrt{10^2 + 5^2}$$



The speed of the guy in the train relative to the ground-based observer is:

1. 2 m/s 2. 9 m/s 3. 11 m/s 4. 7 m/s 5. ???

Relative Velocity



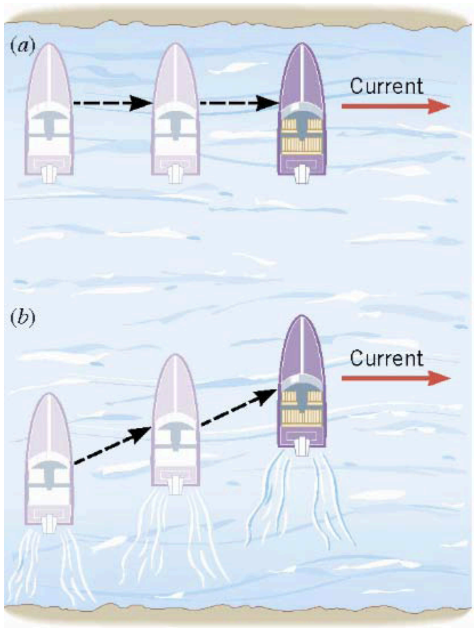
$$\vec{v}_{PG} = \vec{v}_{PT} + \vec{v}_{TG}$$

Example: Crossing a River

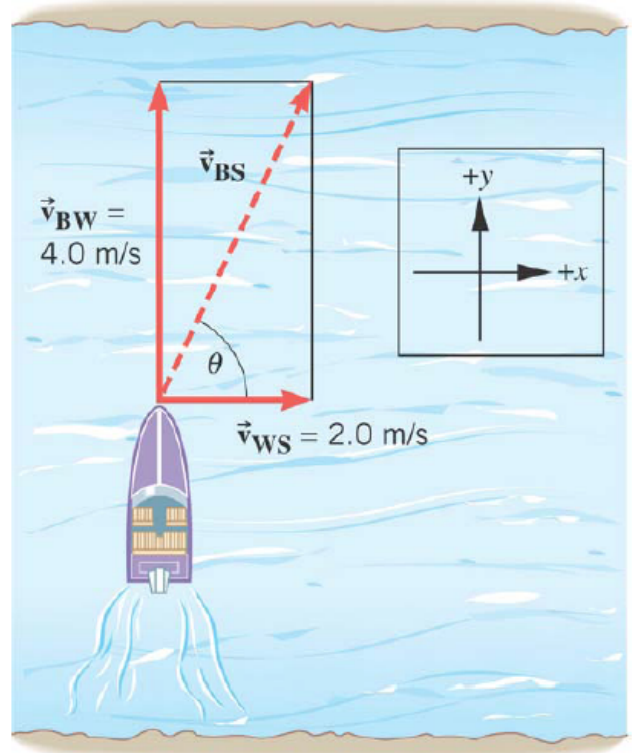
The engine of a boat drives it across a river that is 1800m wide. The velocity of the boat relative to the water is 4.0m/s directed perpendicular to the current. The velocity of the water relative to the shore is 2.0m/s.

(a) What is the velocity of the boat relative to the shore?

(b) How long does it take for the boat to cross the river?



$$\vec{\mathbf{v}}_{\mathbf{BS}} = \vec{\mathbf{v}}_{\mathbf{BW}} + \vec{\mathbf{v}}_{\mathbf{WS}}$$



$$\theta = \tan^{-1}\left(\frac{4.0}{2.0}\right) = 63^{\circ}$$

$$v_{BS} = \sqrt{v_{BW}^2 + v_{WS}^2} = \sqrt{(4.0\,\text{m/s})^2 + (2.0\,\text{m/s})^2}$$

$$= 4.5\,\text{m/s}$$

$$t = \frac{1800\ \text{m}}{4.0\ \text{m/s}} = 450\ \text{s}$$